Computer Algebra
Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

1-Algebraic-functions/1.2-Trinomial-products/1.2.1-Quadratic/37-1.2.1.6-g+h-x-^m-a+b-x+c-x^2-^p-d+e-x+f-x^2-^q

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [143]. This is test number [37].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Mathematica	100.00 (143)	0.00 (0)
Maple	97.90 (140)	2.10 (3)
Rubi	96.50 (138)	3.50 (5)
Fricas	58.04 (83)	41.96 (60)
Giac	37.06 (53)	62.94 (90)
Mupad	13.29 (19)	86.71 (124)
Maxima	10.49 (15)	89.51 (128)
Sympy	8.39 (12)	91.61 (131)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

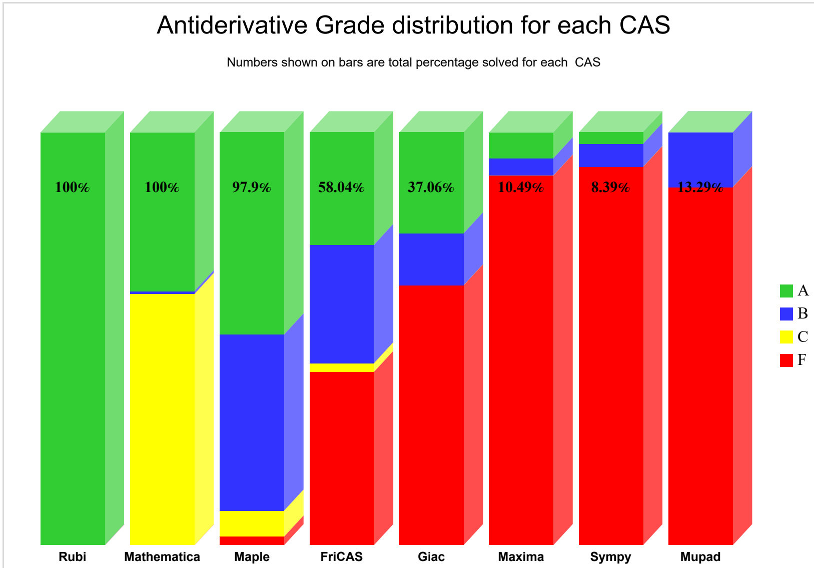
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

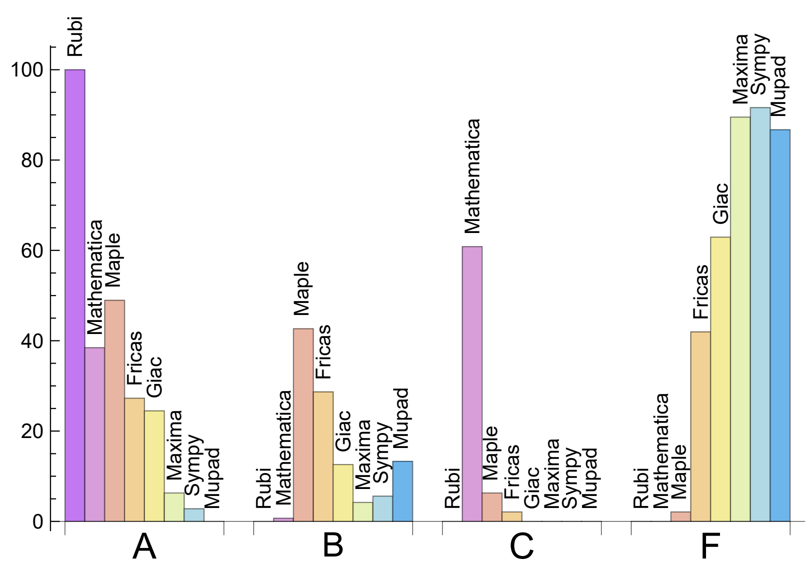
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.503	0.000	0.000	3.497
Maple	48.951	42.657	6.294	2.098
Mathematica	38.462	0.699	60.839	0.000
Fricas	27.273	28.671	2.098	41.958
Giac	24.476	12.587	0.000	62.937
Maxima	6.294	4.196	0.000	89.510
Sympy	2.797	5.594	0.000	91.608
Mupad	0.000	13.287	0.000	86.713

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Maple	3	66.67	33.33	0.00
Rubi	5	100.00	0.00	0.00
Fricas	60	1.67	98.33	0.00
Giac	90	8.89	15.56	75.56
Mupad	124	0.00	100.00	0.00
Maxima	128	52.34	0.00	47.66
Sympy	131	82.44	17.56	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.28
Mathematica	0.81
Rubi	1.10
Maple	1.74
Giac	8.89
Sympy	9.53
Mupad	17.25
Fricas	23.82

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	310.75	0.98	299.50	1.00
Mathematica	361.78	1.04	264.00	0.89
Maxima	388.27	2.32	220.00	1.11
Sympy	738.42	3.40	214.00	2.84
Giac	967.21	5.89	165.00	1.34
Fricas	3703.05	11.25	435.00	2.34
Mupad	7822.16	9.68	187.00	1.25
Maple	49720.61	103.14	533.00	1.56

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

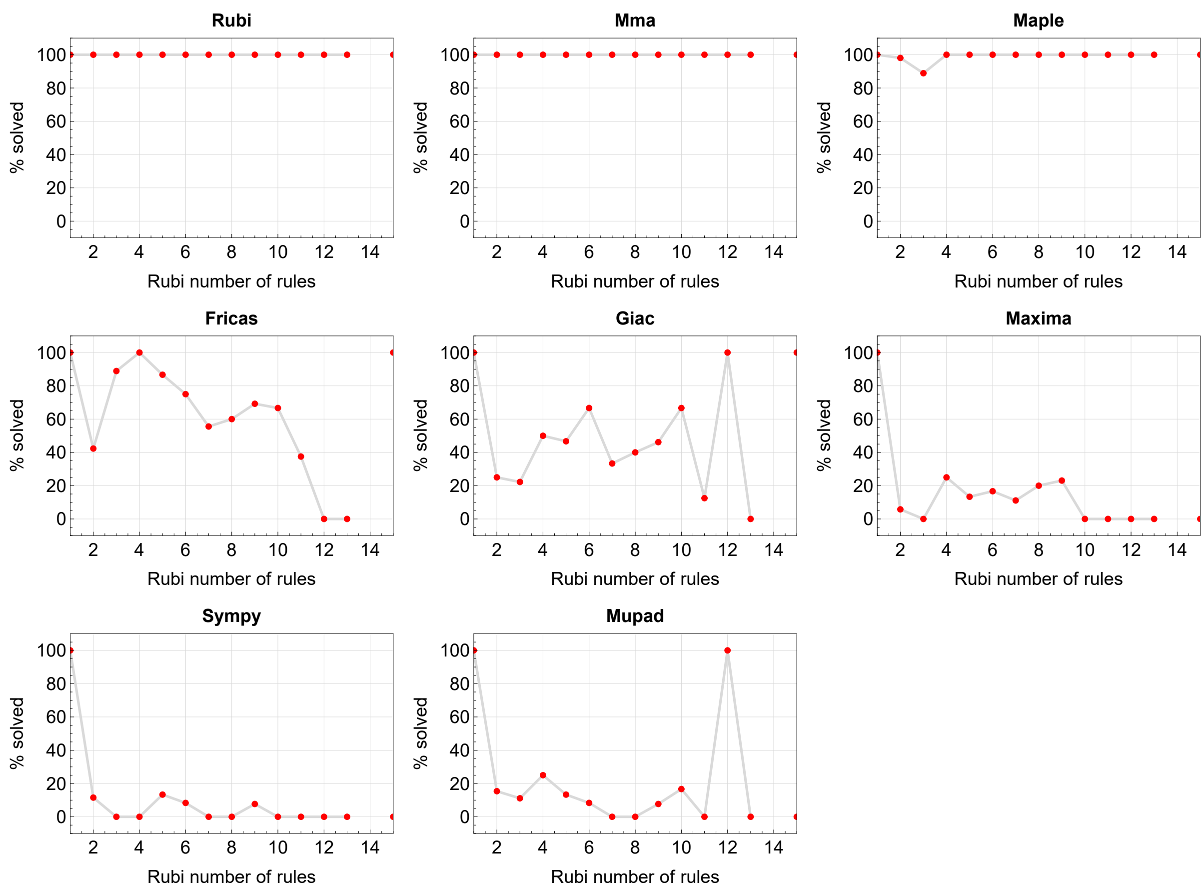


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

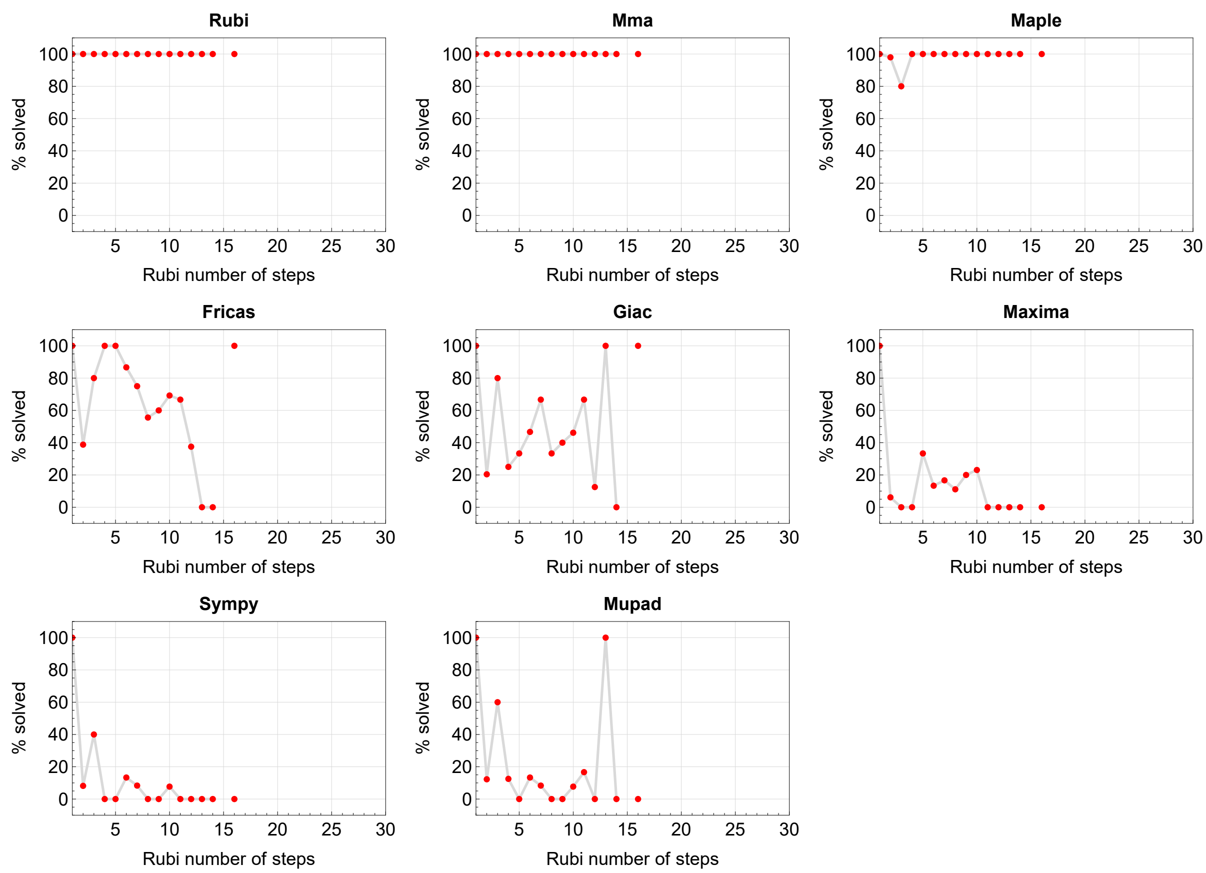


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

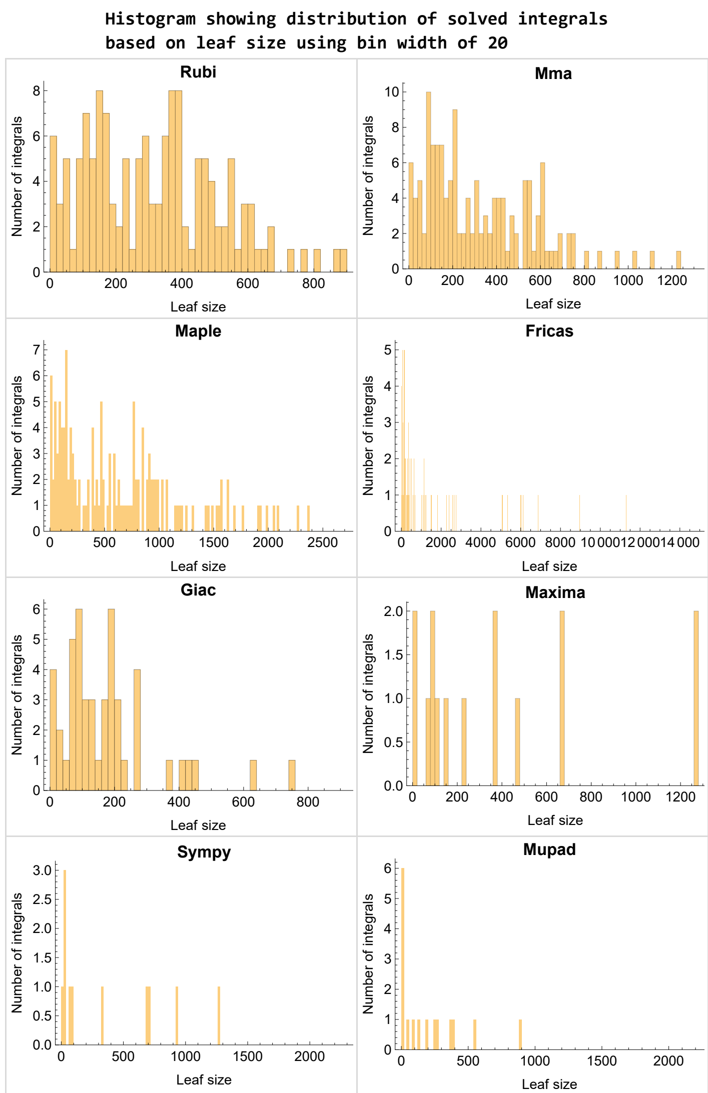


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

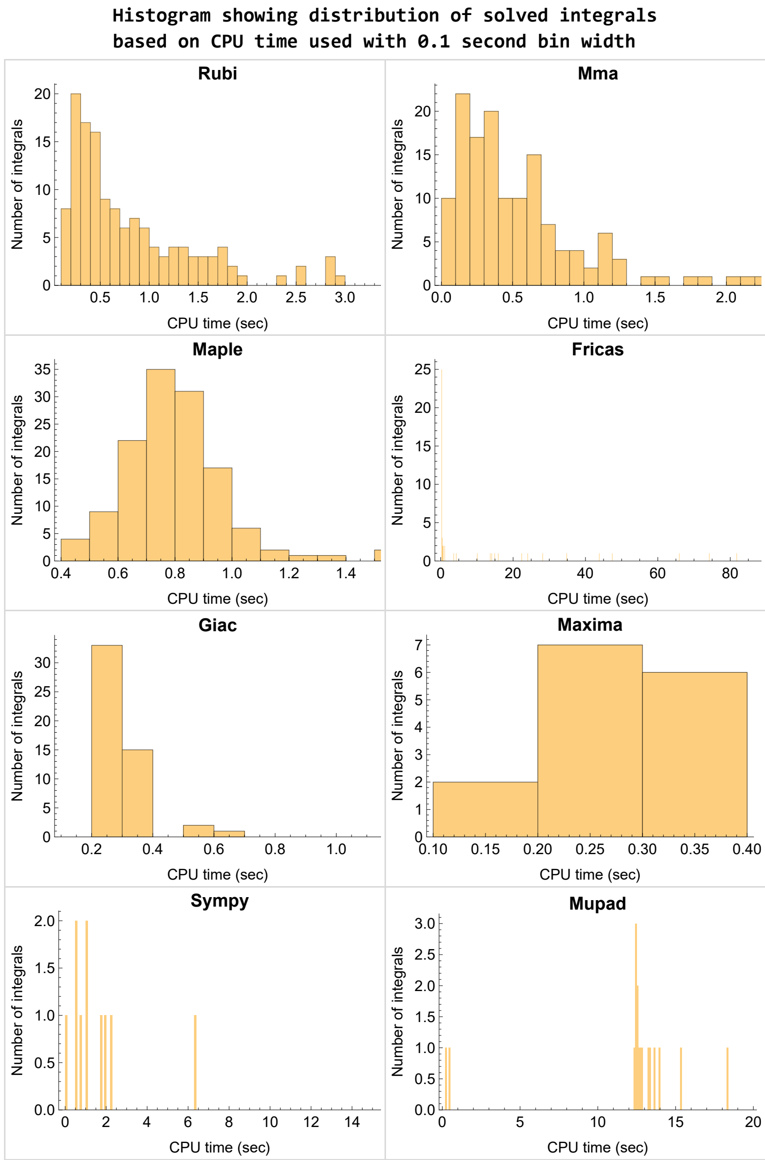


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

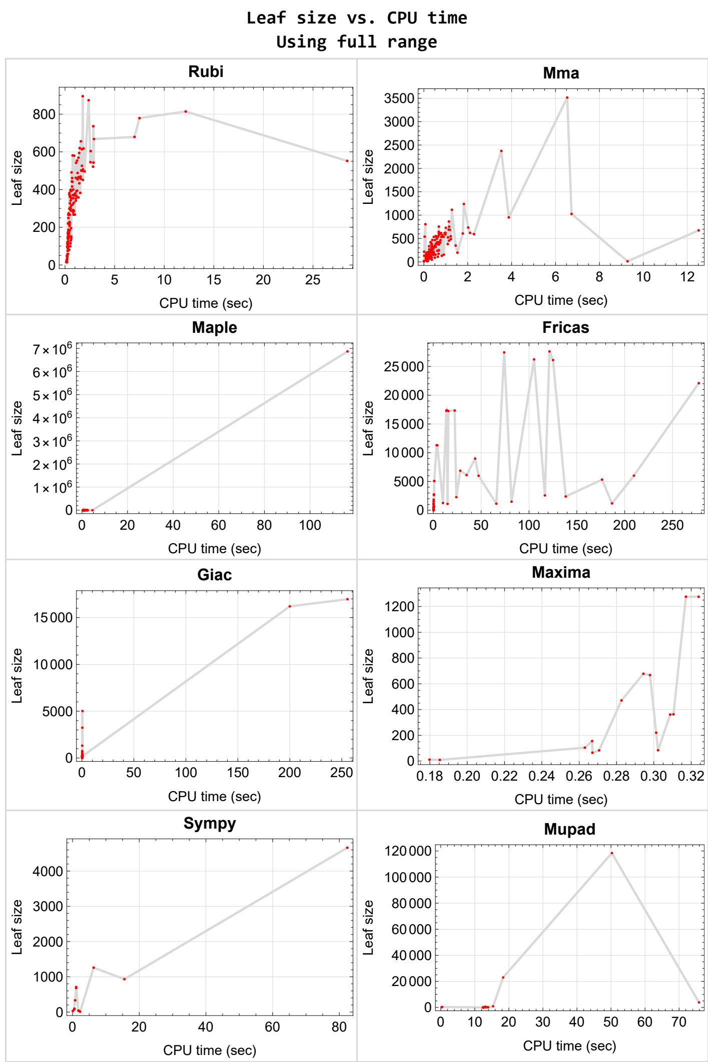


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{19, 38, 128, 129, 130\}$

Mathematica {}

Maple $\{11, 12, 27, 30, 40, 41, 42, 43, 49, 125\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

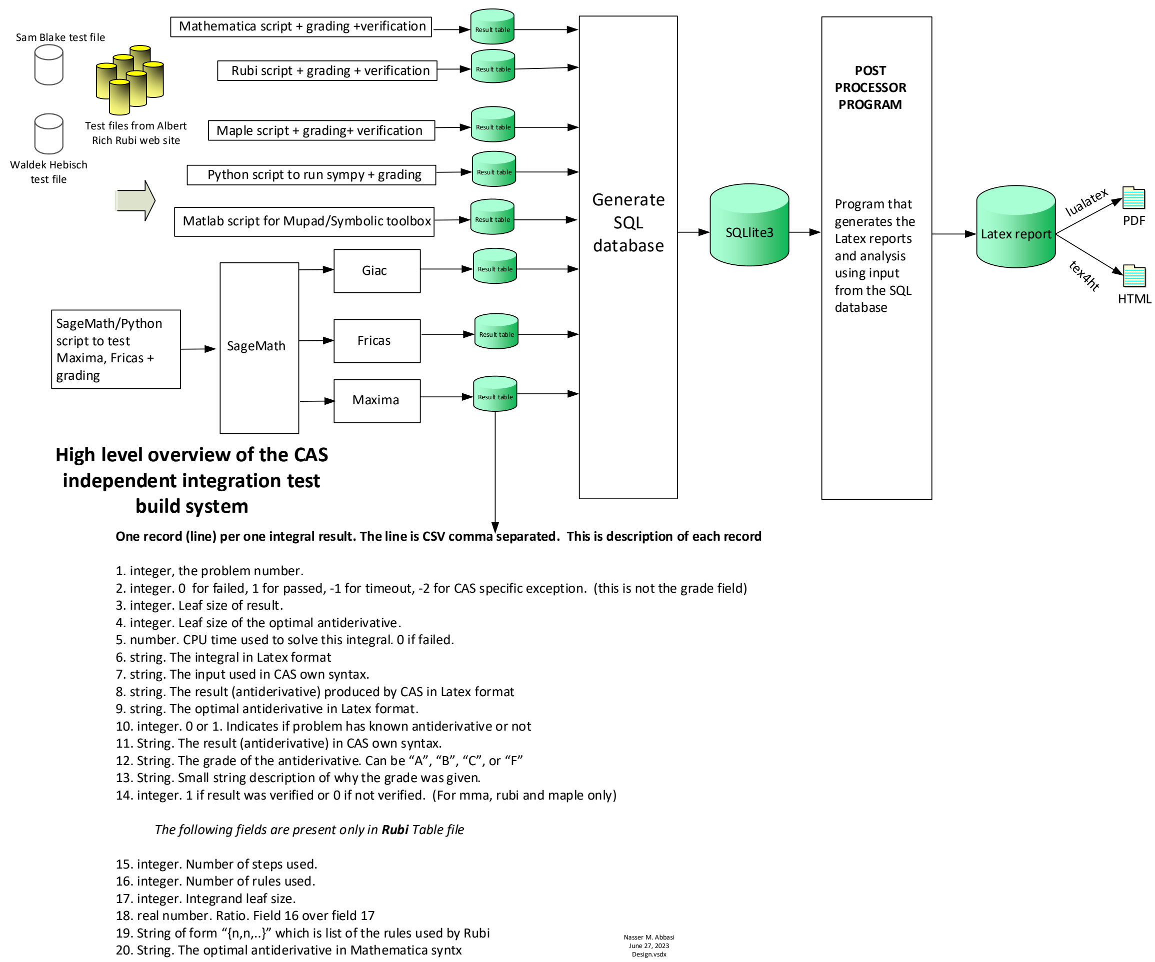
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { }

C grade { }

F normal fail { 16, 20, 109, 124, 125 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.2 Mma

A grade { 1, 2, 3, 4, 5, 9, 10, 13, 14, 15, 16, 17, 18, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143 }

B grade { 24 }

C grade { 6, 7, 8, 11, 12, 19, 20, 21, 22, 23, 25, 26, 27, 34, 35, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

2.1.3 Maple

A grade { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 22, 25, 26, 28, 29, 31, 32, 33, 34, 36, 37, 38, 39, 44, 45, 46, 47, 48, 50, 51, 57, 58, 62, 63, 70, 77, 78, 83, 84, 85, 89, 90, 91, 92, 94, 95, 96, 99, 100, 101, 109, 113, 114, 119, 120, 126, 127, 128, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141 }

B grade { 5, 6, 7, 8, 9, 12, 19, 20, 21, 23, 24, 35, 52, 53, 54, 55, 56, 59, 60, 61, 64, 65, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 79, 80, 81, 82, 86, 87, 88, 93, 97, 98, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 115, 116, 117, 118, 121, 122, 123, 124, 125 }

C grade { 11, 27, 30, 40, 41, 42, 43, 49, 129 }

F normal fail { 142, 143 }

F(-1) timedout fail { 16 }

F(-2) exception fail { }

2.1.4 Fricas

A grade { 1, 2, 3, 10, 12, 13, 14, 33, 36, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 91, 92, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 137, 138, 139, 140, 141 }

B grade { 7, 17, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 37, 54, 55, 56, 66, 67, 71, 72, 73, 74, 79, 80, 81, 82, 83, 97, 98, 99, 100, 103, 104, 105, 106, 116, 117, 135 }

C grade { 11, 34, 93 }

F normal fail { 39 }

F(-1) timedout fail { 4, 5, 6, 8, 9, 15, 16, 19, 20, 35, 52, 53, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 75, 76, 77, 78, 84, 85, 86, 87, 88, 89, 90, 94, 95, 96, 101, 102, 107, 108, 109, 110, 111, 112, 113, 114, 115, 118, 119, 120, 121, 122, 123, 124, 125, 142, 143 }

F(-2) exception fail { }

2.1.5 Maxima

A grade { 1, 2, 3, 10, 92, 133, 134, 138, 139 }

B grade { 25, 26, 27, 28, 29, 30 }

C grade { }

F normal fail { 11, 12, 22, 24, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 55, 56, 57, 61, 62, 63, 68, 69, 70, 75, 76, 81, 82, 83, 88, 89, 90, 93, 99, 100, 101, 107, 108, 112, 113, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

F(-1) timedout fail { }

F(-2) exception fail { 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 23, 39, 52, 53, 54, 58, 59, 60, 64, 65, 66, 67, 71, 72, 73, 74, 77, 78, 79, 80, 84, 85, 86, 87, 91, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 109, 110, 111, 114, 115, 116, 117, 121, 122, 123, 124 }

2.1.6 Giac

A grade { 1, 2, 3, 4, 10, 13, 14, 15, 17, 18, 28, 29, 30, 36, 40, 41, 42, 44, 45, 46, 47, 48, 50, 92, 126, 127, 129, 131, 133, 134, 137, 138, 139, 140, 141 }

B grade { 5, 11, 16, 25, 26, 31, 32, 33, 34, 35, 37, 38, 51, 128, 130, 132, 135, 136 }

C grade { }

F normal fail { 12, 39, 81, 99, 107, 125, 142, 143 }

F(-1) timedout fail { 24, 27, 66, 67, 68, 70, 75, 113, 116, 117, 118, 119, 120, 121 }

F(-2) exception fail { 6, 7, 8, 9, 19, 20, 21, 22, 23, 43, 49, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 71, 72, 73, 74, 76, 77, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 95, 96, 97, 98, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 114, 115, 122, 123, 124 }

2.1.7 Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 13, 14, 16, 17, 18, 31, 33, 36, 133, 134, 138, 139, 140, 141 }

C grade { }

F normal fail { }

F(-1) timedout fail { 6, 7, 8, 9, 10, 11, 12, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 142, 143 }

F(-2) exception fail { }

2.1.8 Sympy

A grade { 33, 133, 134, 139 }

B grade { 1, 2, 13, 14, 17, 18, 31, 138 }

C grade { }

F normal fail { 6, 7, 10, 11, 12, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 107, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118, 119, 120, 125, 126, 127, 128, 129, 130, 131, 132, 135, 136, 137, 140, 141, 142, 143 }

F(-1) timedout fail { 3, 4, 5, 8, 9, 15, 16, 20, 35, 58, 62, 63, 84, 102, 103, 104, 105, 106, 113, 121, 122, 123, 124 }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	86	84	84	200	333	87	97
N.S.	1	1.00	0.91	0.89	0.89	2.13	3.54	0.93	1.03
time (sec)	N/A	0.266	0.093	0.645	0.302	0.304	0.754	0.261	12.889

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	204	235	220	500	933	263	253
N.S.	1	1.00	0.89	1.03	0.96	2.19	4.09	1.15	1.11
time (sec)	N/A	0.442	0.152	0.734	0.301	0.288	15.556	0.258	0.265

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	441	441	422	592	471	1014	0	623	552
N.S.	1	1.00	0.96	1.34	1.07	2.30	0.00	1.41	1.25
time (sec)	N/A	0.764	0.303	0.754	0.283	0.306	0.000	0.261	13.203

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	224	212	239	0	0	0	266	3888
N.S.	1	0.82	0.77	0.87	0.00	0.00	0.00	0.97	14.19
time (sec)	N/A	0.470	0.219	0.950	0.000	0.000	0.000	0.259	75.857

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	596	594	523	1254	0	0	0	1313	23006
N.S.	1	1.00	0.88	2.10	0.00	0.00	0.00	2.20	38.60
time (sec)	N/A	1.425	1.100	1.549	0.000	0.000	0.000	0.276	18.332

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	331	371	619	541	0	0	0	0	0
N.S.	1	1.12	1.87	1.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.778	0.947	0.819	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	219	389	0	6113	0	0	0
N.S.	1	1.00	0.88	1.56	0.00	24.55	0.00	0.00	0.00
time (sec)	N/A	0.334	0.385	0.786	0.000	34.857	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	381	443	753	934	0	0	0	0	0
N.S.	1	1.16	1.98	2.45	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.695	1.151	0.786	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	797	895	674	1768	0	0	0	0	0
N.S.	1	1.12	0.85	2.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.763	12.518	0.753	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	37	46	65	46	0	48	0
N.S.	1	1.00	0.79	0.98	1.38	0.98	0.00	1.02	0.00
time (sec)	N/A	0.203	0.112	0.639	0.267	0.268	0.000	0.279	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	117	125	106	246	0	93	0	457	0
N.S.	1	1.07	0.91	2.10	0.00	0.79	0.00	3.91	0.00
time (sec)	N/A	0.323	0.127	2.078	0.000	0.414	0.000	0.340	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	484	552	210	6870946	0	307	0	0	0
N.S.	1	1.14	0.43	14196.17	0.00	0.63	0.00	0.00	0.00
time (sec)	N/A	28.426	0.363	116.444	0.000	0.368	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	183	175	190	0	583	1260	189	273
N.S.	1	0.99	0.95	1.03	0.00	3.17	6.85	1.03	1.48
time (sec)	N/A	0.469	0.104	0.846	0.000	0.312	6.326	0.264	13.661

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	542	542	535	800	0	1837	4663	743	893
N.S.	1	1.00	0.99	1.48	0.00	3.39	8.60	1.37	1.65
time (sec)	N/A	1.109	0.453	0.925	0.000	0.418	82.394	0.275	15.368

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	406	314	267	384	0	0	0	405	0
N.S.	1	0.77	0.66	0.95	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.592	0.242	1.569	0.000	0.000	0.000	0.275	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	A	F(-1)	F(-2)	F(-1)	F(-1)	B	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	1075	0	952	0	0	0	0	3236	118429
N.S.	1	0.00	0.89	0.00	0.00	0.00	0.00	3.01	110.17
time (sec)	N/A	0.000	3.869	180.000	0.000	0.000	0.000	0.332	50.268

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	131	141	0	1150	709	219	395
N.S.	1	0.99	0.94	1.01	0.00	8.21	5.06	1.56	2.82
time (sec)	N/A	0.307	0.087	0.902	0.000	0.347	1.085	0.275	0.422

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	139	131	141	0	1130	680	207	375
N.S.	1	0.99	0.94	1.01	0.00	8.07	4.86	1.48	2.68
time (sec)	N/A	0.285	0.018	0.950	0.000	0.341	1.083	0.269	12.763

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	620	1115	1176	0	0	0	0	0
N.S.	1	1.00	1.81	1.91	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.478	1.276	0.994	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	1092	0	3516	2278	0	0	0	0	0
N.S.	1	0.00	3.22	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.532	1.081	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	416	278	805	0	22103	0	0	0
N.S.	1	1.00	0.67	1.94	0.00	53.13	0.00	0.00	0.00
time (sec)	N/A	0.702	0.435	1.011	0.000	277.457	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	780	874	218	425	0	6861	0	0	0
N.S.	1	1.12	0.28	0.54	0.00	8.80	0.00	0.00	0.00
time (sec)	N/A	2.361	0.373	0.773	0.000	28.212	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	302	302	195	639	0	8977	0	0	0
N.S.	1	1.00	0.65	2.12	0.00	29.73	0.00	0.00	0.00
time (sec)	N/A	0.480	0.378	0.773	0.000	43.725	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	B	F	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	349	337	0	1515	0	0	0
N.S.	1	1.00	3.46	3.34	0.00	15.00	0.00	0.00	0.00
time (sec)	N/A	0.273	1.447	0.575	0.000	0.375	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	153	149	176	361	309	0	16965	0
N.S.	1	1.10	1.07	1.27	2.60	2.22	0.00	122.05	0.00
time (sec)	N/A	0.361	0.188	2.369	0.309	0.294	0.000	255.510	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	177	137	195	678	361	0	16200	0
N.S.	1	1.07	0.83	1.17	4.08	2.17	0.00	97.59	0.00
time (sec)	N/A	0.307	0.330	2.395	0.294	0.308	0.000	199.977	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	C	B	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	193	207	183	484	1276	439	0	0	0
N.S.	1	1.07	0.95	2.51	6.61	2.27	0.00	0.00	0.00
time (sec)	N/A	0.442	0.448	2.271	0.324	0.307	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	161	109	186	363	245	0	93	0
N.S.	1	1.07	0.72	1.23	2.40	1.62	0.00	0.62	0.00
time (sec)	N/A	0.327	0.636	2.316	0.311	0.308	0.000	0.314	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	187	130	209	668	365	0	112	0
N.S.	1	1.07	0.75	1.20	3.84	2.10	0.00	0.64	0.00
time (sec)	N/A	0.327	0.817	2.460	0.298	0.323	0.000	0.314	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	197	213	154	471	1276	435	0	121	0
N.S.	1	1.08	0.78	2.39	6.48	2.21	0.00	0.61	0.00
time (sec)	N/A	0.441	0.908	2.517	0.317	0.299	0.000	0.319	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	0	49	32	31	13
N.S.	1	1.00	1.00	0.93	0.00	3.27	2.13	2.07	0.87
time (sec)	N/A	0.157	0.149	0.835	0.000	0.301	1.958	0.283	12.559

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	55	40	0	106	0	108	0
N.S.	1	1.00	1.25	0.91	0.00	2.41	0.00	2.45	0.00
time (sec)	N/A	0.233	0.200	0.823	0.000	0.323	0.000	0.285	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	24	20	0	34	39	39	19
N.S.	1	1.00	1.00	0.83	0.00	1.42	1.62	1.62	0.79
time (sec)	N/A	0.157	0.193	1.776	0.000	0.287	1.740	0.276	12.551

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	92	45	0	203	0	133	0
N.S.	1	1.00	1.64	0.80	0.00	3.62	0.00	2.38	0.00
time (sec)	N/A	0.221	0.118	0.986	0.000	0.301	0.000	0.299	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	280	2374	1430	0	0	0	5022	0
N.S.	1	1.12	9.53	5.74	0.00	0.00	0.00	20.17	0.00
time (sec)	N/A	0.788	3.530	1.378	0.000	0.000	0.000	0.532	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	46	48	0	85	0	81	49
N.S.	1	1.00	0.96	1.00	0.00	1.77	0.00	1.69	1.02
time (sec)	N/A	0.190	0.361	1.046	0.000	0.515	0.000	0.300	12.421

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	17	18	0	56	0	98	0
N.S.	1	1.00	1.00	1.06	0.00	3.29	0.00	5.76	0.00
time (sec)	N/A	0.159	0.153	0.819	0.000	0.312	0.000	0.280	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	80	51	123	0	132	0	163	0
N.S.	1	0.93	0.59	1.43	0.00	1.53	0.00	1.90	0.00
time (sec)	N/A	0.400	0.181	0.820	0.000	0.291	0.000	0.303	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	111	108	121	0	0	0	0	0
N.S.	1	0.82	0.79	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.289	0.233	0.896	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	212	150	107	103	0	175	0	117	0
N.S.	1	0.71	0.50	0.49	0.00	0.83	0.00	0.55	0.00
time (sec)	N/A	0.293	0.167	0.454	0.000	0.284	0.000	0.291	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	161	122	95	83	0	157	0	98	0
N.S.	1	0.76	0.59	0.52	0.00	0.98	0.00	0.61	0.00
time (sec)	N/A	0.238	0.128	0.510	0.000	0.287	0.000	0.276	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	148	96	84	65	0	128	0	79	0
N.S.	1	0.65	0.57	0.44	0.00	0.86	0.00	0.53	0.00
time (sec)	N/A	0.212	0.140	0.507	0.000	0.308	0.000	0.286	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	160	109	118	92	0	341	0	0	0
N.S.	1	0.68	0.74	0.58	0.00	2.13	0.00	0.00	0.00
time (sec)	N/A	0.273	0.170	0.487	0.000	0.298	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	103	109	112	0	333	0	126	0
N.S.	1	0.66	0.70	0.72	0.00	2.13	0.00	0.81	0.00
time (sec)	N/A	0.268	0.180	0.503	0.000	0.294	0.000	0.288	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	112	124	107	0	377	0	199	0
N.S.	1	0.70	0.77	0.66	0.00	2.34	0.00	1.24	0.00
time (sec)	N/A	0.294	0.247	0.451	0.000	0.283	0.000	0.290	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	224	236	266	0	517	0	368	0
N.S.	1	0.71	0.74	0.84	0.00	1.63	0.00	1.16	0.00
time (sec)	N/A	0.416	0.607	0.673	0.000	0.354	0.000	0.314	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	165	176	199	0	391	0	265	0
N.S.	1	0.73	0.78	0.88	0.00	1.72	0.00	1.17	0.00
time (sec)	N/A	0.312	0.418	0.674	0.000	0.323	0.000	0.286	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	144	172	146	0	287	0	180	0
N.S.	1	0.73	0.87	0.74	0.00	1.45	0.00	0.91	0.00
time (sec)	N/A	0.279	0.671	0.667	0.000	0.328	0.000	0.502	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD
size	211	164	149	215	0	651	0	0	0
N.S.	1	0.78	0.71	1.02	0.00	3.09	0.00	0.00	0.00
time (sec)	N/A	0.388	0.440	0.699	0.000	0.779	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	149	151	201	0	647	0	203	0
N.S.	1	0.74	0.75	1.00	0.00	3.20	0.00	1.00	0.00
time (sec)	N/A	0.359	0.452	0.591	0.000	0.450	0.000	0.658	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	170	155	157	0	693	0	434	0
N.S.	1	0.79	0.72	0.73	0.00	3.22	0.00	2.02	0.00
time (sec)	N/A	0.401	0.530	0.608	0.000	0.527	0.000	0.339	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	452	462	615	842	0	0	0	0	0
N.S.	1	1.02	1.36	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.042	0.691	0.816	0.000	0.000	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	395	400	379	769	0	0	0	0	0
N.S.	1	1.01	0.96	1.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	0.339	0.793	0.000	0.000	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	327	264	1212	0	2384	0	0	0
N.S.	1	1.10	0.89	4.07	0.00	8.00	0.00	0.00	0.00
time (sec)	N/A	0.501	0.288	0.740	0.000	138.398	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	310	1316	0	2266	0	0	0
N.S.	1	1.00	0.87	3.68	0.00	6.33	0.00	0.00	0.00
time (sec)	N/A	1.263	0.363	0.680	0.000	24.096	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	382	382	467	776	0	5324	0	0	0
N.S.	1	1.00	1.22	2.03	0.00	13.94	0.00	0.00	0.00
time (sec)	N/A	1.453	0.384	0.759	0.000	176.365	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	507	507	533	789	0	0	0	0	0
N.S.	1	1.00	1.05	1.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.775	0.792	0.840	0.000	0.000	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	795	656	1239	1183	0	0	0	0	0
N.S.	1	0.83	1.56	1.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.560	1.822	0.655	0.000	0.000	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	553	569	755	1039	0	0	0	0	0
N.S.	1	1.03	1.37	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.350	0.683	0.756	0.000	0.000	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	484	488	542	886	0	0	0	0	0
N.S.	1	1.01	1.12	1.83	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.221	0.552	0.776	0.000	0.000	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	496	496	552	2379	0	0	0	0	0
N.S.	1	1.00	1.11	4.80	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.971	0.547	0.652	0.000	0.000	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	604	604	497	957	0	0	0	0	0
N.S.	1	1.00	0.82	1.58	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.630	0.598	0.783	0.000	0.000	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	668	668	617	911	0	0	0	0	0
N.S.	1	1.00	0.92	1.36	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.693	0.783	0.701	0.000	0.000	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	380	380	312	713	0	0	0	0	0
N.S.	1	1.00	0.82	1.88	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.015	0.356	0.783	0.000	0.000	0.000	0.000	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	342	261	663	0	0	0	0	0
N.S.	1	0.99	0.76	1.93	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.557	0.356	0.744	0.000	0.000	0.000	0.000	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	274	156	622	0	5085	0	0	0
N.S.	1	0.93	0.53	2.12	0.00	17.30	0.00	0.00	0.00
time (sec)	N/A	0.354	0.283	0.737	0.000	1.029	0.000	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	131	589	0	5073	0	0	0
N.S.	1	1.00	0.49	2.21	0.00	19.07	0.00	0.00	0.00
time (sec)	N/A	0.324	0.283	0.634	0.000	1.030	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	236	681	0	0	0	0	0
N.S.	1	1.00	0.72	2.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.897	0.314	0.674	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	298	733	0	0	0	0	0
N.S.	1	1.00	0.81	2.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.138	0.394	0.812	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	457	457	422	777	0	0	0	0	0
N.S.	1	1.00	0.92	1.70	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.594	0.612	0.853	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	499	499	401	1576	0	27621	0	0	0
N.S.	1	1.00	0.80	3.16	0.00	55.35	0.00	0.00	0.00
time (sec)	N/A	1.789	0.590	0.645	0.000	121.373	0.000	0.000	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	410	393	385	1525	0	26116	0	0	0
N.S.	1	0.96	0.94	3.72	0.00	63.70	0.00	0.00	0.00
time (sec)	N/A	0.639	0.546	0.654	0.000	125.127	0.000	0.000	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	411	395	330	1490	0	26234	0	0	0
N.S.	1	0.96	0.80	3.63	0.00	63.83	0.00	0.00	0.00
time (sec)	N/A	0.545	0.499	0.748	0.000	105.280	0.000	0.000	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	416	399	346	1457	0	27447	0	0	0
N.S.	1	0.96	0.83	3.50	0.00	65.98	0.00	0.00	0.00
time (sec)	N/A	0.569	0.503	0.788	0.000	74.110	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	526	526	542	1565	0	0	0	0	0
N.S.	1	1.00	1.03	2.98	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.715	0.820	0.679	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	618	618	684	1639	0	0	0	0	0
N.S.	1	1.00	1.11	2.65	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.877	1.186	0.792	0.000	0.000	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	392	392	409	525	0	0	0	0	0
N.S.	1	1.00	1.04	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.153	0.689	0.905	0.000	0.000	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	316	354	319	483	0	0	0	0	0
N.S.	1	1.12	1.01	1.53	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.809	0.541	0.724	0.000	0.000	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	288	349	460	0	1192	0	0	0
N.S.	1	1.02	1.24	1.63	0.00	4.23	0.00	0.00	0.00
time (sec)	N/A	0.570	0.370	0.747	0.000	186.758	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	298	389	772	0	1139	0	0	0
N.S.	1	1.12	1.46	2.90	0.00	4.28	0.00	0.00	0.00
time (sec)	N/A	0.453	0.355	0.678	0.000	65.905	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	334	850	0	1253	0	0	0
N.S.	1	1.00	1.25	3.18	0.00	4.69	0.00	0.00	0.00
time (sec)	N/A	0.998	0.235	0.730	0.000	10.140	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	288	464	0	1094	0	0	0
N.S.	1	1.00	1.01	1.62	0.00	3.83	0.00	0.00	0.00
time (sec)	N/A	0.894	0.390	0.902	0.000	14.916	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	353	353	381	479	0	1485	0	0	0
N.S.	1	1.00	1.08	1.36	0.00	4.21	0.00	0.00	0.00
time (sec)	N/A	1.089	0.604	0.961	0.000	81.785	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	501	501	734	758	0	0	0	0	0
N.S.	1	1.00	1.47	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.619	2.019	0.802	0.000	0.000	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	417	433	608	648	0	0	0	0	0
N.S.	1	1.04	1.46	1.55	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.417	1.774	0.763	0.000	0.000	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	349	369	619	602	0	0	0	0	0
N.S.	1	1.06	1.77	1.72	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.943	0.985	0.734	0.000	0.000	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	321	729	545	0	0	0	0	0
N.S.	1	1.02	2.31	1.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.752	1.001	0.844	0.000	0.000	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	469	469	601	1639	0	0	0	0	0
N.S.	1	1.00	1.28	3.49	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.549	0.755	0.894	0.000	0.000	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	463	541	585	0	0	0	0	0
N.S.	1	1.00	1.17	1.26	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.394	0.649	0.850	0.000	0.000	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	614	614	587	549	0	0	0	0	0
N.S.	1	1.00	0.96	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.733	0.976	0.915	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	195	196	308	0	2579	0	0	0
N.S.	1	1.03	1.04	1.63	0.00	13.65	0.00	0.00	0.00
time (sec)	N/A	0.613	1.529	0.712	0.000	116.634	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	57	102	83	70	0	73	0
N.S.	1	1.00	0.76	1.36	1.11	0.93	0.00	0.97	0.00
time (sec)	N/A	0.270	0.089	0.694	0.271	0.311	0.000	0.279	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	C	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	150	121	267	0	132	0	0	0
N.S.	1	1.15	0.93	2.05	0.00	1.02	0.00	0.00	0.00
time (sec)	N/A	0.429	0.110	4.599	0.000	0.327	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	369	369	252	463	0	0	0	0	0
N.S.	1	1.00	0.68	1.25	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.928	0.682	0.841	0.000	0.000	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	287	287	210	409	0	0	0	0	0
N.S.	1	1.00	0.73	1.43	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.829	0.347	0.743	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	266	266	194	399	0	0	0	0	0
N.S.	1	1.00	0.73	1.50	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.496	0.263	0.800	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	149	354	0	2753	0	0	0
N.S.	1	1.00	0.68	1.61	0.00	12.51	0.00	0.00	0.00
time (sec)	N/A	0.309	0.203	0.822	0.000	0.648	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	160	358	0	2641	0	0	0
N.S.	1	1.00	0.73	1.63	0.00	12.00	0.00	0.00	0.00
time (sec)	N/A	0.300	0.234	0.786	0.000	0.717	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	193	391	0	5995	0	0	0
N.S.	1	1.00	0.72	1.46	0.00	22.45	0.00	0.00	0.00
time (sec)	N/A	0.824	0.229	0.764	0.000	47.358	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	216	426	0	6018	0	0	0
N.S.	1	1.00	0.74	1.46	0.00	20.68	0.00	0.00	0.00
time (sec)	N/A	0.802	0.443	0.809	0.000	209.662	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	241	457	0	0	0	0	0
N.S.	1	1.00	0.64	1.22	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.973	0.672	0.866	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	466	466	453	1064	0	0	0	0	0
N.S.	1	1.00	0.97	2.28	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.351	1.149	0.827	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	410	960	0	17339	0	0	0
N.S.	1	1.00	1.20	2.82	0.00	50.85	0.00	0.00	0.00
time (sec)	N/A	1.068	0.743	0.871	0.000	22.372	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	297	354	371	946	0	17285	0	0	0
N.S.	1	1.19	1.25	3.19	0.00	58.20	0.00	0.00	0.00
time (sec)	N/A	0.637	0.684	0.770	0.000	13.619	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	361	407	899	0	17258	0	0	0
N.S.	1	1.21	1.36	3.01	0.00	57.72	0.00	0.00	0.00
time (sec)	N/A	0.494	0.722	0.684	0.000	15.820	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	310	365	464	903	0	17397	0	0	0
N.S.	1	1.18	1.50	2.91	0.00	56.12	0.00	0.00	0.00
time (sec)	N/A	0.532	0.758	0.769	0.000	14.101	0.000	0.000	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	394	394	486	990	0	0	0	0	0
N.S.	1	1.00	1.23	2.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.192	1.233	0.747	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	454	454	620	1065	0	0	0	0	0
N.S.	1	1.00	1.37	2.35	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.250	2.103	0.948	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	761	0	864	1148	0	0	0	0	0
N.S.	1	0.00	1.14	1.51	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	1.142	0.915	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	549	554	642	1022	0	0	0	0	0
N.S.	1	1.01	1.17	1.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.211	0.678	0.907	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	431	491	539	1547	0	0	0	0	0
N.S.	1	1.14	1.25	3.59	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.672	0.035	0.935	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	523	521	467	1691	0	0	0	0	0
N.S.	1	1.00	0.89	3.23	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.778	0.502	1.048	0.000	0.000	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	736	736	582	976	0	0	0	0	0
N.S.	1	1.00	0.79	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.864	0.690	0.952	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	545	545	423	914	0	0	0	0	0
N.S.	1	1.00	0.78	1.68	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.518	0.712	1.073	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	463	461	318	844	0	0	0	0	0
N.S.	1	1.00	0.69	1.82	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.712	0.436	0.878	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	402	382	204	794	0	11311	0	0	0
N.S.	1	0.95	0.51	1.98	0.00	28.14	0.00	0.00	0.00
time (sec)	N/A	0.448	0.364	0.898	0.000	3.566	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	B	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	374	374	218	761	0	11287	0	0	0
N.S.	1	1.00	0.58	2.03	0.00	30.18	0.00	0.00	0.00
time (sec)	N/A	0.424	0.014	0.826	0.000	4.272	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	451	451	319	859	0	0	0	0	0
N.S.	1	1.00	0.71	1.90	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	1.835	0.455	0.888	0.000	0.000	0.000	0.000	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	543	543	423	927	0	0	0	0	0
N.S.	1	1.00	0.78	1.71	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	2.916	0.676	1.216	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-1)	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	679	679	547	991	0	0	0	0	0
N.S.	1	1.00	0.81	1.46	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	7.010	1.224	1.151	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	779	779	690	2083	0	0	0	0	0
N.S.	1	1.00	0.89	2.67	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	7.387	1.129	0.996	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	580	534	1992	0	0	0	0	0
N.S.	1	0.95	0.88	3.27	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.925	0.896	0.859	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	609	581	570	1939	0	0	0	0	0
N.S.	1	0.95	0.94	3.18	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.755	0.849	0.857	0.000	0.000	0.000	0.000	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	666	0	806	1906	0	0	0	0	0
N.S.	1	0.00	1.21	2.86	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	0.070	1.086	0.000	0.000	0.000	0.000	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	F	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	N/A	Yes	No	TBD	TBD	TBD	TBD	TBD
size	816	0	1025	2059	0	0	0	0	0
N.S.	1	0.00	1.26	2.52	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.000	6.728	1.156	0.000	0.000	0.000	0.000	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	99	155	0	178	0	188	0
N.S.	1	1.00	0.71	1.11	0.00	1.27	0.00	1.34	0.00
time (sec)	N/A	0.613	0.225	0.976	0.000	0.280	0.000	0.295	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	144	0	175	0	185	0
N.S.	1	1.00	0.82	1.25	0.00	1.52	0.00	1.61	0.00
time (sec)	N/A	0.550	0.179	0.819	0.000	0.331	0.000	0.297	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	99	77	130	0	161	0	171	0
N.S.	1	1.01	0.79	1.33	0.00	1.64	0.00	1.74	0.00
time (sec)	N/A	0.526	0.156	0.727	0.000	0.287	0.000	0.293	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F	A	F	A	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	34	33	81	0	50	0	68	0
N.S.	1	0.50	0.49	1.19	0.00	0.74	0.00	1.00	0.00
time (sec)	N/A	0.216	0.112	0.688	0.000	0.313	0.000	0.288	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	58	54	121	0	132	0	165	0
N.S.	1	0.61	0.57	1.27	0.00	1.39	0.00	1.74	0.00
time (sec)	N/A	0.321	0.153	0.684	0.000	0.304	0.000	0.287	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	90	152	0	170	0	199	0
N.S.	1	1.00	0.69	1.17	0.00	1.31	0.00	1.53	0.00
time (sec)	N/A	0.578	0.174	0.768	0.000	0.295	0.000	0.292	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	113	169	0	194	0	269	0
N.S.	1	1.00	0.75	1.12	0.00	1.28	0.00	1.78	0.00
time (sec)	N/A	0.613	0.219	0.831	0.000	0.299	0.000	0.306	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	169	89	80	155	88	95	85	187
N.S.	1	1.13	0.60	0.54	1.04	0.59	0.64	0.57	1.26
time (sec)	N/A	0.323	0.501	0.658	0.267	0.275	0.531	0.304	13.916

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	113	74	65	104	73	76	70	136
N.S.	1	1.10	0.72	0.63	1.01	0.71	0.74	0.68	1.32
time (sec)	N/A	0.234	0.272	0.529	0.263	0.288	0.519	0.312	13.332

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	30	32	0	53	0	63	0
N.S.	1	1.00	1.07	1.14	0.00	1.89	0.00	2.25	0.00
time (sec)	N/A	0.183	0.139	0.636	0.000	0.317	0.000	0.312	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	89	80	57	0	126	0	159	0
N.S.	1	1.06	0.95	0.68	0.00	1.50	0.00	1.89	0.00
time (sec)	N/A	0.261	0.263	0.770	0.000	0.269	0.000	0.323	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	154	95	72	0	186	0	232	0
N.S.	1	1.11	0.68	0.52	0.00	1.34	0.00	1.67	0.00
time (sec)	N/A	0.346	0.357	0.808	0.000	0.269	0.000	0.318	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	12	11	11	31	11	15
N.S.	1	1.00	0.87	0.80	0.73	0.73	2.07	0.73	1.00
time (sec)	N/A	0.137	9.279	0.552	0.180	0.281	0.084	0.278	12.694

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	13	10	9	11	10	11	13
N.S.	1	1.00	0.81	0.62	0.56	0.69	0.62	0.69	0.81
time (sec)	N/A	0.146	0.001	0.520	0.185	0.275	2.276	0.277	12.354

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	14	0	11	0	11	15
N.S.	1	1.00	0.87	0.93	0.00	0.73	0.00	0.73	1.00
time (sec)	N/A	0.180	0.004	0.512	0.000	0.270	0.000	0.303	12.442

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	13	14	0	11	0	11	13
N.S.	1	1.00	0.87	0.93	0.00	0.73	0.00	0.73	0.87
time (sec)	N/A	0.202	0.003	0.451	0.000	0.262	0.000	0.270	12.488

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	242	169	381	0	0	0	0	0	0
N.S.	1	0.70	1.57	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.279	1.088	0.000	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	488	301	593	0	0	0	0	0	0
N.S.	1	0.62	1.22	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.555	2.284	0.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [93] had the largest ratio of [.647059000000000051]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	25	0.080
2	A	2	2	1.00	27	0.074
3	A	2	2	1.00	27	0.074
4	A	11	10	0.82	27	0.370
5	A	13	12	1.00	27	0.444
6	A	12	11	1.12	30	0.367
7	A	6	5	1.00	30	0.167
8	A	8	7	1.16	30	0.233
9	A	10	9	1.12	30	0.300
10	A	6	5	1.00	23	0.217
11	A	6	5	1.07	23	0.217
12	A	6	5	1.14	30	0.167
13	A	2	2	0.99	28	0.071
14	A	2	2	1.00	30	0.067
15	A	7	6	0.77	30	0.200
16	F	0	0	N/A	0.000	N/A
17	A	6	5	0.99	34	0.147
18	A	6	5	0.99	34	0.147
19	A	10	9	1.00	32	0.281
20	F	0	0	N/A	0.000	N/A
21	A	4	3	1.00	32	0.094
22	A	5	4	1.12	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
23	A	4	3	1.00	29	0.103
24	A	7	6	1.00	26	0.231
25	A	6	5	1.10	30	0.167
26	A	8	7	1.07	30	0.233
27	A	10	9	1.07	30	0.300
28	A	5	4	1.07	30	0.133
29	A	7	6	1.07	30	0.200
30	A	9	8	1.08	30	0.267
31	A	3	2	1.00	26	0.077
32	A	7	6	1.00	26	0.231
33	A	3	2	1.00	24	0.083
34	A	6	5	1.00	20	0.250
35	A	8	7	1.12	36	0.194
36	A	2	2	1.00	36	0.056
37	A	3	2	1.00	32	0.062
38	A	12	11	0.93	32	0.344
39	A	7	6	0.82	38	0.158
40	A	11	10	0.71	35	0.286
41	A	8	7	0.76	33	0.212
42	A	7	6	0.65	32	0.188
43	A	10	9	0.68	35	0.257
44	A	10	9	0.66	35	0.257
45	A	11	10	0.70	35	0.286
46	A	9	8	0.71	38	0.211
47	A	7	6	0.73	36	0.167
48	A	7	6	0.73	35	0.171
49	A	10	9	0.78	38	0.237
50	A	10	9	0.74	38	0.237
51	A	10	9	0.79	38	0.237
52	A	9	8	1.02	27	0.296
53	A	11	10	1.01	25	0.400
54	A	7	6	1.10	24	0.250
55	A	2	2	1.00	27	0.074
56	A	2	2	1.00	27	0.074

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	2	2	1.00	27	0.074
58	A	12	11	0.83	27	0.407
59	A	12	11	1.03	25	0.440
60	A	9	8	1.01	24	0.333
61	A	2	2	1.00	27	0.074
62	A	2	2	1.00	27	0.074
63	A	2	2	1.00	27	0.074
64	A	2	2	1.00	27	0.074
65	A	8	7	0.99	27	0.259
66	A	4	3	0.93	25	0.120
67	A	4	3	1.00	24	0.125
68	A	2	2	1.00	27	0.074
69	A	2	2	1.00	27	0.074
70	A	2	2	1.00	27	0.074
71	A	2	2	1.00	27	0.074
72	A	6	5	0.96	27	0.185
73	A	6	5	0.96	25	0.200
74	A	6	5	0.96	24	0.208
75	A	2	2	1.00	27	0.074
76	A	2	2	1.00	27	0.074
77	A	2	2	1.00	28	0.071
78	A	12	11	1.12	28	0.393
79	A	12	11	1.02	26	0.423
80	A	9	8	1.12	25	0.320
81	A	2	2	1.00	28	0.071
82	A	2	2	1.00	28	0.071
83	A	2	2	1.00	28	0.071
84	A	2	2	1.00	28	0.071
85	A	14	13	1.04	28	0.464
86	A	14	13	1.06	26	0.500
87	A	12	11	1.02	25	0.440
88	A	2	2	1.00	28	0.071
89	A	2	2	1.00	28	0.071
90	A	2	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
91	A	11	10	1.03	24	0.417
92	A	10	9	1.00	22	0.409
93	A	12	11	1.15	17	0.647
94	A	2	2	1.00	28	0.071
95	A	2	2	1.00	28	0.071
96	A	10	9	1.00	28	0.321
97	A	6	5	1.00	26	0.192
98	A	5	4	1.00	25	0.160
99	A	2	2	1.00	28	0.071
100	A	2	2	1.00	28	0.071
101	A	2	2	1.00	28	0.071
102	A	2	2	1.00	28	0.071
103	A	2	2	1.00	28	0.071
104	A	8	7	1.19	28	0.250
105	A	8	7	1.21	26	0.269
106	A	8	7	1.18	25	0.280
107	A	2	2	1.00	28	0.071
108	A	2	2	1.00	28	0.071
109	F	0	0	N/A	0.000	N/A
110	A	10	9	1.01	28	0.321
111	A	7	6	1.14	27	0.222
112	A	2	2	1.00	30	0.067
113	A	2	2	1.00	30	0.067
114	A	2	2	1.00	30	0.067
115	A	8	7	1.00	30	0.233
116	A	4	3	0.95	28	0.107
117	A	4	3	1.00	27	0.111
118	A	2	2	1.00	30	0.067
119	A	2	2	1.00	30	0.067
120	A	2	2	1.00	30	0.067
121	A	2	2	1.00	30	0.067
122	A	6	5	0.95	30	0.167
123	A	6	5	0.95	28	0.179

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2.3. Detailed conclusion table specific for Rubi results

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
124	F	0	0	N/A	0.000	N/A
125	F	0	0	N/A	0.000	N/A
126	A	2	2	1.00	30	0.067
127	A	2	2	1.00	30	0.067
128	A	16	15	1.01	30	0.500
129	A	6	5	0.50	28	0.179
130	A	10	9	0.61	27	0.333
131	A	2	2	1.00	30	0.067
132	A	2	2	1.00	30	0.067
133	A	10	9	1.13	34	0.265
134	A	7	6	1.10	30	0.200
135	A	4	3	1.00	34	0.088
136	A	7	6	1.06	34	0.176
137	A	11	10	1.11	34	0.294
138	A	1	1	1.00	17	0.059
139	A	1	1	1.00	15	0.067
140	A	3	3	1.00	23	0.130
141	A	4	4	1.00	21	0.190
142	A	2	2	0.70	40	0.050
143	A	3	3	0.62	104	0.029

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2} \, dx$

3.1.1 Optimal result

Integrand size = 25, antiderivative size = 94

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2} \, dx = \frac{(bB+Ac)x}{f}+\frac{Bcx^2}{2f}-\frac{(bBd+Acd-aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{3/2}} - \frac{(Bcd-Abf-aBf)\log\left(d+fx^2\right)}{2f^2}$$

output
```
(A*c+B*b)*x/f+1/2*B*c*x^2/f-1/2*(-A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/f^2-(-A*a
*f+A*c*d+B*b*d)*arctan(x*f^(1/2)/d^(1/2))/f^(3/2)/d^(1/2)
```

3.1.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2} \, dx = \frac{fx(2bB+2Ac+Bcx)-\frac{2\sqrt{f}(bBd+Acd-aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}}+(-Bcd+Abf+aBf)\log\left(d+fx^2\right)}{2f^2}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

output
```
(f*x*(2*b*B + 2*A*c + B*c*x) - (2*Sqrt[f]*(b*B*d + A*c*d - a*A*f)*ArcTan[(
Sqrt[f]*x)/Sqrt[d]])/Sqrt[d] + (-(B*c*d) + A*b*f + a*B*f)*Log[d + f*x^2])/
(2*f^2)
```

3.1.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2160, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$$

$$\Big\downarrow \text{2160}$$

$$\int \left(-\frac{x(-aBf-Abf+Bcd)-aAf+Acd+bBd}{f\left(d+fx^2\right)}+\frac{Ac+bB}{f}+\frac{Bcx}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(-aAf+Acd+bBd)}{\sqrt{d}f^{3/2}}-\frac{\log\left(d+fx^2\right)(-aBf-Abf+Bcd)}{2f^2}+\frac{x(Ac+bB)}{f}+\frac{Bcx^2}{2f}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x]
```

output
```
((b*B + A*c)*x)/f + (B*c*x^2)/(2*f) - ((b*B*d + A*c*d - a*A*f)*ArcTan[(Sqr
t[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(3/2)) - ((B*c*d - A*b*f - a*B*f)*Log[d + f*x
^2])/(2*f^2)
```

3.1.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2160
```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.1.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

method	result
default	$\frac{\frac{1}{2}Bc\,x^2+Acx+Bbx}{f}+\frac{\frac{(Abf+Baf-Bcd)\ln\left(f\,x^2+d\right)}{2f}+\frac{(Aaf-Acd-Bbd)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}}{f}$
risch	$\frac{Bc\,x^2}{2f}+\frac{Acx}{f}+\frac{Bbx}{f}+\frac{\ln\left(Aafd-Ac\,d^2-Bb\,d^2-\sqrt{-df(Aaf-Acd-Bbd)^2}\,x\right)Ab}{2f}+\frac{\ln\left(Aafd-Ac\,d^2-Bb\,d^2-\sqrt{-df(Aaf-A\right.}{2f}$

input
```
int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/f*(1/2*B*c*x^2+A*c*x+B*b*x)+1/f*(1/2*(A*b*f+B*a*f-B*c*d)/f*ln(f*x^2+d)+(
A*a*f-A*c*d-B*b*d)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))
```

3.1.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.13

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx$$
$$=\left[\frac{Bcdfx^2+2\,(Bb+Ac)dfx-(Aaf-(Bb+Ac)d)\sqrt{-df}\log\left(\frac{fx^2-2\sqrt{-df}x-d}{fx^2+d}\right)-(Bcd^2-(Ba+Ab)df)\,\mathrm{lo}}{2\,df^2}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

output
```
[1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x - (A*a*f - (B*b + A*c)*d)*sqrt(-d*
f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 + d)) - (B*c*d^2 - (B*a + A*b)*
d*f)*log(f*x^2 + d))/(d*f^2), 1/2*(B*c*d*f*x^2 + 2*(B*b + A*c)*d*f*x + 2*(
A*a*f - (B*b + A*c)*d)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - (B*c*d^2 - (B*a +
 A*b)*d*f)*log(f*x^2 + d))/(d*f^2)]
```

3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(90) = 180$.

Time = 0.75 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.54

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2}\,dx = \frac{Bcx^2}{2f} + x\left(\frac{Ac}{f}+\frac{Bb}{f}\right) + \left(\frac{Abf+Baf-Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} - \frac{\sqrt{-df^5}(Aaf-Acd-}{2df^4}\right.}{Aaf^2-Acdf-Bbdf}\right. + \left(\frac{Abf+Baf-Bcd}{2f^2} + \frac{\sqrt{-df^5}(Aaf-Acd-Bbd)}{2df^4}\right)\log\left(x + \frac{-Abdf-Badf+Bcd^2+2df^2\left(\frac{Abf+Baf-Bcd}{2f^2} + \frac{\sqrt{-df^5}(Aaf-Acd-}{2df^4}\right.}{Aaf^2-Acdf-Bbdf}\right.$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+d),x)
```

output
```
B*c*x**2/(2*f) + x*(A*c/f + B*b/f) + ((A*b*f + B*a*f - B*c*d)/(2*f**2) - s
qrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4))*log(x + (-A*b*d*f - B*a*d
*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*f - B*c*d)/(2*f**2) - sqrt(-d*f**5)
*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(A*a*f**2 - A*c*d*f - B*b*d*f)) + ((
A*b*f + B*a*f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2
*d*f**4))*log(x + (-A*b*d*f - B*a*d*f + B*c*d**2 + 2*d*f**2*((A*b*f + B*a*
f - B*c*d)/(2*f**2) + sqrt(-d*f**5)*(A*a*f - A*c*d - B*b*d)/(2*d*f**4)))/(
A*a*f**2 - A*c*d*f - B*b*d*f))
```

3.1.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+fx^2}\,dx = \frac{(Aaf-(Bb+Ac)d)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} + \frac{Bcx^2+2\,(Bb+Ac)x}{2\,f} - \frac{(Bcd-(Ba+Ab)f)\log\left(fx^2+d\right)}{2\,f^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

output
```
(A*a*f - (B*b + A*c)*d)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) + 1/2*(B*c*x^2
 + 2*(B*b + A*c)*x)/f - 1/2*(B*c*d - (B*a + A*b)*f)*log(f*x^2 + d)/f^2
```

3.1.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = -\frac{(Bbd+Acd-Aaf)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f} - \frac{(Bcd-Baf-Abf)\log\left(fx^2+d\right)}{2\,f^2} + \frac{Bcfx^2+2\,Bbfx+2\,Acfx}{2\,f^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

output
```
-(B*b*d + A*c*d - A*a*f)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f) - 1/2*(B*c*d
- B*a*f - A*b*f)*log(f*x^2 + d)/f^2 + 1/2*(B*c*f*x^2 + 2*B*b*f*x + 2*A*c*f
*x)/f^2
```

3.1.9 Mupad [B] (verification not implemented)

Time = 12.89 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+fx^2}\,dx = \frac{x\,(A\,c+B\,b)}{f} - \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,(A\,c\,d-A\,a\,f+B\,b\,d)}{\sqrt{d}\,f^{3/2}} + \frac{B\,c\,x^2}{2\,f} + \frac{\ln\left(f\,x^2+d\right)\,\left(4\,A\,b\,d\,f^3+4\,B\,a\,d\,f^3-4\,B\,c\,d^2\,f^2\right)}{8\,d\,f^4}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/(d + f*x^2),x)
```

output
```
(x*(A*c + B*b))/f - (atan((f^(1/2)*x)/d^(1/2))*(A*c*d - A*a*f + B*b*d))/(d
^(1/2)*f^(3/2)) + (B*c*x^2)/(2*f) + (log(d + f*x^2)*(4*A*b*d*f^3 + 4*B*a*d
*f^3 - 4*B*c*d^2*f^2))/(8*d*f^4)
```

3.2 $\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+fx^2} dx$

3.2.1 Optimal result

Integrand size = 27, antiderivative size = 228

$$\begin{aligned}\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx &= \frac{(Ab^2f-Ac(cd-2af)-bB(2cd-2af))\,x}{f^2}+\frac{(2Abcf-B(c^2d-b^2f-2acf))\,x^2}{2f^2}\\ &\quad+\frac{c(2bB+Ac)x^3}{3f}+\frac{Bc^2x^4}{4f}-\frac{(Ab^2df-2bBd(cd-af)-A(cd-af)^2)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{5/2}}\\ &\quad-\frac{(2Abf(cd-af)-B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(d+fx^2\right)}{2f^3}\end{aligned}$$

output
```
(A*b^2*f-A*c*(-2*a*f+c*d)-b*B*(-2*a*f+2*c*d))*x/f^2+1/2*(2*A*b*c*f-B*(-2*a
*c*f-b^2*f+c^2*d))*x^2/f^2+1/3*c*(A*c+2*B*b)*x^3/f+1/4*B*c^2*x^4/f-1/2*(2*
A*b*f*(-a*f+c*d)-B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/f^3-(
A*b^2*d*f-2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^2)*arctan(x*f^(1/2)/d^(1/2))/f^(
5/2)/d^(1/2)
```

3.2.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx=\frac{(-Ab^2df+2bBd(cd-af)+A(cd-af)^2)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{5/2}}$$
$$+\frac{fx(12Abcfx+6b^2f(2A+Bx)+3Bcx(-2cd+4af+cfx^2)+4Ac(-3cd+6af+cfx^2)+4bB(-6cd}{12f^3}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

output
```
((-(A*b^2*d*f) + 2*b*B*d*(c*d - a*f) + A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)
/Sqrt[d]])/(Sqrt[d]*f^(5/2)) + (f*x*(12*A*b*c*f*x + 6*b^2*f*(2*A + B*x) +
3*B*c*x*(-2*c*d + 4*a*f + c*f*x^2) + 4*A*c*(-3*c*d + 6*a*f + c*f*x^2) + 4*
b*B*(-6*c*d + 6*a*f + 2*c*f*x^2)) + 6*(2*A*b*f*(-(c*d) + a*f) + B*(c^2*d^2
 - b^2*d*f - 2*a*c*d*f + a^2*f^2))*Log[d + f*x^2])/(12*f^3)
```

3.2.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1345, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$

$$\Big\downarrow \ 1345$$

$$\int \left(\frac{-x\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)+A(cd-af)^2+2bBd(cd-af)-Ab^2df}{f^2\left(d+fx^2\right)}+\frac{x(2A}{}\right.$$

$$\Big\downarrow \ 2009$$

$$-\frac{\log(d+fx^2)\left(2Abf(cd-af)-B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^3}-\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(-A(cd-af)^2-2bBd(cd-af)+Ab^2df\right)}{\sqrt{d}f^{5/2}}+\frac{x^2\left(2Abcf-B\left(-2acf+b^2(-f)+c^2d\right)\right)}{2f^2}+\frac{x\left(-Ac(cd-2af)-bB(2cd-2af)+Ab^2f\right)}{f^2}+\frac{cx^3(Ac+2bB)}{3f}+\frac{Bc^2x^4}{4f}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x]
```

output
```
((A*b^2*f - A*c*(c*d - 2*a*f) - b*B*(2*c*d - 2*a*f))*x)/f^2 + ((2*A*b*c*f
- B*(c^2*d - b^2*f - 2*a*c*f))*x^2)/(2*f^2) + (c*(2*b*B + A*c)*x^3)/(3*f)
+ (B*c^2*x^4)/(4*f) - ((A*b^2*d*f - 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)
*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(5/2)) - ((2*A*b*f*(c*d - a*f) -
B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[d + f*x^2])/(2*f^3)
```

3.2.3.1 Defintions of rubi rules used

rule 1345
```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.2.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{4}Bc^2x^4f+\frac{1}{3}Ac^2fx^3+\frac{2}{3}Bbcfx^3+Abcfx^2+Bacfx^2+\frac{1}{2}Bb^2fx^2-\frac{1}{2}Bc^2dx^2+2Aacfx+Ab^2fx-Ac^2dx+2Babfx-2Bbcdx}{f^2}+\frac{(2}{}$
risch	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(1/4*B*c^2*x^4*f+1/3*A*c^2*f*x^3+2/3*B*b*c*f*x^3+A*b*c*f*x^2+B*a*c*f
*x^2+1/2*B*b^2*f*x^2-1/2*B*c^2*d*x^2+2*A*a*c*f*x+A*b^2*f*x-A*c^2*d*x+2*B*a
*b*f*x-2*B*b*c*d*x)+1/f^2*(1/2*(2*A*a*b*f^2-2*A*b*c*d*f+B*a^2*f^2-2*B*a*c*
d*f-B*b^2*d*f+B*c^2*d^2)/f*ln(f*x^2+d)+(A*a^2*f^2-2*A*a*c*d*f-A*b^2*d*f+A*
c^2*d^2-2*B*a*b*d*f+2*B*b*c*d^2)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))
```

3.2.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.19

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$
$$= \left[\frac{3\,Bc^2df^2x^4 + 4\,(2\,Bbc + Ac^2)df^2x^3 - 6\,(Bc^2d^2f - (Bb^2 + 2\,(Ba + Ab)c)df^2)x^2 - 6\,(Aa^2f^2 + (2\,Bbc + \ldots}{\ldots}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

output
```
[1/12*(3*B*c^2*d*f^2*x^4 + 4*(2*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f
- (B*b^2 + 2*(B*a + A*b)*c)*d*f^2)*x^2 - 6*(A*a^2*f^2 + (2*B*b*c + A*c^2)*
d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f
)*x - d)/(f*x^2 + d)) - 12*((2*B*b*c + A*c^2)*d^2*f - (2*B*a*b + A*b^2 + 2
*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b)*c)*d^2*f + (B*a^2
 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3), 1/12*(3*B*c^2*d*f^2*x^4 + 4*(2
*B*b*c + A*c^2)*d*f^2*x^3 - 6*(B*c^2*d^2*f - (B*b^2 + 2*(B*a + A*b)*c)*d*f
^2)*x^2 + 12*(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a
*c)*d*f)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) - 12*((2*B*b*c + A*c^2)*d^2*f - (
2*B*a*b + A*b^2 + 2*A*a*c)*d*f^2)*x + 6*(B*c^2*d^3 - (B*b^2 + 2*(B*a + A*b
)*c)*d^2*f + (B*a^2 + 2*A*a*b)*d*f^2)*log(f*x^2 + d))/(d*f^3)]
```

3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. 2(209) = 418.

Time = 15.56 (sec) , antiderivative size = 933, normalized size of antiderivative = 4.09

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx = \frac{Bc^2x^4}{4f} + x^3\left(\frac{Ac^2}{3f} + \frac{2Bbc}{3f}\right) + x^2\left(\frac{Abc}{f} + \frac{Bac}{f} + \frac{Bb^2}{2f} - \frac{Bc^2d}{2f^2}\right) + x\left(\frac{2Aac}{f} + \frac{Ab^2}{f} - \frac{Ac^2d}{f^2} + \frac{2Bab}{f} - \frac{2Bbcd}{f^2}\right) + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3} - \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - B\dots}{\dots}\right) + \left(\frac{2Aabf^2 - 2Abcdf + Ba^2f^2 - 2Bacdf - Bb^2df + Bc^2d^2}{2f^3} + \frac{\sqrt{-df^7}\left(Aa^2f^2 - 2Aacdf - Ab^2df + Ac^2d^2 - 2Babdf + 2Bbcd^2\right)}{2df^6}\right)\log\left(x + \frac{-2Aabdf^2 + 2Abcd^2f - B\dots}{\dots}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

output
```
B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f)) + x**2*(A*b*c/f +
B*a*c/f + B*b**2/(2*f) - B*c**2*d/(2*f**2)) + x*(2*A*a*c/f + A*b**2/f - A*
c**2*d/f**2 + 2*B*a*b/f - 2*B*b*c*d/f**2) + ((2*A*a*b*f**2 - 2*A*b*c*d*f +
 B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) - sqrt(-d*
f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f
+ 2*B*b*c*d**2)/(2*d*f**6))*log(x + (-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*
a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*
A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2
*d**2)/(2*f**3) - sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*b**2*d*f +
A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*
a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2
*f)) + ((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**2 - 2*B*a*c*d*f - B*b**2*d
*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a**2*f**2 - 2*A*a*c*d*f - A*
b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d**2)/(2*d*f**6))*log(x + (
-2*A*a*b*d*f**2 + 2*A*b*c*d**2*f - B*a**2*d*f**2 + 2*B*a*c*d**2*f + B*b**2
*d**2*f - B*c**2*d**3 + 2*d*f**3*((2*A*a*b*f**2 - 2*A*b*c*d*f + B*a**2*f**
2 - 2*B*a*c*d*f - B*b**2*d*f + B*c**2*d**2)/(2*f**3) + sqrt(-d*f**7)*(A*a*
*2*f**2 - 2*A*a*c*d*f - A*b**2*d*f + A*c**2*d**2 - 2*B*a*b*d*f + 2*B*b*c*d
**2)/(2*d*f**6)))/(A*a**2*f**3 - 2*A*a*c*d*f**2 - A*b**2*d*f**2 + A*c**2*d
**2*f - 2*B*a*b*d*f**2 + 2*B*b*c*d**2*f))
```

3.2. $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$

3.2.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$
$$= \frac{(Aa^2f^2 + (2\,Bbc + Ac^2)d^2 - (2\,Bab + Ab^2 + 2\,Aac)df)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}$$
$$+ \frac{3\,Bc^2fx^4 + 4\,(2\,Bbc + Ac^2)fx^3 - 6\,(Bc^2d - (Bb^2 + 2\,(Ba + Ab)c)f)x^2 - 12\,((2\,Bbc + Ac^2)d - (2\,Bab}{12\,f^2}$$
$$+ \frac{(Bc^2d^2 - (Bb^2 + 2\,(Ba + Ab)c)df + (Ba^2 + 2\,Aab)f^2)\log\left(fx^2 + d\right)}{2\,f^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

output
```
(A*a^2*f^2 + (2*B*b*c + A*c^2)*d^2 - (2*B*a*b + A*b^2 + 2*A*a*c)*d*f)*arct
an(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/12*(3*B*c^2*f*x^4 + 4*(2*B*b*c + A*c
^2)*f*x^3 - 6*(B*c^2*d - (B*b^2 + 2*(B*a + A*b)*c)*f)*x^2 - 12*((2*B*b*c +
 A*c^2)*d - (2*B*a*b + A*b^2 + 2*A*a*c)*f)*x)/f^2 + 1/2*(B*c^2*d^2 - (B*b^
2 + 2*(B*a + A*b)*c)*d*f + (B*a^2 + 2*A*a*b)*f^2)*log(f*x^2 + d)/f^3
```

3.2.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+fx^2}\,dx$$
$$= \frac{(2\,Bbcd^2 + Ac^2d^2 - 2\,Babdf - Ab^2df - 2\,Aacdf + Aa^2f^2)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}\,f^2}$$
$$+ \frac{(Bc^2d^2 - Bb^2df - 2\,Bacdf - 2\,Abcdf + Ba^2f^2 + 2\,Aabf^2)\log\left(fx^2 + d\right)}{2\,f^3}$$
$$+ \frac{3\,Bc^2f^3x^4 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 - 6\,Bc^2df^2x^2 + 6\,Bb^2f^3x^2 + 12\,Bacf^3x^2 + 12\,Abcf^3x^2 - 24\,Bbcd}{12\,f^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

output
```
(2*B*b*c*d^2 + A*c^2*d^2 - 2*B*a*b*d*f - A*b^2*d*f - 2*A*a*c*d*f + A*a^2*f
^2)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^2) + 1/2*(B*c^2*d^2 - B*b^2*d*f - 2
*B*a*c*d*f - 2*A*b*c*d*f + B*a^2*f^2 + 2*A*a*b*f^2)*log(f*x^2 + d)/f^3 + 1
/12*(3*B*c^2*f^3*x^4 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*x^3 - 6*B*c^2*d*f^2*x
^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 24*B*b*c*d*f^
2*x - 12*A*c^2*d*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)
/f^4
```

3.2.9 Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\,(a+bx+cx^2)^2}{d+fx^2}\,dx = x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\,(A\,c^2+2\,B\,b\,c)}{f^2}\right) + x^2\left(\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{2\,f}-\frac{B\,c^2\,d}{2\,f^2}\right)+\frac{x^3\,(A\,c^2+2\,B\,b\,c)}{3\,f}+\frac{B\,c^2\,x^4}{4\,f} + \frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,(A\,a^2\,f^2-2\,B\,a\,b\,d\,f-2\,A\,a\,c\,d\,f-A\,b^2\,d\,f+2\,B\,b\,c\,d^2+A\,c^2\,d^2)}{\sqrt{d}\,f^{5/2}} + \frac{\ln\left(f\,x^2+d\right)\,(4\,B\,a^2\,d\,f^5+8\,A\,a\,b\,d\,f^5-8\,B\,a\,c\,d^2\,f^4-4\,B\,b^2\,d^2\,f^4-8\,A\,b\,c\,d^2\,f^4+4\,B\,c^2\,d^3\,f^3)}{8\,d\,f^6}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + f*x^2),x)
```

output
```
x*((A*b^2 + 2*A*a*c + 2*B*a*b)/f - (d*(A*c^2 + 2*B*b*c))/f^2) + x^2*((B*b^
2 + 2*A*b*c + 2*B*a*c)/(2*f) - (B*c^2*d)/(2*f^2)) + (x^3*(A*c^2 + 2*B*b*c)
)/(3*f) + (B*c^2*x^4)/(4*f) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^2*f^2 + A*c^
2*d^2 + 2*B*b*c*d^2 - A*b^2*d*f - 2*A*a*c*d*f - 2*B*a*b*d*f))/(d^(1/2)*f^(
5/2)) + (log(d + f*x^2)*(4*B*a^2*d*f^5 - 4*B*b^2*d^2*f^4 + 4*B*c^2*d^3*f^3
 + 8*A*a*b*d*f^5 - 8*A*b*c*d^2*f^4 - 8*B*a*c*d^2*f^4))/(8*d*f^6)
```

3.3 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

3.3.1 Optimal result

Integrand size = 27, antiderivative size = 441

$$\begin{aligned}
&\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx \\
&= -\frac{(b^3Bdf+3Ab^2f(cd-af)-3bB(cd-af)^2-Ac(c^2d^2-3acdf+3a^2f^2))\,x}{f^3} \\
&\quad -\frac{(Abf(3c^2d-b^2f-6acf)-B(c^3d^2-3ac^2df+3ab^2f^2-3cf(b^2d-a^2f)))\,x^2}{2f^3} \\
&\quad +\frac{(b^3Bf+3Ab^2cf-Ac^2(cd-3af)-3bBc(cd-2af))\,x^3}{3f^2} \\
&\quad +\frac{c(3Abcf-B(c^2d-3b^2f-3acf))\,x^4}{4f^2}+\frac{c^2(3bB+Ac)x^5}{5f}+\frac{Bc^3x^6}{6f} \\
&\quad +\frac{(b^3Bd^2f+3Ab^2df(cd-af)-3bBd(cd-af)^2-A(cd-af)^3)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{7/2}} \\
&\quad +\frac{(Abf(3c^2d^2-6acdf-f(b^2d-3a^2f))-B(cd-af)\,(c^2d^2-2acdf-f(3b^2d-a^2f)))\log\left(d+fx^2\right)}{2f^4}
\end{aligned}$$

output
```
-(b^3*B*d*f+3*A*b^2*f*(-a*f+c*d)-3*b*B*(-a*f+c*d)^2-A*c*(3*a^2*f^2-3*a*c*d
*f+c^2*d^2))*x/f^3-1/2*(A*b*f*(-6*a*c*f-b^2*f+3*c^2*d)-B*(c^3*d^2-3*a*c^2*
d*f+3*a*b^2*f^2-3*c*f*(-a^2*f+b^2*d)))*x^2/f^3+1/3*(b^3*B*f+3*A*b^2*c*f-A*
c^2*(-3*a*f+c*d)-3*b*B*c*(-2*a*f+c*d))*x^3/f^2+1/4*c*(3*A*b*c*f-B*(-3*a*c*
f-3*b^2*f+c^2*d))*x^4/f^2+1/5*c^2*(A*c+3*B*b)*x^5/f+1/6*B*c^3*x^6/f+1/2*(A
*b*f*(3*c^2*d^2-6*a*c*d*f-f*(-3*a^2*f+b^2*d))-B*(-a*f+c*d)*(c^2*d^2-2*a*c*
d*f-f*(-a^2*f+3*b^2*d)))*ln(f*x^2+d)/f^4+(b^3*B*d^2*f+3*A*b^2*d*f*(-a*f+c*
d)-3*b*B*d*(-a*f+c*d)^2-A*(-a*f+c*d)^3)*arctan(x*f^(1/2)/d^(1/2))/f^(7/2)/
d^(1/2)
```

3.3.2 Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$
$$= \frac{(b^3Bd^2f+3Ab^2df(cd-af)-3bBd(cd-af)^2-A(cd-af)^3)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}f^{7/2}}$$
$$+ \frac{fx(10b^3f(-6Bd+3Afx+2Bfx^2)+15b^2f(3Bx(-2cd+2af+cfx^2)+4A(-3cd+3af+cfx^2))+$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

output
```
((b^3*B*d^2*f + 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d -
 a*f)^3)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + (f*x*(10*b^3*f*(
-6*B*d + 3*A*f*x + 2*B*f*x^2) + 15*b^2*f*(3*B*x*(-2*c*d + 2*a*f + c*f*x^2)
 + 4*A*(-3*c*d + 3*a*f + c*f*x^2)) + 3*b*(15*A*c*f*x*(-2*c*d + 4*a*f + c*f
*x^2) + 4*B*(15*a^2*f^2 + 10*a*c*f*(-3*d + f*x^2) + c^2*(15*d^2 - 5*d*f*x^
2 + 3*f^2*x^4))) + c*(5*B*x*(18*a^2*f^2 + 9*a*c*f*(-2*d + f*x^2) + c^2*(6*
d^2 - 3*d*f*x^2 + 2*f^2*x^4)) + 4*A*(45*a^2*f^2 + 15*a*c*f*(-3*d + f*x^2)
+ c^2*(15*d^2 - 5*d*f*x^2 + 3*f^2*x^4)))) - 30*(A*b*f*(-3*c^2*d^2 + b^2*d*
f + 6*a*c*d*f - 3*a^2*f^2) + B*(c*d - a*f)*(c^2*d^2 - 3*b^2*d*f - 2*a*c*d*
f + a^2*f^2))*Log[d + f*x^2])/(60*f^4)
```

3.3.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {1345, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$

$$\Big\downarrow \; 1345$$

$$\int \left(-\frac{x\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{f^3}-\frac{-x\left(Abf\left(-f\left(b^2d-\right.\right.\right.}{}\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{\log\left(d+fx^2\right)\left(Abf\left(-f\left(b^2d-3a^2f\right)-6acdf+3c^2d^2\right)-B(cd-af)\left(-f\left(3b^2d-a^2f\right)-2acdf+c^2d^2\right)\right)}{2f^4}-\frac{x^2\left(Abf\left(-6acf+b^2(-f)+3c^2d\right)-B\left(-3cf\left(b^2d-a^2f\right)+3ab^2f^2-3ac^2df+c^3d^2\right)\right)}{2f^3}-\frac{x\left(-Ac\left(3a^2f^2-3acdf+c^2d^2\right)+3Ab^2f(cd-af)-3bB(cd-af)^2+b^3Bdf\right)}{f^3}+\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)\left(3Ab^2df(cd-af)-A(cd-af)^3-3bBd(cd-af)^2+b^3Bd^2f\right)}{\sqrt{d}f^{7/2}}+\frac{cx^4\left(3Abcf-B\left(-3acf-3b^2f+c^2d\right)\right)}{4f^2}+\frac{x^3\left(-Ac^2(cd-3af)-3bBc(cd-2af)+3Ab^2cf+b^3Bf\right)}{3f^2}+\frac{c^2x^5(Ac+3bB)}{5f}+\frac{Bc^3x^6}{6f}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x]
```

output
```
-(((b^3*B*d*f + 3*A*b^2*f*(c*d - a*f) - 3*b*B*(c*d - a*f)^2 - A*c*(c^2*d^2
 - 3*a*c*d*f + 3*a^2*f^2))*x)/f^3) - ((A*b*f*(3*c^2*d - b^2*f - 6*a*c*f) -
 B*(c^3*d^2 - 3*a*c^2*d*f + 3*a*b^2*f^2 - 3*c*f*(b^2*d - a^2*f)))*x^2)/(2*
f^3) + ((b^3*B*f + 3*A*b^2*c*f - A*c^2*(c*d - 3*a*f) - 3*b*B*c*(c*d - 2*a*
f))*x^3)/(3*f^2) + (c*(3*A*b*c*f - B*(c^2*d - 3*b^2*f - 3*a*c*f))*x^4)/(4*
f^2) + (c^2*(3*b*B + A*c)*x^5)/(5*f) + (B*c^3*x^6)/(6*f) + ((b^3*B*d^2*f +
 3*A*b^2*d*f*(c*d - a*f) - 3*b*B*d*(c*d - a*f)^2 - A*(c*d - a*f)^3)*ArcTan
[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*f^(7/2)) + ((A*b*f*(3*c^2*d^2 - 6*a*c*d*f
- f*(b^2*d - 3*a^2*f)) - B*(c*d - a*f)*(c^2*d^2 - 2*a*c*d*f - f*(3*b^2*d -
 a^2*f)))*Log[d + f*x^2])/(2*f^4)
```

3.3.3.1 Defintions of rubi rules used

rule 1345
```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + c*x^2)^p*(d + e*x +
 f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, c, d, e, f, g, h}, x] && NeQ[e^2 -
 4*d*f, 0] && IntegersQ[p, q] && (GtQ[p, 0] || GtQ[q, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.3.4 Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 592, normalized size of antiderivative = 1.34

method	result
default	$\frac{A b^2 c f^2 x^3+A a c^2 f^2 x^3-3 A a c^2 d f x-3 A b^2 c d f x-\frac{3}{2} B b^2 c d f x^2-\frac{3}{2} B a c^2 d f x^2-\frac{3}{2} A b c^2 d f x^2+3 A a b c f^2 x^2-B b c^2 d f x^3+2 B a b c f^2 x^3+A}{}$
risch	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/f^3*(A*b^2*c*f^2*x^3+A*a*c^2*f^2*x^3-3*A*a*c^2*d*f*x-3*A*b^2*c*d*f*x-3/2
*B*b^2*c*d*f*x^2-3/2*B*a*c^2*d*f*x^2-3/2*A*b*c^2*d*f*x^2+3*A*a*b*c*f^2*x^2
-B*b*c^2*d*f*x^3+2*B*a*b*c*f^2*x^3+A*c^3*d^2*x+1/2*B*c^3*d^2*x^2+1/2*A*b^3
*f^2*x^2+1/3*B*b^3*f^2*x^3+1/6*B*c^3*x^6*f^2+1/5*A*c^3*f^2*x^5-6*B*a*b*c*d
*f*x+3/5*B*b*c^2*f^2*x^5+3/4*A*b*c^2*f^2*x^4+3/4*B*a*c^2*f^2*x^4+3/4*B*b^2
*c*f^2*x^4-1/4*B*c^3*d*f*x^4-1/3*A*c^3*d*f*x^3+3/2*B*a^2*c*f^2*x^2+3/2*B*a
*b^2*f^2*x^2+3*A*a^2*c*f^2*x+3*A*a*b^2*f^2*x+3*B*a^2*b*f^2*x-b^3*B*d*f*x+3
*B*b*c^2*d^2*x)+1/f^3*(1/2*(3*A*a^2*b*f^3-6*A*a*b*c*d*f^2-A*b^3*d*f^2+3*A*
b*c^2*d^2*f+B*a^3*f^3-3*B*a^2*c*d*f^2-3*B*a*b^2*d*f^2+3*B*a*c^2*d^2*f+3*B*
b^2*c*d^2*f-B*c^3*d^3)/f*ln(f*x^2+d)+(A*a^3*f^3-3*A*a^2*c*d*f^2-3*A*a*b^2*
d*f^2+3*A*a*c^2*d^2*f+3*A*b^2*c*d^2*f-A*c^3*d^3-3*B*a^2*b*d*f^2+6*B*a*b*c*
d^2*f+B*b^3*d^2*f-3*B*b*c^2*d^3)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))
```

3.3.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1014, normalized size of antiderivative = 2.30

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$

$$= \left[\frac{10\,Bc^3df^3x^6 + 12\,(3\,Bbc^2 + Ac^3)df^3x^5 - 15\,(Bc^3d^2f^2 - 3\,(Bb^2c + (Ba + Ab)c^2)df^3)x^4 - 20\,((3\,Bbc^2 + \right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="fricas")
```

output
```
[1/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d
^2*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3
)*d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B
*c^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 +
3*(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 - 30*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^
3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b
^2 + A*a^2*c)*d*f^2)*sqrt(-d*f)*log((f*x^2 - 2*sqrt(-d*f)*x - d)/(f*x^2 +
d)) + 60*((3*B*b*c^2 + A*c^3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*
b^2)*c)*d^2*f^2 + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4
 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^3*f + (3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2
*A*a*b)*c)*d^2*f^2 - (B*a^3 + 3*A*a^2*b)*d*f^3)*log(f*x^2 + d))/(d*f^4), 1
/60*(10*B*c^3*d*f^3*x^6 + 12*(3*B*b*c^2 + A*c^3)*d*f^3*x^5 - 15*(B*c^3*d^2
*f^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f^3)*x^4 - 20*((3*B*b*c^2 + A*c^3)*
d^2*f^2 - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f^3)*x^3 + 30*(B*c
^3*d^3*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f^2 + (3*B*a*b^2 + A*b^3 + 3*
(B*a^2 + 2*A*a*b)*c)*d*f^3)*x^2 + 60*(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3
+ (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2
 + A*a^2*c)*d*f^2)*sqrt(d*f)*arctan(sqrt(d*f)*x/d) + 60*((3*B*b*c^2 + A*c^
3)*d^3*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d^2*f^2 + 3*(B*a^2*
b + A*a*b^2 + A*a^2*c)*d*f^3)*x - 30*(B*c^3*d^4 - 3*(B*b^2*c + (B*a + A...
```

3.3.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**3/(f*x**2+d),x)
```

output
```
Timed out
```

3.3.7 Maxima [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$
$$= \frac{(Aa^3f^3-(3\,Bbc^2+Ac^3)d^3+(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)d^2f-3\,(Ba^2b+Aab^2+Aa^2c)df^2)\operatorname{arcta}}{\sqrt{df}\,f^3}$$
$$+\frac{10\,Bc^3f^2x^6+12\,(3\,Bbc^2+Ac^3)f^2x^5-15\,(Bc^3df-3\,(Bb^2c+(Ba+Ab)c^2)f^2)x^4-20\,((3\,Bbc^2+Ac^3)}{}$$
$$-\frac{(Bc^3d^3-3\,(Bb^2c+(Ba+Ab)c^2)d^2f+(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)df^2-(Ba^3+3\,Aa^2b)f^3)\log}{2\,f^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="maxima")
```

output
```
(A*a^3*f^3 - (3*B*b*c^2 + A*c^3)*d^3 + (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A
*b^2)*c)*d^2*f - 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*d*f^2)*arctan(f*x/sqrt(d*
f))/(sqrt(d*f)*f^3) + 1/60*(10*B*c^3*f^2*x^6 + 12*(3*B*b*c^2 + A*c^3)*f^2*
x^5 - 15*(B*c^3*d*f - 3*(B*b^2*c + (B*a + A*b)*c^2)*f^2)*x^4 - 20*((3*B*b*
c^2 + A*c^3)*d*f - (B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*f^2)*x^3 +
30*(B*c^3*d^2 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d*f + (3*B*a*b^2 + A*b^3 + 3
*(B*a^2 + 2*A*a*b)*c)*f^2)*x^2 + 60*((3*B*b*c^2 + A*c^3)*d^2 - (B*b^3 + 3*
A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*d*f + 3*(B*a^2*b + A*a*b^2 + A*a^2*c)*f^2
)*x)/f^3 - 1/2*(B*c^3*d^3 - 3*(B*b^2*c + (B*a + A*b)*c^2)*d^2*f + (3*B*a*b
^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*d*f^2 - (B*a^3 + 3*A*a^2*b)*f^3)*log(f
*x^2 + d)/f^4
```

3.3.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 623, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx =$$
$$-\frac{(3\,Bbc^2d^3+Ac^3d^3-Bb^3d^2f-6\,Babcd^2f-3\,Ab^2cd^2f-3\,Aac^2d^2f+3\,Ba^2bdf^2+3\,Aab^2df^2+3\,Aa^2c}{\sqrt{df}\,f^3}$$
$$-\frac{(Bc^3d^3-3\,Bb^2cd^2f-3\,Bac^2d^2f-3\,Abc^2d^2f+3\,Bab^2df^2+Ab^3df^2+3\,Ba^2cdf^2+6\,Aabcdf^2-Ba^3f}{2\,f^4}$$
$$+\frac{10\,Bc^3f^5x^6+36\,Bbc^2f^5x^5+12\,Ac^3f^5x^5-15\,Bc^3df^4x^4+45\,Bb^2cf^5x^4+45\,Bac^2f^5x^4+45\,Abc^2f^5x^4}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^3/(f*x^2+d),x, algorithm="giac")
```

output
```
-(3*B*b*c^2*d^3 + A*c^3*d^3 - B*b^3*d^2*f - 6*B*a*b*c*d^2*f - 3*A*b^2*c*d^
2*f - 3*A*a*c^2*d^2*f + 3*B*a^2*b*d*f^2 + 3*A*a*b^2*d*f^2 + 3*A*a^2*c*d*f^
2 - A*a^3*f^3)*arctan(f*x/sqrt(d*f))/(sqrt(d*f)*f^3) - 1/2*(B*c^3*d^3 - 3*
B*b^2*c*d^2*f - 3*B*a*c^2*d^2*f - 3*A*b*c^2*d^2*f + 3*B*a*b^2*d*f^2 + A*b^
3*d*f^2 + 3*B*a^2*c*d*f^2 + 6*A*a*b*c*d*f^2 - B*a^3*f^3 - 3*A*a^2*b*f^3)*l
og(f*x^2 + d)/f^4 + 1/60*(10*B*c^3*f^5*x^6 + 36*B*b*c^2*f^5*x^5 + 12*A*c^3
*f^5*x^5 - 15*B*c^3*d*f^4*x^4 + 45*B*b^2*c*f^5*x^4 + 45*B*a*c^2*f^5*x^4 +
45*A*b*c^2*f^5*x^4 - 60*B*b*c^2*d*f^4*x^3 - 20*A*c^3*d*f^4*x^3 + 20*B*b^3*
f^5*x^3 + 120*B*a*b*c*f^5*x^3 + 60*A*b^2*c*f^5*x^3 + 60*A*a*c^2*f^5*x^3 +
30*B*c^3*d^2*f^3*x^2 - 90*B*b^2*c*d*f^4*x^2 - 90*B*a*c^2*d*f^4*x^2 - 90*A*
b*c^2*d*f^4*x^2 + 90*B*a*b^2*f^5*x^2 + 30*A*b^3*f^5*x^2 + 90*B*a^2*c*f^5*x
^2 + 180*A*a*b*c*f^5*x^2 + 180*B*b*c^2*d^2*f^3*x + 60*A*c^3*d^2*f^3*x - 60
*B*b^3*d*f^4*x - 360*B*a*b*c*d*f^4*x - 180*A*b^2*c*d*f^4*x - 180*A*a*c^2*d
*f^4*x + 180*B*a^2*b*f^5*x + 180*A*a*b^2*f^5*x + 180*A*a^2*c*f^5*x)/f^6
```

3.3.9 Mupad [B] (verification not implemented)

Time = 13.20 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$$
$$= x^2\left(\frac{3\,B\,c\,a^2+3\,B\,a\,b^2+6\,A\,c\,a\,b+A\,b^3}{2\,f}-\frac{d\left(\frac{3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2}{f}-\frac{B\,c^3\,d}{f^2}\right)}{2\,f}\right)$$
$$+x\left(\frac{3\,B\,a^2\,b+3\,A\,c\,a^2+3\,A\,a\,b^2}{f}-\frac{d\left(\frac{B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2}{f}-\frac{d\,(A\,c^3+3\,B\,b\,c^2)}{f^2}\right)}{f}\right)$$
$$+x^3\left(\frac{B\,b^3+3\,A\,b^2\,c+6\,B\,a\,b\,c+3\,A\,a\,c^2}{3\,f}-\frac{d\,(A\,c^3+3\,B\,b\,c^2)}{3\,f^2}\right)$$
$$+x^4\left(\frac{3\,B\,b^2\,c+3\,A\,b\,c^2+3\,B\,a\,c^2}{4\,f}-\frac{B\,c^3\,d}{4\,f^2}\right)+\frac{x^5\,(A\,c^3+3\,B\,b\,c^2)}{5\,f}+\frac{B\,c^3\,x^6}{6\,f}$$
$$+\frac{\ln\left(f\,x^2+d\right)\,\left(4\,B\,a^3\,d\,f^7+12\,A\,a^2\,b\,d\,f^7-12\,B\,a^2\,c\,d^2\,f^6-12\,B\,a\,b^2\,d^2\,f^6-24\,A\,a\,b\,c\,d^2\,f^6+12\,B\,a\,c\right.}{8\,d\,f^8}$$
$$+\frac{\mathrm{atan}\left(\frac{\sqrt{f}\,x}{\sqrt{d}}\right)\,\left(A\,a^3\,f^3-3\,B\,a^2\,b\,d\,f^2-3\,A\,a^2\,c\,d\,f^2-3\,A\,a\,b^2\,d\,f^2+6\,B\,a\,b\,c\,d^2\,f+3\,A\,a\,c^2\,d^2\,f+B\,b^3\right.}{\sqrt{d}\,f^{7/2}}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^3)/(d + f*x^2),x)
```

output
```
x^2*((A*b^3 + 3*B*a*b^2 + 3*B*a^2*c + 6*A*a*b*c)/(2*f) - (d*((3*A*b*c^2 +
3*B*a*c^2 + 3*B*b^2*c)/f - (B*c^3*d)/f^2))/(2*f)) + x*((3*A*a*b^2 + 3*A*a^
2*c + 3*B*a^2*b)/f - (d*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*B*a*b*c)/f - (
d*(A*c^3 + 3*B*b*c^2))/f^2))/f) + x^3*((B*b^3 + 3*A*a*c^2 + 3*A*b^2*c + 6*
B*a*b*c)/(3*f) - (d*(A*c^3 + 3*B*b*c^2))/(3*f^2)) + x^4*((3*A*b*c^2 + 3*B*
a*c^2 + 3*B*b^2*c)/(4*f) - (B*c^3*d)/(4*f^2)) + (x^5*(A*c^3 + 3*B*b*c^2))/
(5*f) + (B*c^3*x^6)/(6*f) + (log(d + f*x^2)*(4*B*a^3*d*f^7 - 4*A*b^3*d^2*f
^6 - 4*B*c^3*d^4*f^4 - 12*B*a*b^2*d^2*f^6 + 12*A*b*c^2*d^3*f^5 + 12*B*a*c^
2*d^3*f^5 - 12*B*a^2*c*d^2*f^6 + 12*B*b^2*c*d^3*f^5 + 12*A*a^2*b*d*f^7 - 2
4*A*a*b*c*d^2*f^6))/(8*d*f^8) + (atan((f^(1/2)*x)/d^(1/2))*(A*a^3*f^3 - A*
c^3*d^3 - 3*B*b*c^2*d^3 + B*b^3*d^2*f - 3*A*a*b^2*d*f^2 + 3*A*a*c^2*d^2*f
- 3*A*a^2*c*d*f^2 - 3*B*a^2*b*d*f^2 + 3*A*b^2*c*d^2*f + 6*B*a*b*c*d^2*f))/
(d^(1/2)*f^(7/2))
```

3.3. $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{d+fx^2}\,dx$

3.4 $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$

3.4.1 Optimal result

Integrand size = 27, antiderivative size = 274

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \frac{\sqrt{f}(bBd-Acd+aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\,(c^2d^2-2acdf+f(b^2d+a^2f))} - \frac{(Ab^2f+2Ac(cd-af)-bB(cd+af))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(c^2d^2-2acdf+f(b^2d+a^2f))} + \frac{(Bcd+Abf-aBf)\log(a+bx+cx^2)}{2(c^2d^2-2acdf+f(b^2d+a^2f))} - \frac{(Bcd+Abf-aBf)\log(d+fx^2)}{2(c^2d^2-2acdf+f(b^2d+a^2f))}$$

output
```
1/2*(A*b*f-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d)
)-1/2*(A*b*f-B*a*f+B*c*d)*ln(f*x^2+d)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))-
(A*b^2*f+2*A*c*(-a*f+c*d)-b*B*(a*f+c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))/(-4*a*c+b^2)^(1/2)+(A*a*f-A*c*d+B
*b*d)*arctan(x*f^(1/2)/d^(1/2))*f^(1/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d)
)/d^(1/2)
```

3.4.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$$
$$=\frac{2\sqrt{-b^2+4ac}\sqrt{f}(bBd-Acd+aAf)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)+\sqrt{d}\Big(2(Ab^2f+2Ac(cd-af)-bB(cd+af))\,\text{arctan}}{2\sqrt{-b^2+4ac}\sqrt{d}\,(c^2d^2-2acdf\,-}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

output
```
(2*Sqrt[-b^2 + 4*a*c]*Sqrt[f]*(b*B*d - A*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/S
qrt[d]] + Sqrt[d]*(2*(A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTa
n[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(B*c*d + A*b*f - a*
B*f)*(-Log[d + f*x^2] + Log[a + x*(b + c*x)])))/(2*Sqrt[-b^2 + 4*a*c]*Sqrt
[d]*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)))
```

3.4.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.370, Rules used = {1356, 25, 27, 452, 218, 240, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+fx^2)(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \ 1356$$
$$\frac{\int -\frac{abBf-A(fb^2+c^2d-acf)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}+\frac{\int \frac{f(bBd-Acd+aAf-(Bcd+Abf-aBf)x)}{fx^2+d}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{f(bBd-Acd+aAf-(Bcd+Abf-aBf)x)}{fx^2+d}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}-\frac{\int \frac{abBf-A(fb^2+c^2d-acf)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}$$
$$\Big\downarrow \ 27$$

$$\frac{f\int \frac{bBd-Acd+aAf-(Bcd+Abf-aBf)x}{fx^2+d}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{\int \frac{abBf-A\left(fb^2+c^2d-acf\right)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 452$$

$$\frac{f\left((aAf-Acd+bBd)\int \frac{1}{fx^2+d}dx-(-aBf+Abf+Bcd)\int \frac{x}{fx^2+d}dx\right)}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{\int \frac{abBf-A\left(fb^2+c^2d-acf\right)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 218$$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-(-aBf+Abf+Bcd)\int \frac{x}{fx^2+d}dx\right)}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{\int \frac{abBf-A\left(fb^2+c^2d-acf\right)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 240$$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2f}\right)}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{\int \frac{abBf-A\left(fb^2+c^2d-acf\right)-c(Bcd+Abf-aBf)x}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 1142$$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2f}\right)}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{-\frac{1}{2}\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)\int \frac{1}{cx^2+bx+a}dx-\frac{1}{2}(-aBf+Abf+Bcd)\int \frac{b+2cx}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 1083$$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-\frac{\log\left(d+fx^2\right)(-aBf+Abf+Bcd)}{2f}\right)}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}-\frac{\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)-\frac{1}{2}(-aBf+Abf+Bcd)\int \frac{b+2cx}{cx^2+bx+a}dx}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}$$

$$\Big\downarrow \; 219$$

3.4. $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-\frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2f}\right)}{f(a^2f+b^2d)-2acdf+c^2d^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2Ac(cd-af)-bB(af+cd)+Ab^2f)}{\sqrt{b^2-4ac}}-\frac{1}{2}(-aBf+Abf+Bcd)\int\frac{b+2cx}{cx^2+bx+a}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}$$

$$\Big\downarrow \text{1103}$$

$$\frac{f\left(\frac{\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)(aAf-Acd+bBd)}{\sqrt{d}\sqrt{f}}-\frac{\log(d+fx^2)(-aBf+Abf+Bcd)}{2f}\right)}{f(a^2f+b^2d)-2acdf+c^2d^2}-\frac{\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)(2Ac(cd-af)-bB(af+cd)+Ab^2f)}{\sqrt{b^2-4ac}}-\frac{1}{2}\log(a+bx+cx^2)(-aBf+Abf+Bcd)}{f(a^2f+b^2d)-2acdf+c^2d^2}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)*(d + f*x^2)),x]
```

output
```
-((((A*b^2*f + 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*ArcTanh[(b + 2*c*x)/Sq
rt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c] - ((B*c*d + A*b*f - a*B*f)*Log[a + b*x
 + c*x^2])/2)/(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))) + (f*(((b*B*d - A
*c*d + a*A*f)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) - ((B*c*d + A
*b*f - a*B*f)*Log[d + f*x^2])/(2*f)))/(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^
2*f))
```

3.4.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1356
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*
(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 + b^2*d*f - 2*a*c*d*f + a
^2*f^2]}, Simp[1/q   Int[Simp[g*c^2*d + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*
c*d + g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[Simp[
b*h*d*f - g*c*d*f + a*g*f^2 - f*(h*c*d + g*b*f - a*h*f)*x, x]/(d + f*x^2),
x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c,
 0]
```

3.4. $\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)} \, dx$

3.4.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.87

method	result
default	$\frac{f\left(\frac{(-Abf+Baf-Bcd)\ln\left(f\,x^2+d\right)}{2f}+\frac{(Aaf-Acd+Bbd)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{\sqrt{df}}\right)}{a^2 f^2-2acdf+b^2 df+c^2 d^2}+\frac{\frac{\left(Abcf-Bacf+B\,c^2 d\right)\ln\left(c\,x^2+bx+a\right)}{2c}+\frac{2\left(-Aacf+A\,b^2 f+A\,c^2 d-\cdots\right.}{\cdots}}{a^2 f^2-2acdf+b^2 df-\cdots}$
risch	Expression too large to display

input
```
int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
f/(a^2*f^2-2*a*c*d*f+b^2*d*f+c^2*d^2)*(1/2*(-A*b*f+B*a*f-B*c*d)/f*ln(f*x^2
+d)+(A*a*f-A*c*d+B*b*d)/(d*f)^(1/2)*arctan(f*x/(d*f)^(1/2)))+1/(a^2*f^2-2*
a*c*d*f+b^2*d*f+c^2*d^2)*(1/2*(A*b*c*f-B*a*c*f+B*c^2*d)/c*ln(c*x^2+b*x+a)+
2*(-A*a*c*f+A*b^2*f+A*c^2*d-B*a*b*f-1/2*(A*b*c*f-B*a*c*f+B*c^2*d)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2)))
```

3.4.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.4.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d),x)
```

output
```
Timed out
```

3.4.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.4.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx &= \frac{(Bcd-Baf+Abf)\log(cx^2+bx+a)}{2(c^2d^2+b^2df-2\,acdf+a^2f^2)} - \frac{(Bcd-Baf+Abf)\log(fx^2+d)}{2(c^2d^2+b^2df-2\,acdf+a^2f^2)}\\ &\quad+\frac{(Bbdf-Acdf+Aaf^2)\arctan\left(\frac{fx}{\sqrt{df}}\right)}{(c^2d^2+b^2df-2\,acdf+a^2f^2)\sqrt{df}}\\ &\quad-\frac{(Bbcd-2\,Ac^2d+Babf-Ab^2f+2\,Aacf)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(c^2d^2+b^2df-2\,acdf+a^2f^2)\sqrt{-b^2+4\,ac}}\end{aligned}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d),x, algorithm="giac")
```

output
```
1/2*(B*c*d - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 + b^2*d*f - 2*a*
c*d*f + a^2*f^2) - 1/2*(B*c*d - B*a*f + A*b*f)*log(f*x^2 + d)/(c^2*d^2 + b
^2*d*f - 2*a*c*d*f + a^2*f^2) + (B*b*d*f - A*c*d*f + A*a*f^2)*arctan(f*x/s
qrt(d*f))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(d*f)) - (B*b*c*d
 - 2*A*c^2*d + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2
 + 4*a*c))/((c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2)*sqrt(-b^2 + 4*a*c))
```

3.4.9 Mupad [B] (verification not implemented)

Time = 75.86 (sec) , antiderivative size = 3888, normalized size of antiderivative = 14.19

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)),x)
```

output
```
(log(B^3*c^2*f^2*x + (((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-
d*f)^(1/2))/2 + (A*c*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*((((B*c*d
^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/2 + (A*c*d*(-d*f)
^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(4*A*a^2*c^2*f^4 + 4*A*c^4*d^2*f^2 - c
*f^2*x*(3*A*b^3*f^2 + 4*B*c^3*d^2 - B*a*b^2*f^2 + 4*B*a^2*c*f^2 - 12*A*a*b
*c*f^2 + 12*A*b*c^2*d*f - 8*B*a*c^2*d*f - 3*B*b^2*c*d*f) - 3*A*b^2*c^2*d*f
^3 - 4*B*b*c^3*d^2*f^2 - A*a*b^2*c*f^4 - 8*A*a*c^3*d*f^3 + B*b^3*c*d*f^3 +
 (2*c*f^2*((B*c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 - (A*a*f*(-d*f)^(1/2))/
2 + (A*c*d*(-d*f)^(1/2))/2 - (B*b*d*(-d*f)^(1/2))/2)*(2*b*c^3*d^3 + 4*c^4*
d^3*x - a^2*b^2*f^3*x + 4*a*b^3*d*f^2 - 2*b^3*c*d^2*f + 4*a^3*c*f^3*x + 3*
b^4*d*f^2*x + 12*a*b*c^2*d^2*f - 14*a^2*b*c*d*f^2 - 4*a*c^3*d^2*f*x - 4*a^
2*c^2*d*f^2*x + 3*b^2*c^2*d^2*f*x - 10*a*b^2*c*d*f^2*x))/(d*(a^2*f^2 + c^2
*d^2 + b^2*d*f - 2*a*c*d*f)) + 4*B*a*b*c^2*d*f^3))/(d*(a^2*f^2 + c^2*d^2 +
 b^2*d*f - 2*a*c*d*f)) - c*f^2*x*(2*B^2*c^2*d - 4*A^2*c^2*f - B^2*b^2*f +
2*B^2*a*c*f + 2*A*B*b*c*f) + A^2*b*c^2*f^3 + B^2*b*c^2*d*f^2 - 4*A*B*a*c^2
*f^3 + A*B*b^2*c*f^3 - 4*A*B*c^3*d*f^2))/(d*(a^2*f^2 + c^2*d^2 + b^2*d*f -
 2*a*c*d*f)) + A*B^2*c^2*f^2)*(f*((B*a*d)/2 - (A*b*d)/2 + (A*a*(-d*f)^(1/2
))/2) - (B*c*d^2)/2 - (A*c*d*(-d*f)^(1/2))/2 + (B*b*d*(-d*f)^(1/2))/2))/(c
^2*d^3 + a^2*d*f^2 + b^2*d^2*f - 2*a*c*d^2*f) - (log(B^3*c^2*f^2*x + (((B*
c*d^2)/2 + (A*b*d*f)/2 - (B*a*d*f)/2 + (A*a*f*(-d*f)^(1/2))/2 - (A*c*d*...
```

3.5 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$

3.5.1 Optimal result

Integrand size = 27, antiderivative size = 596

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$$
$$= \frac{Abc(cd+af)-(Ab-aB)\left(2c^2d+b^2f-2acf\right)-c(Ab^2f+2Ac(cd-af)-bB(cd+af))\,x}{(b^2-4ac)\left(b^2df+(cd-af)^2\right)(a+bx+cx^2)}$$
$$- \frac{f^{3/2}(Ab^2df+2bBd(cd-af)-A(cd-af)^2)\arctan\left(\frac{\sqrt{f}x}{\sqrt{d}}\right)}{\sqrt{d}\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$- \frac{(b^5Bdf^2-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd-af)^2+b^3Bf(5c^2d^2-4acdf-a^2f^2)-4Ab^2cf(2c^2}{(b^2-4ac)^{3/2}\left(c^2d^2-2acdf+f\right.}$$
$$- \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(a+bx+cx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$
$$+ \frac{f(2Abf(cd-af)+B(c^2d^2-2acdf-f(b^2d-a^2f)))\log\left(d+fx^2\right)}{2\left(c^2d^2-2acdf+f\left(b^2d+a^2f\right)\right)^2}$$

output
```
(A*b*c*(a*f+c*d)-(A*b-B*a)*(-2*a*c*f+b^2*f+2*c^2*d)-c*(A*b^2*f+2*A*c*(-a*f
+c*d)-b*B*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f+(-a*f+c*d)^2)/(c*x^2+b*x+a)-
(b^5*B*d*f^2-2*A*b^4*f^2*(-a*f+c*d)-4*A*c^2*(-3*a*f+c*d)*(-a*f+c*d)^2+b^3*
B*f*(-a^2*f^2-4*a*c*d*f+5*c^2*d^2)-4*A*b^2*c*f*(3*a^2*f^2-3*a*c*d*f+2*c^2*
d^2)+2*b*B*c*(3*a^3*f^3+3*a^2*c*d*f^2-7*a*c^2*d^2*f+c^3*d^3))*arctanh((2*c
*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b
^2*d))^2-1/2*f*(2*A*b*f*(-a*f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))
*ln(c*x^2+b*x+a)/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2+1/2*f*(2*A*b*f*(-a*
f+c*d)+B*(c^2*d^2-2*a*c*d*f-f*(-a^2*f+b^2*d)))*ln(f*x^2+d)/(c^2*d^2-2*a*c*
d*f+f*(a^2*f+b^2*d))^2-f^(3/2)*(A*b^2*d*f+2*b*B*d*(-a*f+c*d)-A*(-a*f+c*d)^
2)*arctan(x*f^(1/2)/d^(1/2))/(c^2*d^2-2*a*c*d*f+f*(a^2*f+b^2*d))^2/d^(1/2)
```

3.5.2 Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 523, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$$

$$= \frac{-\frac{2(c^2d^2-2acdf+f(b^2d+a^2f))(A(b^3f+bc(cd-3af)+b^2cfx+2c^2(cd-af)x)+B(2a^2cf-bc^2dx-a(2c^2d+b^2f+bcfx)))}{(b^2-4ac)(a+x(b+cx))}+\frac{2f^{3/2}(-Ab^2df+A(c}{}}{}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

output
```
((-2*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))*(A*(b^3*f + b*c*(c*d - 3*a*
f) + b^2*c*f*x + 2*c^2*(c*d - a*f)*x) + B*(2*a^2*c*f - b*c^2*d*x - a*(2*c^
2*d + b^2*f + b*c*f*x))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) + (2*f^(3/2)*(
-(A*b^2*d*f) + A*(c*d - a*f)^2 + 2*b*B*d*(-(c*d) + a*f))*ArcTan[(Sqrt[f]*x
)/Sqrt[d]])/Sqrt[d] - (2*(b^5*B*d*f^2 - 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^
2 + 2*A*b^4*f^2*(-(c*d) + a*f) - b^3*B*f*(-5*c^2*d^2 + 4*a*c*d*f + a^2*f^2
) - 4*A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d^3 - 7
*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTan[(b + 2*c*x)/Sqrt[-b^2 +
4*a*c]])/(-b^2 + 4*a*c)^(3/2) + f*(2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*
c*d*f + f*(-(b^2*d) + a^2*f)))*Log[d + f*x^2] + f*(2*A*b*f*(-(c*d) + a*f)
+ B*(-(c^2*d^2) + 2*a*c*d*f + f*(b^2*d - a^2*f)))*Log[a + x*(b + c*x)])/(2
*(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f))^2)
```

3.5.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {1351, 25, 2142, 25, 27, 452, 218, 240, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d+fx^2)(a+bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1351}$$

$$\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2+b^2df)}-\frac{\int -\frac{-cf\left(Afb^2-B(cd+af)b+2Ac(cd-af)\right)x^2-\left(b^2-4ac\right)f(Bcd+Abf-aBf)x+(bB-2Ac)\left(dfb^2+(cd-af)^2\right)+af\left(Afb^2-B(cd+af)b+2Ac(cd-a\right.}{(cx^2+bx+a)(fx^2+d)}}{(b^2-4ac)((cd-af)^2+b^2df)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{-cf\left(Afb^2-B(cd+af)b+2Ac(cd-af)\right)x^2-\left(b^2-4ac\right)f(Bcd+Abf-aBf)x+(bB-2Ac)\left(dfb^2+(cd-af)^2\right)+af\left(Afb^2-B(cd+af)b+2Ac(cd-af)\right)}{(cx^2+bx+a)(fx^2+d)}}{(b^2-4ac)((cd-af)^2+b^2df)}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2+b^2df)}$$

$$\Big\downarrow \text{ 2142}$$

$$\frac{\frac{\int -\frac{\left(b^2-4ac\right)f^2\left(Adfb^2+2Bd(cd-af)b-A(cd-af)^2-\left(2Abf(cd-af)+B\left(c^2d^2-2acfd-f\left(b^2d-a^2f\right)\right)\right)x\right)}{fx^2+d}dx}{f(a^2f+b^2d)-2acdf+c^2d^2}+\frac{\int \frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd\right.}{}}{}}{(b^2-4ac}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2+b^2df)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd-a^2f^2\right)b^3-2Acf\left(2c^2d^2-5acfd+5a^2f^2\right)b^2+Bc\left(c^3d^3-5ac^2fd^2-a^2cf^2d+5a^3f^3\right)b-2Ac^2(cd-3af)(cd-af)^2-c\left(\right.}{cx^2+bx+a}}{f(a^2f+b^2d)-2acdf+c^2d^2}}{(b^2-4ac)}+\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2+b^2df)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd-a^2f^2\right)b^3-2Acf\left(2c^2d^2-5acfd+5a^2f^2\right)b^2+Bc\left(c^3d^3-5ac^2fd^2-a^2cf^2d+5a^3f^3\right)b-2Ac^2(cd-3af)(cd-af)^2-c(\ldots}{cx^2+bx+a}}{f(a^2f+b^2d)-2acdf+c^2d^2}}{(b^2-4ac)}$$

$$-\frac{(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \text{ 452}$$

$$\frac{\int \frac{\frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd-a^2f^2\right)b^3-2Acf\left(2c^2d^2-5acfd+5a^2f^2\right)b^2+Bc\left(c^3d^3-5ac^2fd^2-a^2cf^2d+5a^3f^3\right)b-2Ac^2(cd-3af)(cd-af)^2-c(\ldots}{cx^2+bx+a}}{f(a^2f+b^2d)-2acdf+c^2d^2}}{\ldots}$$

$$-\frac{(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\int \frac{\frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd-a^2f^2\right)b^3-2Acf\left(2c^2d^2-5acfd+5a^2f^2\right)b^2+Bc\left(c^3d^3-5ac^2fd^2-a^2cf^2d+5a^3f^3\right)b-2Ac^2(cd-3af)(cd-af)^2-c(\ldots}{cx^2+bx+a}}{f(a^2f+b^2d)-2acdf+c^2d^2}}{\ldots}$$

$$-\frac{(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \text{ 240}$$

$$\frac{\int \frac{\frac{Bdf^2b^5-2Af^2(cd-af)b^4+Bf\left(2c^2d^2-3acfd-a^2f^2\right)b^3-2Acf\left(2c^2d^2-5acfd+5a^2f^2\right)b^2+Bc\left(c^3d^3-5ac^2fd^2-a^2cf^2d+5a^3f^3\right)b-2Ac^2(cd-3af)(cd-af)^2-c(\ldots}{cx^2+bx+a}}{f(a^2f+b^2d)-2acdf+c^2d^2}}{\ldots}$$

$$-\frac{(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \text{ 1142}$$

$$\frac{\frac{\frac{1}{2}\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2f^2-4acdf+5c^2d^2\right)+2bBc\left(3a^3f^3+3a^2cdf^2-7ac^2d^2f+c^3d^3\right)-2Ab^4f^2(cd-af)-4Ac^2(cd-3af)(cd\ldots\right.}{f(a^2f+b^2d)-2acdf+c^2d^2}}{\ldots}$$

$$-\frac{(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{(b^2-4ac)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \text{ 1083}$$

3.5. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$

$$\frac{-\frac{1}{2}f\left(b^2-4ac\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2f^2-4acdf+5c^2d^2\right)\cdots\right.}{f\left(a^2f+b^2d\right)-2acdf+c^2\cdots}}{\cdots}$$

$$\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \; 219$$

$$\frac{-\frac{1}{2}f\left(b^2-4ac\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-a^2\cdots\right.\right.}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}}{\cdots}$$

$$\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

$$\Big\downarrow \; 1103$$

$$\frac{-\frac{1}{2}f\left(b^2-4ac\right)\log\left(a+bx+cx^2\right)\left(B\left(-f\left(b^2d-a^2f\right)-2acdf+c^2d^2\right)+2Abf(cd-af)\right)-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-4Ab^2cf\left(3a^2f^2-3acdf+2c^2d^2\right)+b^3Bf\left(-\cdots\right.\right.}{f\left(a^2f+b^2d\right)-2acdf+c^2d^2}}{\cdots}$$

$$\frac{-(Ab-aB)\left(-2acf+b^2f+2c^2d\right)-cx\left(2Ac(cd-af)-bB(af+cd)+Ab^2f\right)+Abc(af+cd)}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)\left((cd-af)^2+b^2df\right)}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + f*x^2)),x]
```

output
```
(A*b*c*(c*d + a*f) - (A*b - a*B)*(2*c^2*d + b^2*f - 2*a*c*f) - c*(A*b^2*f
+ 2*A*c*(c*d - a*f) - b*B*(c*d + a*f))*x)/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
 a*f)^2)*(a + b*x + c*x^2)) + ((-(((b^5*B*d*f^2 - 2*A*b^4*f^2*(c*d - a*f)
- 4*A*c^2*(c*d - 3*a*f)*(c*d - a*f)^2 + b^3*B*f*(5*c^2*d^2 - 4*a*c*d*f - a
^2*f^2) - 4*A*b^2*c*f*(2*c^2*d^2 - 3*a*c*d*f + 3*a^2*f^2) + 2*b*B*c*(c^3*d
^3 - 7*a*c^2*d^2*f + 3*a^2*c*d*f^2 + 3*a^3*f^3))*ArcTanh[(b + 2*c*x)/Sqrt[
b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]) - ((b^2 - 4*a*c)*f*(2*A*b*f*(c*d - a*f)
+ B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d - a^2*f)))*Log[a + b*x + c*x^2])/2)/(c
^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f)) - ((b^2 - 4*a*c)*f^2*(((A*b^2*d*f
+ 2*b*B*d*(c*d - a*f) - A*(c*d - a*f)^2)*ArcTan[(Sqrt[f]*x)/Sqrt[d]])/(Sqr
t[d]*Sqrt[f]) - ((2*A*b*f*(c*d - a*f) + B*(c^2*d^2 - 2*a*c*d*f - f*(b^2*d
- a^2*f)))*Log[d + f*x^2])/(2*f)))/(c^2*d^2 - 2*a*c*d*f + f*(b^2*d + a^2*f
)))/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2))
```

3.5. $\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+fx^2\right)}\,dx$

3.5.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1351
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*S
imp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b
*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*
((g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f))))*(p +
 q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1])
```

rule 2142
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (f_.)*(x_)^2)), x_Sym
bol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px, x, 2]
, q = c^2*d^2 + b^2*d*f - 2*a*c*d*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a
*c*C*d + A*b^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d + A*b*f -
 a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 + b*B*d*f -
A*c*d*f - a*C*d*f + a*A*f^2 - f*(B*c*d - b*C*d + A*b*f - a*B*f)*x)/(d + f*x
^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. 2(580) = 1160.

Time = 1.55 (sec) , antiderivative size = 1254, normalized size of antiderivative = 2.10

method	result	size
default	Expression too large to display	1254
risch	Expression too large to display	3364134

input
```
int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
f^2/(a^4*f^4-4*a^3*c*d*f^3+2*a^2*b^2*d*f^3+6*a^2*c^2*d^2*f^2-4*a*b^2*c*d^2
*f^2-4*a*c^3*d^3*f+b^4*d^2*f^2+2*b^2*c^2*d^3*f+c^4*d^4)*(1/2*(-2*A*a*b*f^2
+2*A*b*c*d*f+B*a^2*f^2-2*B*a*c*d*f-B*b^2*d*f+B*c^2*d^2)/f*ln(f*x^2+d)+(A*a
^2*f^2-2*A*a*c*d*f-A*b^2*d*f+A*c^2*d^2+2*B*a*b*d*f-2*B*b*c*d^2)/(d*f)^(1/2
)*arctan(f*x/(d*f)^(1/2)))-1/(a^4*f^4-4*a^3*c*d*f^3+2*a^2*b^2*d*f^3+6*a^2*
c^2*d^2*f^2-4*a*b^2*c*d^2*f^2-4*a*c^3*d^3*f+b^4*d^2*f^2+2*b^2*c^2*d^3*f+c^
4*d^4)*((c*(2*A*a^3*c*f^3-A*a^2*b^2*f^3-6*A*a^2*c^2*d*f^2+4*A*a*b^2*c*d*f^
2+6*A*a*c^3*d^2*f-A*b^4*d*f^2-3*A*b^2*c^2*d^2*f-2*A*c^4*d^3+B*a^3*b*f^3-B*
a^2*b*c*d*f^2+B*a*b^3*d*f^2-B*a*b*c^2*d^2*f+B*b^3*c*d^2*f+B*b*c^3*d^3)/(4*
a*c-b^2)*x+(3*A*a^3*b*c*f^3-A*a^2*b^3*f^3-7*A*a^2*b*c^2*d*f^2+5*A*a*b^3*c*
d*f^2+5*A*a*b*c^3*d^2*f-A*b^5*d*f^2-2*A*b^3*c^2*d^2*f-A*b*c^4*d^3-2*B*a^4*
c*f^3+B*a^3*b^2*f^3+6*B*a^3*c^2*d*f^2-4*B*a^2*b^2*c*d*f^2-6*B*a^2*c^3*d^2*
f+B*a*b^4*d*f^2+3*B*a*b^2*c^2*d^2*f+2*B*a*c^4*d^3)/(4*a*c-b^2))/(c*x^2+b*x
+a)+1/(4*a*c-b^2)*(1/2*(-8*A*a^2*b*c^2*f^3+2*A*a*b^3*c*f^3+8*A*a*b*c^3*d*f
^2-2*A*b^3*c^2*d*f^2+4*B*a^3*c^2*f^3-B*a^2*b^2*c*f^3-8*B*a^2*c^3*d*f^2-2*B
*a*b^2*c^2*d*f^2+4*B*a*c^4*d^2*f+B*b^4*c*d*f^2-B*b^2*c^3*d^2*f)/c*ln(c*x^2
+b*x+a)+2*(6*A*a^3*c^2*f^3-10*A*a^2*b^2*c*f^3-14*A*a^2*c^3*d*f^2+2*A*a*b^4
*f^3+10*A*a*b^2*c^2*d*f^2+10*A*a*c^4*d^2*f-2*A*b^4*c*d*f^2-4*A*b^2*c^3*d^2
*f-2*A*c^5*d^3+5*B*a^3*b*c*f^3-B*a^2*b^3*f^3-B*a^2*b*c^2*d*f^2-3*B*a*b^3*c
*d*f^2-5*B*a*b*c^3*d^2*f+b^5*B*d*f^2+2*B*b^3*c^2*d^2*f+B*b*c^4*d^3-1/2*...
```

3.5.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.5.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+d),x)
```

output
```
Timed out
```

3.5.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+fx^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.5.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1313 vs. $2(579) = 1158$.

Time = 0.28 (sec) , antiderivative size = 1313, normalized size of antiderivative = 2.20

$$\int \frac{A+Bx}{(a+bx+cx^2)^2\,(d+fx^2)}\,dx=\text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+d),x, algorithm="giac")
```

output
```
-1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^
3 - 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3
*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d
*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) + 1/2*(B*c^2*d^2*f - B*b^2*d*f^2 - 2*B*a*c
*d*f^2 + 2*A*b*c*d*f^2 + B*a^2*f^3 - 2*A*a*b*f^3)*log(f*x^2 + d)/(c^4*d^4
+ 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^
2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 + a^4*f^4) - (2*B*b*c*d^2*
f^2 - A*c^2*d^2*f^2 - 2*B*a*b*d*f^3 + A*b^2*d*f^3 + 2*A*a*c*d*f^3 - A*a^2*
f^4)*arctan(f*x/sqrt(d*f))/((c^4*d^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f + b
^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a
^3*c*d*f^3 + a^4*f^4)*sqrt(d*f)) + (2*B*b*c^4*d^3 - 4*A*c^5*d^3 + 5*B*b^3*
c^2*d^2*f - 14*B*a*b*c^3*d^2*f - 8*A*b^2*c^3*d^2*f + 20*A*a*c^4*d^2*f + B*
b^5*d*f^2 - 4*B*a*b^3*c*d*f^2 - 2*A*b^4*c*d*f^2 + 6*B*a^2*b*c^2*d*f^2 + 12
*A*a*b^2*c^2*d*f^2 - 28*A*a^2*c^3*d*f^2 - B*a^2*b^3*f^3 + 2*A*a*b^4*f^3 +
6*B*a^3*b*c*f^3 - 12*A*a^2*b^2*c*f^3 + 12*A*a^3*c^2*f^3)*arctan((2*c*x + b
)/sqrt(-b^2 + 4*a*c))/((b^2*c^4*d^4 - 4*a*c^5*d^4 + 2*b^4*c^2*d^3*f - 12*a
*b^2*c^3*d^3*f + 16*a^2*c^4*d^3*f + b^6*d^2*f^2 - 8*a*b^4*c*d^2*f^2 + 22*a
^2*b^2*c^2*d^2*f^2 - 24*a^3*c^3*d^2*f^2 + 2*a^2*b^4*d*f^3 - 12*a^3*b^2*c*d
*f^3 + 16*a^4*c^2*d*f^3 + a^4*b^2*f^4 - 4*a^5*c*f^4)*sqrt(-b^2 + 4*a*c)) +
 (2*B*a*c^4*d^3 - A*b*c^4*d^3 + 3*B*a*b^2*c^2*d^2*f - 2*A*b^3*c^2*d^2*f...
```

3.5.9 Mupad [B] (verification not implemented)

Time = 18.33 (sec) , antiderivative size = 23006, normalized size of antiderivative = 38.60

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((d + f*x^2)*(a + b*x + c*x^2)^2),x)
```

output
```
((A*b^3*f + A*b*c^2*d - 2*B*a*c^2*d - B*a*b^2*f + 2*B*a^2*c*f - 3*A*a*b*c*
f)/((4*a*c - b^2)*(a^2*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)) - (x*(2*A*a*c
^2*f - 2*A*c^3*d + B*b*c^2*d - A*b^2*c*f + B*a*b*c*f))/((4*a*c - b^2)*(a^2
*f^2 + c^2*d^2 + b^2*d*f - 2*a*c*d*f)))/(a + b*x + c*x^2) + symsum(log((x*
(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*f^6 + B^3*b^2*
c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*
f^6 + 4*A^2*B*c^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 + 6*B^3*a*b^2*c^4*d*f^5 -
 24*A^2*B*a*c^6*d*f^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8
*A^2*B*a*b^2*c^4*f^6 - 4*A*B^2*b*c^6*d^2*f^4 - 6*A*B^2*b^3*c^4*d*f^5 + 8*A
^2*B*b^2*c^5*d*f^5 + 16*A*B^2*a*b*c^5*d*f^5))/(16*a^2*c^6*d^4 + a^4*b^4*f^
4 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b
^2*c*f^4 + 2*a^2*b^6*d*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^3*d*f^3 + 2*b^6*c
^2*d^3*f + 96*a^4*c^4*d^2*f^2 + 54*a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d
^2*f^2 - 20*a*b^4*c^3*d^3*f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 + 64
*a^2*b^2*c^4*d^3*f + 64*a^4*b^2*c^2*d*f^3) - root(2560*a^3*b^2*c^9*d^8*f*z
^4 - 1152*a^2*b^4*c^8*d^8*f*z^4 + 384*a^5*b^8*c*d^3*f^6*z^4 + 384*a*b^8*c^
5*d^7*f^2*z^4 + 288*a^3*b^10*c*d^4*f^5*z^4 + 288*a*b^10*c^3*d^6*f^3*z^4 +
224*a^7*b^6*c*d^2*f^7*z^4 - 192*a^10*b^2*c^2*d*f^8*z^4 + 224*a*b^6*c^7*d^8
*f*z^4 + 80*a*b^12*c*d^5*f^4*z^4 + 48*a^9*b^4*c*d*f^8*z^4 - 33920*a^6*b^2*
c^6*d^5*f^4*z^4 + 27936*a^5*b^4*c^5*d^5*f^4*z^4 + 26112*a^7*b^2*c^5*d^4...
```

3.5. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+fx^2)}\,dx$

3.6 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.6.1 Optimal result

Integrand size = 30, antiderivative size = 331

$$\begin{aligned}
&\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx \\
&= -\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{(bB+2Ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} \\
&\quad-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}} \\
&\quad+\frac{\left(B\sqrt{d}+A\sqrt{f}\right)\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^{3/2}}
\end{aligned}$$

output
```
-1/2*(2*A*c+B*b)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1
/2)-B*(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*
d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))
*(B*d^(1/2)-A*f^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)+1
/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*
x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))*(c*d+a
*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)/d^(1/2)
```

3.6.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.95 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.87

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx =$$

$$-\frac{2B\sqrt{a+x(b+cx)}-\frac{2(bB+2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}+\text{RootSum}\left[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-\right.}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
-1/2*(2*B*Sqrt[a + x*(b + c*x)] - (2*(b*B + 2*A*c)*ArcTanh[(Sqrt[c]*x)/(Sq
rt[a] - Sqrt[a + x*(b + c*x)])])/Sqrt[c] + RootSum[c^2*d - b^2*f + 4*Sqrt[
a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4 & , (-(A*c^2*d*Log[x]) + A*b^
2*f*Log[x] + a*b*B*f*Log[x] - a*A*c*f*Log[x] + A*c^2*d*Log[-Sqrt[a] + Sqrt
[a + b*x + c*x^2] - x*#1] - A*b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1] - a*b*B*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*A*c*f*Lo
g[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*B*c*d*Log[x]*#1 - 2
*Sqrt[a]*A*b*f*Log[x]*#1 - 2*a^(3/2)*B*f*Log[x]*#1 + 2*Sqrt[a]*B*c*d*Log[-
Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[a]*A*b*f*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*a^(3/2)*B*f*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1]*#1 + b*B*d*Log[x]*#1^2 + A*c*d*Log[x]*#1^2 + a*A*f*L
og[x]*#1^2 - b*B*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - A*c
*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*A*f*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1
- d*#1^3) & ])/f
```

3.6.3 Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.367$, Rules used = {1354, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$

$$\Big\downarrow \; 1354$$

$$\frac{\int \frac{(bB+2Ac)fx^2+2(Bcd+Abf+aBf)x+bBd+2aAf}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(bB+2Ac)fx^2+2(Bcd+Abf+aBf)x+bBd+2aAf}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{2f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 2144$$

$$\frac{-\frac{\int -\frac{2f(bBd+Acd+aAf+(Bcd+Abf+aBf)x)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f}-\left((2Ac+bB)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)}{2f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{bBd+Acd+aAf+(Bcd+Abf+aBf)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-(2Ac+bB)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 1092$$

$$\frac{2\int \frac{bBd+Acd+aAf+(Bcd+Abf+aBf)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-2(2Ac+bB)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{2f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{2\int \frac{bBd+Acd+aAf+(Bcd+Abf+aBf)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-\frac{(2Ac+bB)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2f}-\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 1366$$

$$\frac{2\left(\frac{1}{2}\left(\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\left(-\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf\right.}{2f}$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 25$$

3.6. $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

$$\frac{2\left(\frac{1}{2}\left(\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\int\frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(-\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+\cdots\right.\right.}{2f}$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(\frac{\left(\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\int\frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\left(-\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\int\frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}\cdots\right.}{2f}$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 1154$$

$$2\left(\frac{\left(-\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\int\frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\left(\sqrt{f}(a\cdots\right.}{\cdots}\right.$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{2\left(\frac{\left(\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(-\frac{\sqrt{f}(aAf+Acd+bBd)}{\sqrt{d}}+aBf+Abf+Bcd\right)\operatorname{arctanh}\left(\cdots\right.}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}\cdots}}\right.}{2f}$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

3.6. $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

output
```
-((B*Sqrt[a + b*x + c*x^2])/f) + (-(((b*B + 2*A*c)*ArcTanh[(b + 2*c*x)/(2*
Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]) + 2*(-1/2*((B*c*d + A*b*f + a*B*
f - (Sqrt[f]*(b*B*d + A*c*d + a*A*f))/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sq
rt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/(Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) +
 ((B*c*d + A*b*f + a*B*f + (Sqrt[f]*(b*B*d + A*c*d + a*A*f))/Sqrt[d])*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f])))/(2*f)
```

3.6.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1354
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))   Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(
c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q
+ 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(251) = 502$.

Time = 0.82 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{B\sqrt{c\,x^2+bx+a}}{f}-\frac{\frac{Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}+2A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{\left(A\sqrt{df}\,bf-Aa\,f^2-Acdf+B\sqrt{df}\,af+B\sqrt{df}\,cd-Bbc}{}}{}$
default	$\frac{(-Af-B\sqrt{df})\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{(2c\sqrt{df}+bf)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{d}}{}}\right.}{2f\sqrt{c}}\right.}{}$

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-B*(c*x^2+b*x+a)^(1/2)/f-1/2/f*(B*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(
1/2))/c^(1/2)+2*A*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))-(A*(
d*f)^(1/2)*b*f-A*a*f^2-A*c*d*f+B*(d*f)^(1/2)*a*f+B*(d*f)^(1/2)*c*d-B*b*d*f
)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/
2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(A
*(d*f)^(1/2)*b*f+A*a*f^2+A*c*d*f+B*(d*f)^(1/2)*a*f+B*(d*f)^(1/2)*c*d+B*b*d
*f)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f
*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.6.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.6.6 Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{A\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{Bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(A*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(B*x*sqrt(a
 + b*x + c*x**2)/(-d + f*x**2), x)
```

3.6.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.6.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.7 $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.7.1 Optimal result

Integrand size = 30, antiderivative size = 249

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\left(B+\frac{A\sqrt{f}}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B-A*f^(1/2)/d^(1/2))/f^(1
/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/
2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/
2))^(1/2))*(B+A*f^(1/2)/d^(1/2))/f^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.7.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\Big[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2 -f\#1^4\&,\frac{Ab\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-aB\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-2A\sqrt{c}\log}{b\sqrt{c}d-2cd\#1-af\#}$$

input
```
Integrate[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
f*#1^4 & , (A*b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*B*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*A*Sqrt[c]*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1]*#1 + B*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ]
```

3.7.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1366$$

$$\frac{1}{2}\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+ \frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{A\sqrt{f}}{\sqrt{d}}+B\right)\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(B-\frac{A\sqrt{f}}{\sqrt{d}}\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

input

```
Int[(A + B*x)/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output

```
-1/2*((B - (A*Sqrt[f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sq
rt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((B + (A*Sqrt[
f])/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*
x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt
[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

3.7. $\int \frac{A+Bx}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.7.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.7.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(185) = 370$.

Time = 0.79 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.56

method	result
default	$-\frac{\left(-Af-B\sqrt{df}\right)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\,f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*
ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+
2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2
)/f))-1/2*(A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f
)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2))/(x+(d*f)^(1/2)/f))
```

3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6113 vs. $2(185) = 370$.

Time = 34.86 (sec) , antiderivative size = 6113, normalized size of antiderivative = 24.55

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.7.6 Sympy [F]

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{A}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx - \int \frac{Bx}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(A/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x
) - Integral(B*x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2
)), x)
```

3.7.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.7.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

3.7.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{A+B\,x}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.8 $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.8.1 Optimal result

Integrand size = 30, antiderivative size = 381

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(aB(2c^2d-b^2f+2acf)+A(b^3f-bc(cd+3af))+c(Ab^2f+bB(cd-af)-2Ac(cd+af))x)}{(b^2-4ac)(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\left(B\sqrt{d}-A\sqrt{f}\right)\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\left(B\sqrt{d}+A\sqrt{f}\right)\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)-A*f^(1/
2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2
*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/
2)*f^(1/2))^(1/2))*f^(1/2)*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+a*f+b*d^(1/2
)*f^(1/2))^(3/2)-2*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+
c*(A*b^2*f+b*B*(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c
*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.8.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.15 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.98

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{4A(-b^3f+bc(cd+3af)-b^2cfx+2c^2(cd+af)x)+4B(-2a^2cf-bc$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(4*A*(-(b^3*f) + b*c*(c*d + 3*a*f) - b^2*c*f*x + 2*c^2*(c*d + a*f)*x) + 4*
B*(-2*a^2*c*f - b*c^2*d*x + a*(-2*c^2*d + b^2*f + b*c*f*x)) - (b^2 - 4*a*c
)*f*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d
*#1^2 - 4*a*f*#1^2 + d*#1^4 & , (2*b*B*c*d*Log[x] - A*c^2*d*Log[x] - A*b^2
*f*Log[x] + a*b*B*f*Log[x] - a*A*c*f*Log[x] - 2*b*B*c*d*Log[-Sqrt[a] + Sqr
t[a + b*x + c*x^2] - x*#1] + A*c^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2]
- x*#1] + A*b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*b*B*f*L
og[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*A*c*f*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*B*c*d*Log[x]*#1 + 2*Sqrt[a]*A*b*f*Log
[x]*#1 - 2*a^(3/2)*B*f*Log[x]*#1 + 2*Sqrt[a]*B*c*d*Log[-Sqrt[a] + Sqrt[a +
 b*x + c*x^2] - x*#1]*#1 - 2*Sqrt[a]*A*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1]*#1 + 2*a^(3/2)*B*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*
#1]*#1 - b*B*d*Log[x]*#1^2 + A*c*d*Log[x]*#1^2 + a*A*f*Log[x]*#1^2 + b*B*d
*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - A*c*d*Log[-Sqrt[a] +
Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*A*f*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1]*#1^2)/(Sqrt[a]*b*f - c*d*#1 - 2*a*f*#1 + d*#1^3) & ])/(2*(b^
2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*
x)])
```

3.8.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {1351, 27, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d-fx^2)(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1351$$

$$\frac{2\int \frac{(b^2-4ac)f(bBd-A(cd+af)+(Abf-B(cd+af))x)}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{(b^2-4ac)(b^2df-(af+cd)^2)}-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{f\int \frac{bBd-A(cd+af)+(Abf-B(cd+af))x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{b^2df-(af+cd)^2}-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 1366$$

$$\frac{f\left(\frac{1}{2}\left(\frac{\sqrt{f}(bBd-A(af+cd))}{\sqrt{d}}-B(af+cd)+Abf\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int -\frac{}{\sqrt{f}\left(\sqrt{f}x+\cdots\right.}}{2\sqrt{d}}\cdots\right.}{b^2df-(af+cd)^2}$$
$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{f\left(\frac{1}{2}\left(\frac{\sqrt{f}(bBd-A(af+cd))}{\sqrt{d}}-B(af+cd)+Abf\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{}{\sqrt{f}\left(\sqrt{f}x+\cdots\right.}}{2\sqrt{d}}\cdots\right.}{b^2df-(af+cd)^2}$$
$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{f\left(\frac{\left(\frac{\sqrt{f}(bBd-A(af+cd))}{\sqrt{d}}-B(af+cd)+Abf\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}+\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}\sqrt{f}}\right)}{b^2df-(af+cd)^2}-\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 1154$$

3.8. $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

$$\frac{f\left(-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}\sqrt{f}}-\frac{\left(\frac{\sqrt{f}(bBd-A}{\sqrt{d}}\right.}{}\right.}{b^2df-(af+cd)^2}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\Big\downarrow$ 219

$$\frac{f\left(\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\left(\frac{\sqrt{f}(bBd-A(af+cd))}{\sqrt{d}}-B(af+cd)+Abf\right)\text{arctanh}\left(\frac{2a}{2\sqrt{a}}\right.}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right.}{b^2df-(af+cd)^2}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

input

```
Int[(A + B*x)/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output

```
(-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(
A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*(((B*Sqrt[d] - A*Sqrt[f])*(c
*d + b*Sqrt[d]*Sqrt[f] + a*f)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt
[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
c*x^2])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + ((A*b*
f - B*(c*d + a*f) + (Sqrt[f]*(b*B*d - A*(c*d + a*f)))/Sqrt[d])*ArcTanh[(b*
Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[
d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]))/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]
*Sqrt[f] + a*f])))/(b^2*d*f - (c*d + a*f)^2)
```

3.8. $\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.8.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1351
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*S
imp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b
*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*
((g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
 q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1])
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.8.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(311) = 622$.

Time = 0.79 (sec) , antiderivative size = 934, normalized size of antiderivative = 2.45

method	result
default	$\frac{(Af-B\sqrt{df})\left(\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}}-\frac{(-2c\sqrt{df}+bf}{(-b\sqrt{df}+fa+cd)\left(\frac{4c(-b\sqrt{df}+fa+cd)}{f}-\frac{(-2c\sqrt{df}}{f^2}\right.}\right.}{}$

input
```
int((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c
*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+
c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f
)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(
d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)
^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)
))+1/2*(-A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/
2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x
-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2
*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*
d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.8.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.8.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.8.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.8.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.8.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{A+B\,x}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.9 $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

3.9.1 Optimal result

Integrand size = 30, antiderivative size = 797

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx =$$

$$-\frac{2(aB(2c^2d-b^2f+2acf)+A(b^3f-bc(cd+3af))+c(Ab^2f+bB(cd-af)-2Ac(cd+af))\,x)}{3\,(b^2-4ac)\,(b^2df-(cd+af)^2)\,(a+bx+cx^2)^{3/2}}$$

$$-\frac{2(3b^6Bdf^2+24a^2Bc^2f(cd+af)^2-Ab^5f^2(7cd+6af)-b^4Bf(7c^2d^2+14acdf-3a^2f^2)+Ab^3cf(15c^2d^2}{}$$

$$-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{5/2}}$$

$$+\frac{\left(B\sqrt{d}+A\sqrt{f}\right)f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{5/2}}$$

output
```
-2/3*(a*B*(2*a*c*f-b^2*f+2*c^2*d)+A*(b^3*f-b*c*(3*a*f+c*d))+c*(A*b^2*f+b*B
*(-a*f+c*d)-2*A*c*(a*f+c*d))*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+
b*x+a)^(3/2)-1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)
-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(
1/2)-A*f^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(5/2)+1/2*f^(3/2)*arct
anh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1
/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(B*d^(1/2)+A*f^(1/2))/d^(1/2)/(c*d+
a*f+b*d^(1/2)*f^(1/2))^(5/2)-2/3*(3*b^6*B*d*f^2+24*a^2*B*c^2*f*(a*f+c*d)^2
-A*b^5*f^2*(6*a*f+7*c*d)-b^4*B*f*(-3*a^2*f^2+14*a*c*d*f+7*c^2*d^2)+A*b^3*c
*f*(43*a^2*f^2+46*a*c*d*f+15*c^2*d^2)+2*b^2*B*c*(-11*a^3*f^3+4*a^2*c*d*f^2
+5*a*c^2*d^2*f+2*c^3*d^3)-4*A*b*c^2*(17*a^3*f^3+24*a^2*c*d*f^2+9*a*c^2*d^2
*f+2*c^3*d^3)+c*(3*b^5*B*d*f^2-2*A*b^4*f^2*(3*a*f+4*c*d)-8*A*c^2*(a*f+c*d)
^2*(5*a*f+2*c*d)-b^3*B*f*(-3*a^2*f^2+10*a*c*d*f+17*c^2*d^2)+2*A*b^2*c*f*(1
9*a^2*f^2+22*a*c*d*f+15*c^2*d^2)+4*b*B*c*(-5*a^3*f^3+4*a^2*c*d*f^2+11*a*c^
2*d^2*f+2*c^3*d^3))*x)/(-4*a*c+b^2)^2/(c^2*d^2+2*a*c*d*f-f*(-a^2*f+b^2*d))
^2/(c*x^2+b*x+a)^(1/2)
```

3.9.2 Mathematica [A] (verified)

Time = 12.52 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx = \frac{2\left(\frac{4c(-Ab^2f+bB(-cd+af)+2Ac(cd+af))(b+2cx)}{(b^2-4ac)\sqrt{a+x(b+cx)}} - \frac{3f(b^4Bdf+2c(cd+af)^2(-aB+Acx)+b\right.}{}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

output
```
(2*((4*c*(-(A*b^2*f) + b*B*(-(c*d) + a*f) + 2*A*c*(c*d + a*f))*(b + 2*c*x)
)/((b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]) - (3*f*(b^4*B*d*f + 2*c*(c*d + a*f
)^2*(-(a*B) + A*c*x) + b^3*f*(-(A*(c*d + 2*a*f)) + B*c*d*x) + b*c*(c*d + a
*f)*(A*c*d + 5*a*A*f - 3*B*c*d*x + a*B*f*x) - b^2*(B*(c^2*d^2 + 2*a*c*d*f
- a^2*f^2) + 2*a*A*c*f^2*x)))/((c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f)
)*Sqrt[a + x*(b + c*x)]) + (A*(b^3*f - b*c*(c*d + 3*a*f) + b^2*c*f*x - 2*c
^2*(c*d + a*f)*x) + B*(2*a^2*c*f + b*c^2*d*x + a*(2*c^2*d - b^2*f - b*c*f*
x)))/(a + x*(b + c*x))^(3/2) + (3*(b^2 - 4*a*c)*f^(3/2)*(((-(B*Sqrt[d]) +
A*Sqrt[f])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*a*Sqrt[f] + 2*c*S
qrt[d]*x + b*(Sqrt[d] - Sqrt[f]*x))/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] - ((B*Sqrt[d
] + A*Sqrt[f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(-2*(a*Sqrt[f] +
c*Sqrt[d]*x) - b*(Sqrt[d] + Sqrt[f]*x))/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] +
a*f]*Sqrt[a + x*(b + c*x)])])/Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/(4*Sqr
t[d]*(-(b^2*d*f) + (c*d + a*f)^2))))/(3*(b^2 - 4*a*c)*(-(b^2*d*f) + (c*d +
 a*f)^2))
```

3.9.3 Rubi [A] (verified)

Time = 1.76 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1351, 27, 2137, 27, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(d-fx^2)\left(a+bx+cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1351}$$

$$\frac{2\int \frac{3Bdfb^3-Af(7cd+3af)b^2-4Bcd(cd+2af)b+4cf\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x^2+4Ac\left(2c^2d^2+5acfd+3a^2f^2\right)+3\left(b^2-4ac\right)f\left(Abf-B(cd\right.}{2\left(cx^2+bx+a\right)^{3/2}\left(d-fx^2\right)}}{3\left(b^2-4ac\right)\left(b^2df-(af+cd)^2\right)}$$
$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(b^2df-(af+cd)^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3Bdfb^3-Af(7cd+3af)b^2-4Bcd(cd+2af)b+4cf(Afb^2+B(cd-af)b-2Ac(cd+af))x^2+4Ac(2c^2d^2+5acfd+3a^2f^2)+3(b^2-4ac)f(Abf-B(cd+\cdots}{(cx^2+bx+a)^{3/2}(d-fx^2)}}{3(b^2-4ac)(b^2df-(af+cd)^2)}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(b^2df-(af+cd)^2)}$$

$\downarrow$ 2137

$$\frac{2\int \frac{3\left(b^2-4ac\right)^2f^2\left(Adfb^2-2Bd(cd+af)b+A(cd+af)^2-\left(2Abf(cd+af)-B\left(c^2d^2+2acfd+f\left(fa^2+b^2d\right)\right)\right)x\right)}{2\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{(b^2-4ac)(b^2df-(af+cd)^2)}-\frac{2\left(Ab^3cf\left(43a^2f^2+46acdf+15c^2d^2\right)-b^4B\cdots\right.}{\cdots}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(b^2df-(af+cd)^2)}$$

$\downarrow$ 27

$$\frac{3f^2(b^2-4ac)\int \frac{Adfb^2-2Bd(cd+af)b+A(cd+af)^2-\left(2Abf(cd+af)-B\left(c^2d^2+2acfd+f\left(fa^2+b^2d\right)\right)\right)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{b^2df-(af+cd)^2}-\frac{2\left(Ab^3cf\left(43a^2f^2+46acdf+15c^2d^2\right)-b^4Bf\left(\cdots\right.\right.}{\cdots}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(b^2df-(af+cd)^2)}$$

$\downarrow$ 1366

$$\frac{3f^2(b^2-4ac)\left(\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}+\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{b^2df-(af+cd)^2}\cdots$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(b^2df-(af+cd)^2)}$$

$\downarrow$ 25

$$\frac{3f^2(b^2-4ac)\left(\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{b^2df-(af+cd)^2}-\cdots$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3(b^2-4ac)(a+bx+cx^2)^{3/2}(b^2df-(af+cd)^2)}$$

$\downarrow$ 27

3.9. $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

$$\frac{3f^2(b^2-4ac)\left(\frac{\left(A\sqrt{f}+B\sqrt{d}\right)\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int\frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}\sqrt{f}}-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int\frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}\sqrt{f}}\right)}{b^2df-(af+cd)^2} - \frac{2(A}{}$$

$$\frac{2\left(cx\left(-2Ac(af+cd)+bB(cd-af)+Ab^2f\right)-Abc(3af+cd)+aB\left(2acf+b^2(-f)+2c^2d\right)+Ab^3f\right)}{3\left(b^2-4ac\right)\left(a+bx+cx^2\right)^{3/2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 1154

$$\frac{3(b^2-4ac)f^2\left(\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(\sqrt{d}\sqrt{f}b+cd+af\right)^2\int\frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}\sqrt{f}}-\frac{\left(\sqrt{f}A+B\sqrt{\right.}}{}\right.}{b^2df-(cd+af)^2}$$

$$\frac{2\left(Afb^3-Ac(cd+3af)b+aB\left(-fb^2+2c^2d+2acf\right)+c\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x\right)}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(cx^2+bx+a\right)^{3/2}}$$

$\downarrow$ 219

$$\frac{3(b^2-4ac)f^2\left(\frac{\left(\sqrt{f}A+B\sqrt{d}\right)\left(-\sqrt{d}\sqrt{f}b+cd+af\right)^2\text{arctanh}\left(\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{2\sqrt{\sqrt{d}\sqrt{f}b+cd+af}\sqrt{cx^2+bx+a}}\right)}{2\sqrt{d}\sqrt{f}\sqrt{\sqrt{d}\sqrt{f}b+cd+af}}-\frac{\left(B\sqrt{d}-A\sqrt{f}\right)\left(\sqrt{d}\sqrt{f}b+cd+af\right)^2\text{arctanh}\left(\frac{-2\sqrt{f}a+\left(2\right.}{2\sqrt{-\sqrt{d}\sqrt{f}b-}}\right.}{2\sqrt{d}\sqrt{f}\sqrt{-\sqrt{d}\sqrt{f}b+cd+af}}\right.}{b^2df-(cd+af)^2}$$

$$\frac{2\left(Afb^3-Ac(cd+3af)b+aB\left(-fb^2+2c^2d+2acf\right)+c\left(Afb^2+B(cd-af)b-2Ac(cd+af)\right)x\right)}{3\left(b^2-4ac\right)\left(b^2df-(cd+af)^2\right)\left(cx^2+bx+a\right)^{3/2}}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)^(5/2)*(d - f*x^2)),x]
```

3.9. $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

output
```
(-2*(A*b^3*f - A*b*c*(c*d + 3*a*f) + a*B*(2*c^2*d - b^2*f + 2*a*c*f) + c*(
A*b^2*f + b*B*(c*d - a*f) - 2*A*c*(c*d + a*f))*x))/(3*(b^2 - 4*a*c)*(b^2*d
*f - (c*d + a*f)^2)*(a + b*x + c*x^2)^(3/2)) + ((-2*(3*b^6*B*d*f^2 + 24*a^
2*B*c^2*f*(c*d + a*f)^2 - A*b^5*f^2*(7*c*d + 6*a*f) - b^4*B*f*(7*c^2*d^2 +
 14*a*c*d*f - 3*a^2*f^2) + A*b^3*c*f*(15*c^2*d^2 + 46*a*c*d*f + 43*a^2*f^2
) + 2*b^2*B*c*(2*c^3*d^3 + 5*a*c^2*d^2*f + 4*a^2*c*d*f^2 - 11*a^3*f^3) - 4
*A*b*c^2*(2*c^3*d^3 + 9*a*c^2*d^2*f + 24*a^2*c*d*f^2 + 17*a^3*f^3) + c*(3*
b^5*B*d*f^2 - 2*A*b^4*f^2*(4*c*d + 3*a*f) - 8*A*c^2*(c*d + a*f)^2*(2*c*d +
 5*a*f) - b^3*B*f*(17*c^2*d^2 + 10*a*c*d*f - 3*a^2*f^2) + 2*A*b^2*c*f*(15*
c^2*d^2 + 22*a*c*d*f + 19*a^2*f^2) + 4*b*B*c*(2*c^3*d^3 + 11*a*c^2*d^2*f +
 4*a^2*c*d*f^2 - 5*a^3*f^3))*x))/((b^2 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*
Sqrt[a + b*x + c*x^2]) + (3*(b^2 - 4*a*c)*f^2*(-1/2*((B*Sqrt[d] - A*Sqrt[f
])*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 b*x + c*x^2])])/(Sqrt[d]*Sqrt[f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (
(B*Sqrt[d] + A*Sqrt[f])*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^2*ArcTanh[(b*Sqrt[
d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]*Sqrt[f]*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f])))/(b^2*d*f - (c*d + a*f)^2))/(3*(b^2 - 4*a*c)*(b^2*d*f
 - (c*d + a*f)^2))
```

3.9.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1351
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*S
imp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b
*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*
((g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
 q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1])
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2137
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*
c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*((-b)*(c*d + a*f)) + (A
*b - a*B)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)
) - B*(b*c*d + a*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2
- 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)
*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*
(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d +
 a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d
 + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f
*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f
) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; FreeQ[{a, b, c,
 d, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)
^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

3.9. $\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx$

3.9.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. $2(721) = 1442$.

Time = 0.75 (sec) , antiderivative size = 1768, normalized size of antiderivative = 2.22

method	result	size
default	Expression too large to display	1768

input
```
int((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/3/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-
(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2
)+f*a+c*d)/f)^(3/2)-1/2*(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2/3
*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c
*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(3/2)+16/3*c/(4*c*(b*
(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)^2*(2*c*(x-(d*f)^(1/2)/
f)+(2*c*(d*f)^(1/2)+b*f)/f)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f
*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))+1/(b*(d*f)^(1/2)+f*a+
c*d)*f*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/
2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*
f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f
)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^
(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^
(1/2)/f)))+1/2*(A*f-B*(d*f)^(1/2))/(d*f)^(1/2)/f*(1/3*f/(-b*(d*f)^(1/2)+f
*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2...
```

3.9.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.9.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**(5/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.9.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.9.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^(5/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.9.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^{5/2}\,(d-fx^2)}\,dx=\int \frac{A+B\,x}{(d-f\,x^2)\,\left(c\,x^2+b\,x+a\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/((d - f*x^2)*(a + b*x + c*x^2)^(5/2)), x)
```

3.10 $\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx$

3.10.1 Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx = -\frac{1}{2}\arctan\left(\frac{3+x}{2\sqrt{-1+x+x^2}}\right) + \frac{3}{2}\text{arctanh}\left(\frac{1-3x}{2\sqrt{-1+x+x^2}}\right)$$

output
```
-1/2*arctan(1/2*(3+x)/(x^2+x-1)^(1/2))+3/2*arctanh(1/2*(1-3*x)/(x^2+x-1)^(
1/2))
```

3.10.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.79

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}} dx = -\arctan\left(1+x-\sqrt{-1+x+x^2}\right) - 3\text{arctanh}\left(1-x+\sqrt{-1+x+x^2}\right)$$

input
```
Integrate[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

output
```
-ArcTan[1 + x - Sqrt[-1 + x + x^2]] - 3*ArcTanh[1 - x + Sqrt[-1 + x + x^2]
]
```

3.10.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1366, 25, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{(x^2-1)\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{1366}$$

$$\frac{3}{2}\int -\frac{1}{(1-x)\sqrt{x^2+x-1}}dx+\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2+x-1}}dx-\frac{3}{2}\int \frac{1}{(1-x)\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{1154}$$

$$3\int \frac{1}{4-\frac{(1-3x)^2}{x^2+x-1}}d\frac{1-3x}{\sqrt{x^2+x-1}}-\int \frac{1}{-\frac{(x+3)^2}{x^2+x-1}-4}d\left(-\frac{x+3}{\sqrt{x^2+x-1}}\right)$$

$$\Big\downarrow \text{217}$$

$$3\int \frac{1}{4-\frac{(1-3x)^2}{x^2+x-1}}d\frac{1-3x}{\sqrt{x^2+x-1}}-\frac{1}{2}\arctan\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{3}{2}\text{arctanh}\left(\frac{1-3x}{2\sqrt{x^2+x-1}}\right)-\frac{1}{2}\arctan\left(\frac{x+3}{2\sqrt{x^2+x-1}}\right)$$

input
```
Int[(1 + 2*x)/((-1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

output
```
-1/2*ArcTan[(3 + x)/(2*Sqrt[-1 + x + x^2])] + (3*ArcTanh[(1 - 3*x)/(2*Sqrt
[-1 + x + x^2])])/2
```

3.10.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.10.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{3\,\mathrm{arctanh}\left(\frac{-1+3x}{2\sqrt{(-1+x)^2-2+3x}}\right)}{2}+\frac{\arctan\left(\frac{-3-x}{2\sqrt{(1+x)^2-2-x}}\right)}{2}$	46
trager	$-\frac{3\ln\left(-\frac{2\sqrt{x^2+x-1}-1+3x}{-1+x}\right)}{2}-\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{-\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{x^2+x-1}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)}{1+x}\right)}{2}$	70

input
```
int(1/(x^2-1)/(x^2+x-1)^(1/2)*(1+2*x),x,method=_RETURNVERBOSE)
```

output
```
-3/2*arctanh(1/2*(-1+3*x)/((-1+x)^2-2+3*x)^(1/2))+1/2*arctan(1/2*(-3-x)/((
1+x)^2-2-x)^(1/2))
```

3.10.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}+2\right) + \frac{3}{2}\log\left(-x+\sqrt{x^2+x-1}\right)$$

input
```
integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

output
```
arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(-x + sqrt(x^2 + x - 1) + 2) +
 3/2*log(-x + sqrt(x^2 + x - 1))
```

3.10.6 Sympy [F]

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2x+1}{(x-1)(x+1)\sqrt{x^2+x-1}}\,dx$$

input
```
integrate((1+2*x)/(x**2-1)/(x**2+x-1)**(1/2),x)
```

output
```
Integral((2*x + 1)/((x - 1)*(x + 1)*sqrt(x**2 + x - 1)), x)
```

3.10.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = -\frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5\,|2\,x+2|}+\frac{6\sqrt{5}}{5\,|2\,x+2|}\right) - \frac{3}{2}\log\left(\frac{2\sqrt{x^2+x-1}}{|2\,x-2|}+\frac{2}{|2\,x-2|}+\frac{3}{2}\right)$$

input
```
integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(2/5*sqrt(5)*x/abs(2*x + 2) + 6/5*sqrt(5)/abs(2*x + 2)) - 3/2*l
og(2*sqrt(x^2 + x - 1)/abs(2*x - 2) + 2/abs(2*x - 2) + 3/2)
```

3.10.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.02

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \arctan\left(-x+\sqrt{x^2+x-1}-1\right) - \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}+2\right|\right) + \frac{3}{2}\log\left(\left|-x+\sqrt{x^2+x-1}\right|\right)$$

input
```
integrate((1+2*x)/(x^2-1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

output
```
arctan(-x + sqrt(x^2 + x - 1) - 1) - 3/2*log(abs(-x + sqrt(x^2 + x - 1) +
2)) + 3/2*log(abs(-x + sqrt(x^2 + x - 1)))
```

3.10.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(-1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{(x^2-1)\,\sqrt{x^2+x-1}}\,dx$$

input
```
int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)),x)
```

output
```
int((2*x + 1)/((x^2 - 1)*(x + x^2 - 1)^(1/2)), x)
```

3.11 $\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$

3.11.1 Optimal result

Integrand size = 23, antiderivative size = 117

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = -\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\arctan\left(\frac{5+2\sqrt{5}-\sqrt{5}x}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right) + \sqrt{\frac{1}{2}\left(-2+\sqrt{5}\right)}\operatorname{arctanh}\left(\frac{5-2\sqrt{5}+\sqrt{5}x}{\sqrt{10\left(-2+\sqrt{5}\right)}\sqrt{-1+x+x^2}}\right)$$

output
```
1/2*arctanh((5-2*5^(1/2)+x*5^(1/2))/(x^2+x-1)^(1/2)/(-20+10*5^(1/2))^(1/2)
)*(-4+2*5^(1/2))^(1/2)-1/2*arctan((5+2*5^(1/2)-x*5^(1/2))/(x^2+x-1)^(1/2)/
(20+10*5^(1/2))^(1/2))*(4+2*5^(1/2))^(1/2)
```

3.11.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.13 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.91

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[2-4\#1+6\#1^2 +\#1^4\&, \frac{3\log\left(-x+\sqrt{-1+x+x^2}-\#1\right)-2\log\left(-x+\sqrt{-1+x+x^2}-\#1\right)\#1+2\log\left(-x+\sqrt{-1\right.}}{-1+3\#1+\#1^3}$$

input
```
Integrate[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

output
```
RootSum[2 - 4*#1 + 6*#1^2 + #1^4 & , (3*Log[-x + Sqrt[-1 + x + x^2] - #1]
- 2*Log[-x + Sqrt[-1 + x + x^2] - #1]*#1 + 2*Log[-x + Sqrt[-1 + x + x^2] -
 #1]*#1^2)/(-1 + 3*#1 + #1^3) & ]/2
```

3.11.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1369, 25, 1363, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}}dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{\int \frac{\sqrt{5}-\left(5-2\sqrt{5}\right)x}{(x^2+1)\sqrt{x^2+x-1}}dx}{2\sqrt{5}}-\frac{\int -\frac{\left(5+2\sqrt{5}\right)x+\sqrt{5}}{(x^2+1)\sqrt{x^2+x-1}}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\sqrt{5}-\left(5-2\sqrt{5}\right)x}{(x^2+1)\sqrt{x^2+x-1}}dx}{2\sqrt{5}}+\frac{\int \frac{\left(5+2\sqrt{5}\right)x+\sqrt{5}}{(x^2+1)\sqrt{x^2+x-1}}dx}{2\sqrt{5}}$$

$$\Big\downarrow \text{1363}$$

$$-\sqrt{5}\left(2+\sqrt{5}\right)\int \frac{1}{\frac{\left(-\sqrt{5}x+2\sqrt{5}+5\right)^2}{x^2+x-1}+10\left(2+\sqrt{5}\right)}d\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{x^2+x-1}}-\sqrt{5}\left(2-\sqrt{5}\right)\int \frac{1}{\frac{\left(\sqrt{5}x-2\sqrt{5}+5\right)^2}{x^2+x-1}+10\left(2-\sqrt{5}\right)}d\left(-\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{x^2+x-1}}\right)$$

$$\Big\downarrow \text{216}$$

$$-\sqrt{5}\left(2-\sqrt{5}\right)\int \frac{1}{\frac{\left(\sqrt{5}x-2\sqrt{5}+5\right)^2}{x^2+x-1}+10\left(2-\sqrt{5}\right)}d\left(-\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{x^2+x-1}}\right)-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\arctan\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)$$

$$\Big\downarrow \text{220}$$

$$-\sqrt{\frac{1}{2}\left(2+\sqrt{5}\right)}\arctan\left(\frac{-\sqrt{5}x+2\sqrt{5}+5}{\sqrt{10\left(2+\sqrt{5}\right)}\sqrt{x^2+x-1}}\right)-\frac{\left(2-\sqrt{5}\right)\text{arctanh}\left(\frac{\sqrt{5}x-2\sqrt{5}+5}{\sqrt{10\left(\sqrt{5}-2\right)}\sqrt{x^2+x-1}}\right)}{\sqrt{2\left(\sqrt{5}-2\right)}}$$

input
```
Int[(1 + 2*x)/((1 + x^2)*Sqrt[-1 + x + x^2]),x]
```

output
```
-(Sqrt[(2 + Sqrt[5])/2]*ArcTan[(5 + 2*Sqrt[5] - Sqrt[5]*x)/(Sqrt[10*(2 + S
qrt[5])]*Sqrt[-1 + x + x^2])]) - ((2 - Sqrt[5])*ArcTanh[(5 - 2*Sqrt[5] + S
qrt[5]*x)/(Sqrt[10*(-2 + Sqrt[5])]*Sqrt[-1 + x + x^2])])/Sqrt[2*(-2 + Sqrt
[5])]
```

3.11.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

3.11.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.08 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.10

method	result
trager	$\operatorname{RootOf}\left(\operatorname{RootOf}\left(16\_Z^4+16\_Z^2+5\right)^2+\_Z^2+1\right)\ln\left(-\frac{4\operatorname{RootOf}\left(\operatorname{RootOf}\left(16\_Z^4+16\_Z^2+5\right)^2+\_Z\right.}{}\right.$
default	$\frac{\sqrt{\frac{10\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}-\frac{5\sqrt{5}\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}+10+5\sqrt{5}}\,\sqrt{5}\left(\arctan\left(\frac{\sqrt{5}\sqrt{\left(-2+\sqrt{5}\right)\left(-\frac{\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}+4\sqrt{5}+9\right)}\sqrt{20+10\sqrt{5}}\left(\frac{\sqrt{5}\left(-\sqrt{5}-2+x\right)^2}{\left(-\sqrt{5}+2-x\right)^2}\right.}{5\left(-\sqrt{5}+2-x\right)\left(\frac{\left(-\sqrt{5}-2+x\right)^4}{\left(-\sqrt{5}+2-x\right)^4}-\frac{18\left(-\sqrt{}\right.}{\left(-\sqrt{}\right.}\right.}\right.\right.}{}$

input
```
int((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
RootOf(RootOf(16*_Z^4+16*_Z^2+5)^2+_Z^2+1)*ln(-(4*RootOf(RootOf(16*_Z^4+16
*_Z^2+5)^2+_Z^2+1)*RootOf(16*_Z^4+16*_Z^2+5)^2*x+2*RootOf(RootOf(16*_Z^4+1
6*_Z^2+5)^2+_Z^2+1)*x+(x^2+x-1)^(1/2)+RootOf(RootOf(16*_Z^4+16*_Z^2+5)^2+_
Z^2+1))/(4*x*RootOf(16*_Z^4+16*_Z^2+5)^2+2*x-1))-RootOf(16*_Z^4+16*_Z^2+5)
*ln(-(4*RootOf(16*_Z^4+16*_Z^2+5)^3*x+2*RootOf(16*_Z^4+16*_Z^2+5)*x+(x^2+x
-1)^(1/2)-RootOf(16*_Z^4+16*_Z^2+5))/(4*x*RootOf(16*_Z^4+16*_Z^2+5)^2+2*x+
1))
```

3.11.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \frac{1}{2}\sqrt{i-2}\log\left(-x+\sqrt{i-2}+\sqrt{x^2+x-1}+i\right) - \frac{1}{2}\sqrt{i-2}\log\left(-x-\sqrt{i-2}+\sqrt{x^2+x-1}+i\right) + \frac{1}{2}\sqrt{-i-2}\log\left(-x+\sqrt{-i-2}+\sqrt{x^2+x-1}-i\right) - \frac{1}{2}\sqrt{-i-2}\log\left(-x-\sqrt{-i-2}+\sqrt{x^2+x-1}-i\right)$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(I - 2)*log(-x + sqrt(I - 2) + sqrt(x^2 + x - 1) + I) - 1/2*sqrt(I
 - 2)*log(-x - sqrt(I - 2) + sqrt(x^2 + x - 1) + I) + 1/2*sqrt(-I - 2)*log
(-x + sqrt(-I - 2) + sqrt(x^2 + x - 1) - I) - 1/2*sqrt(-I - 2)*log(-x - sq
rt(-I - 2) + sqrt(x^2 + x - 1) - I)
```

3.11.6 Sympy [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2x+1}{(x^2+1)\sqrt{x^2+x-1}}\,dx$$

input
```
integrate((1+2*x)/(x**2+1)/(x**2+x-1)**(1/2),x)
```

output
```
Integral((2*x + 1)/((x**2 + 1)*sqrt(x**2 + x - 1)), x)
```

3.11.7 Maxima [F]

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+x-1}(x^2+1)}\,dx$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 1)/(sqrt(x^2 + x - 1)*(x^2 + 1)), x)
```

3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(86) = 172.

Time = 0.34 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.91

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx$$
$$= \frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33\,x+5\sqrt{5}-33\sqrt{x^2+x-1}+2\sqrt{5\sqrt{5}+11}+11\right.\right.$$
$$+16\left(5\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+11\,x-5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}-11\sqrt{5\sqrt{5}+11}\right.$$
$$-\frac{1}{4}\sqrt{2\sqrt{5}-4}\log\left(16\left(15\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+33\,x+5\sqrt{5}-33\sqrt{x^2+x-1}-2\sqrt{5\sqrt{5}+11}+\right.\right.$$
$$+16\left(5\sqrt{5}\left(x-\sqrt{x^2+x-1}\right)+11\,x+5\sqrt{5}\sqrt{5\sqrt{5}+11}-15\sqrt{5}-11\sqrt{x^2+x-1}+11\sqrt{5\sqrt{5}+11}\right.$$
$$+\frac{\sqrt{2\sqrt{5}-4}\left(\arctan(3)+\arctan\left(\frac{1}{10}\left(x-\sqrt{x^2+x-1}\right)\left(\sqrt{5}\sqrt{5\sqrt{5}+11}+4\sqrt{5}-5\sqrt{5\sqrt{5}+11}\right)-\right.\right.}{2\left(\sqrt{5}-2\right)}$$
$$-\frac{\sqrt{2\sqrt{5}-4}\left(\arctan(3)+\arctan\left(-\frac{1}{10}\left(x-\sqrt{x^2+x-1}\right)\left(\sqrt{5}\sqrt{5\sqrt{5}+11}-4\sqrt{5}-5\sqrt{5\sqrt{5}+11}\right)+\right.\right.}{2\left(\sqrt{5}-2\right)}$$

input
```
integrate((1+2*x)/(x^2+1)/(x^2+x-1)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x
+ 5*sqrt(5) - 33*sqrt(x^2 + x - 1) + 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(
5*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 11*x - 5*sqrt(5)*sqrt(5*sqrt(5) + 11)
- 15*sqrt(5) - 11*sqrt(x^2 + x - 1) - 11*sqrt(5*sqrt(5) + 11) - 33)^2) - 1
/4*sqrt(2*sqrt(5) - 4)*log(16*(15*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 33*x +
 5*sqrt(5) - 33*sqrt(x^2 + x - 1) - 2*sqrt(5*sqrt(5) + 11) + 11)^2 + 16*(5
*sqrt(5)*(x - sqrt(x^2 + x - 1)) + 11*x + 5*sqrt(5)*sqrt(5*sqrt(5) + 11) -
 15*sqrt(5) - 11*sqrt(x^2 + x - 1) + 11*sqrt(5*sqrt(5) + 11) - 33)^2) + 1/
2*sqrt(2*sqrt(5) - 4)*(arctan(3) + arctan(1/10*(x - sqrt(x^2 + x - 1))*(sq
rt(5)*sqrt(5*sqrt(5) + 11) + 4*sqrt(5) - 5*sqrt(5*sqrt(5) + 11)) - 7/10*sq
rt(5)*sqrt(5*sqrt(5) + 11) + 1/5*sqrt(5) + 3/2*sqrt(5*sqrt(5) + 11)))/(sqr
t(5) - 2) - 1/2*sqrt(2*sqrt(5) - 4)*(arctan(3) + arctan(-1/10*(x - sqrt(x^
2 + x - 1))*(sqrt(5)*sqrt(5*sqrt(5) + 11) - 4*sqrt(5) - 5*sqrt(5*sqrt(5) +
 11)) + 7/10*sqrt(5)*sqrt(5*sqrt(5) + 11) + 1/5*sqrt(5) - 3/2*sqrt(5*sqrt(
5) + 11)))/(sqrt(5) - 2)
```

3.11.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1+2x}{(1+x^2)\sqrt{-1+x+x^2}}\,dx = \int \frac{2\,x+1}{(x^2+1)\,\sqrt{x^2+x-1}}\,dx$$

input
```
int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)),x)
```

output
```
int((2*x + 1)/((x^2 + 1)*(x + x^2 - 1)^(1/2)), x)
```

3.12 $\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx$

3.12.1 Optimal result

Integrand size = 30, antiderivative size = 484

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx =$$

$$-\frac{\sqrt{a^2+b^2+c\left(c-\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c-\sqrt{a^2+b^2-2ac+c^2}\right)}\arctan\left(\frac{b\sqrt{a^2\cdots}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

$$-\frac{\sqrt{a^2+b^2+c\left(c+\sqrt{a^2+b^2-2ac+c^2}\right)-a\left(2c+\sqrt{a^2+b^2-2ac+c^2}\right)}\operatorname{arctanh}\left(\frac{b\sqrt{a^2\cdots}}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+\cdots}}\right.}{\sqrt{2}\sqrt[4]{a^2+b^2-2ac+c^2}}$$

output
```
-1/2*arctan(1/2*(b*(a^2-2*a*c+b^2+c^2)^(1/2)-x*(b^2+(a-c)*(a-c+(a^2-2*a*c+
b^2+c^2)^(1/2))))/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a
^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)^(1/2)))^
(1/2))*(a^2+b^2+c*(c-(a^2-2*a*c+b^2+c^2)^(1/2))-a*(2*c-(a^2-2*a*c+b^2+c^2)
^(1/2)))^(1/2)/(a^2-2*a*c+b^2+c^2)^(1/4)*2^(1/2)-1/2*arctanh(1/2*(x*(b^2+(
a-c)*(a-c-(a^2-2*a*c+b^2+c^2)^(1/2)))+b*(a^2-2*a*c+b^2+c^2)^(1/2))/(a^2-2*
a*c+b^2+c^2)^(1/4)*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(a^2+b^2+c*(c+(a^2-2*a*c+b^
2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2))*(a^2+b^2+c*(c+(a^2
-2*a*c+b^2+c^2)^(1/2))-a*(2*c+(a^2-2*a*c+b^2+c^2)^(1/2)))^(1/2)/(a^2-2*a*c
+b^2+c^2)^(1/4)*2^(1/2)
```

3.12.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.43

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \frac{1}{2}\text{RootSum}\Big[a^2+b^2-4b\sqrt{c}\#1-2a\#1^2+4c\#1^2$$
$$+\#1^4\&, \frac{bc\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+2a\sqrt{c}\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1-2c^{3/2}\text{lo}}{b\sqrt{c}+a\#1-2c\#1}$$

input
```
Integrate[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

output
```
RootSum[a^2 + b^2 - 4*b*Sqrt[c]*#1 - 2*a*#1^2 + 4*c*#1^2 + #1^4 & , (b*c*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*a*Sqrt[c]*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/2)*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1 - b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
*#1^2)/(b*Sqrt[c] + a*#1 - 2*c*#1 - #1^3) & ]/2
```

3.12.3 Rubi [A] (verified)

Time = 28.43 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1369, 25, 1363, 218, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx-c}{(x^2+1)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{\int -\frac{b^2-\sqrt{a^2-2ca+b^2+c^2}xb+(a-c)\left(a-c-\sqrt{a^2-2ca+b^2+c^2}\right)}{(x^2+1)\sqrt{cx^2+bx+a}}dx}{2\sqrt{a^2-2ac+b^2+c^2}}-\frac{\int -\frac{b^2+\sqrt{a^2-2ca+b^2+c^2}xb+(a-c)\left(a-c+\sqrt{a^2-2ca+b^2+c^2}\right)}{(x^2+1)\sqrt{cx^2+bx+a}}dx}{2\sqrt{a^2-2ac+b^2+c^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2+\sqrt{a^2-2ca+b^2+c^2}xb+(a-c)\left(a-c+\sqrt{a^2-2ca+b^2+c^2}\right)}{(x^2+1)\sqrt{cx^2+bx+a}}dx}{2\sqrt{a^2-2ac+b^2+c^2}} - \frac{\int \frac{b^2-\sqrt{a^2-2ca+b^2+c^2}xb+(a-c)\left(a-c-\sqrt{a^2-2ca+b^2+c^2}\right)}{(x^2+1)\sqrt{cx^2+bx+a}}dx}{2\sqrt{a^2-2ac+b^2+c^2}}$$

$\downarrow$ 1363

$$-b\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\int \frac{}{\frac{b\left(\sqrt{a^2-2ca+b^2+c^2}b+\left(b^2+(a-c)\left(a-c-\sqrt{a^2-2ca+b^2+c^2}\right)\right)x\right)^2}{cx^2+bx+a}-2b\sqrt{}}$$

$$b\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\int \frac{1}{\frac{b\left(b\sqrt{a^2-2ca+b^2+c^2}-\left(b^2+(a-c)\left(a-c+\sqrt{a^2-2ca+b^2+c^2}\right)\right)x\right)^2}{cx^2+bx+a}+2b\sqrt{a^2-}}$$

$\downarrow$ 218

$$-b\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\int \frac{}{\frac{b\left(\sqrt{a^2-2ca+b^2+c^2}b+\left(b^2+(a-c)\left(a-c-\sqrt{a^2-2ca+b^2+c^2}\right)\right)x\right)^2}{cx^2+bx+a}-2b\sqrt{}}$$

$$\frac{\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\arctan\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}+a\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)+}}$$

$\downarrow$ 221

$$-\frac{\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\arctan\left(\frac{b\sqrt{a^2-2ac+b^2+c^2}-x\left((a-c)\left(\sqrt{a^2-2ac+b^2+c^2}\right.\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(2c-\sqrt{a^2-2ac+b^2+c^2}\right)+c\left(c-\sqrt{a^2-2ac+b^2+c^2}\right)-}}$$

$$\frac{\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)\text{arctanh}\left(\frac{x\left((a-c)\left(-\sqrt{a^2-2ac+b^2+c^2}+a-c\right)+b^2\right)+b\sqrt{a}}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-}\right.}}\right.}{\sqrt{2}\sqrt[4]{a^2-2ac+b^2+c^2}\sqrt{-a\left(\sqrt{a^2-2ac+b^2+c^2}+2c\right)+c\left(\sqrt{a^2-2ac+b^2+c^2}+c\right)+}}$$

input
```
Int[(a - c + b*x)/((1 + x^2)*Sqrt[a + b*x + c*x^2]),x]
```

3.12. $\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx$

output
```
-(((b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c + c^2]))*ArcTan[(b*Sqrt[
a^2 + b^2 - 2*a*c + c^2] - (b^2 + (a - c)*(a - c + Sqrt[a^2 + b^2 - 2*a*c
+ c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c
 - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^2 - 2*a*c + c^2]
)]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)*Sqrt[
a^2 + b^2 + c*(c - Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c - Sqrt[a^2 + b^
2 - 2*a*c + c^2])])) - ((b^2 + (a - c)*(a - c - Sqrt[a^2 + b^2 - 2*a*c + c
^2]))*ArcTanh[(b*Sqrt[a^2 + b^2 - 2*a*c + c^2] + (b^2 + (a - c)*(a - c - S
qrt[a^2 + b^2 - 2*a*c + c^2]))*x)/(Sqrt[2]*(a^2 + b^2 - 2*a*c + c^2)^(1/4)
*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) - a*(2*c + Sqrt[a^
2 + b^2 - 2*a*c + c^2])]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*(a^2 + b^2 - 2*
a*c + c^2)^(1/4)*Sqrt[a^2 + b^2 + c*(c + Sqrt[a^2 + b^2 - 2*a*c + c^2]) -
a*(2*c + Sqrt[a^2 + b^2 - 2*a*c + c^2])])
```

3.12.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

3.12.4 Maple [B] (warning: unable to verify)

result has leaf size over 500,000. Avoiding possible recursion issues.

Time = 116.44 (sec) , antiderivative size = 6870946, normalized size of antiderivative = 14196.17

method	result	size
default	Expression too large to display	6870946

input
```
int((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

3.12.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.63

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = -\frac{1}{4}\sqrt{-a+c+\sqrt{-b^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}\sqrt{-a+c+\sqrt{-b^2}}b+b^2+\sqrt{-b^2}(bx+2\,a)}{x}\right) + \frac{1}{4}\sqrt{-a+c+\sqrt{-b^2}}\log\left(\frac{2\,bcx-2\,\sqrt{cx^2+bx+a}\sqrt{-a+c+\sqrt{-b^2}}b+b^2+\sqrt{-b^2}(bx+2\,a)}{x}\right) - \frac{1}{4}\sqrt{-a+c-\sqrt{-b^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}\sqrt{-a+c-\sqrt{-b^2}}b+b^2-\sqrt{-b^2}(bx+2\,a)}{x}\right) + \frac{1}{4}\sqrt{-a+c-\sqrt{-b^2}}\log\left(\frac{2\,bcx-2\,\sqrt{cx^2+bx+a}\sqrt{-a+c-\sqrt{-b^2}}b+b^2-\sqrt{-b^2}(bx+2\,a)}{x}\right)$$

input
```
integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-a + c + sqrt(-b^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*sqrt
(-a + c + sqrt(-b^2))*b + b^2 + sqrt(-b^2)*(b*x + 2*a))/x) + 1/4*sqrt(-a +
 c + sqrt(-b^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*sqrt(-a + c + sqrt
(-b^2))*b + b^2 + sqrt(-b^2)*(b*x + 2*a))/x) - 1/4*sqrt(-a + c - sqrt(-b^2
))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*sqrt(-a + c - sqrt(-b^2))*b + b^
2 - sqrt(-b^2)*(b*x + 2*a))/x) + 1/4*sqrt(-a + c - sqrt(-b^2))*log((2*b*c*
x - 2*sqrt(c*x^2 + b*x + a)*sqrt(-a + c - sqrt(-b^2))*b + b^2 - sqrt(-b^2)
*(b*x + 2*a))/x)
```

3.12.6 Sympy [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx=\int \frac{a+bx-c}{(x^2+1)\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate((b*x+a-c)/(x**2+1)/(c*x**2+b*x+a)**(1/2),x)
```

output
```
Integral((a + b*x - c)/((x**2 + 1)*sqrt(a + b*x + c*x**2)), x)
```

3.12.7 Maxima [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx=\int \frac{bx+a-c}{\sqrt{cx^2+bx+a}(x^2+1)}\,dx$$

input
```
integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

3.12.8 Giac [F]

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{bx+a-c}{\sqrt{cx^2+bx+a}(x^2+1)}\,dx$$

input
```
integrate((b*x+a-c)/(x^2+1)/(c*x^2+b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate((b*x + a - c)/(sqrt(c*x^2 + b*x + a)*(x^2 + 1)), x)
```

3.12.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a-c+bx}{(1+x^2)\sqrt{a+bx+cx^2}}\,dx = \int \frac{a-c+b\,x}{(x^2+1)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int((a - c + b*x)/((x^2 + 1)*(a + b*x + c*x^2)^(1/2)), x)
```

3.13 $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

3.13.1 Optimal result

Integrand size = 28, antiderivative size = 184

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = -\frac{(Bce-bBf-Acf)x}{f^2}+\frac{Bcx^2}{2f} - \frac{\left(Af\left(ce^2-2cdf-bef+2af^2\right)+B\left(f\left(be^2-2bdf-aef\right)-c\left(e^3-3def\right)\right)\right)\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{f^3\sqrt{e^2-4df}} - \frac{\left(Af(ce-bf)-B\left(ce^2-cdf-bef+af^2\right)\right)\log\left(d+ex+fx^2\right)}{2f^3}$$

output
```
-(-A*c*f-B*b*f+B*c*e)*x/f^2+1/2*B*c*x^2/f-1/2*(A*f*(-b*f+c*e)-B*(a*f^2-b*e
*f-c*d*f+c*e^2))*ln(f*x^2+e*x+d)/f^3-(A*f*(2*a*f^2-b*e*f-2*c*d*f+c*e^2)+B*
(f*(-a*e*f-2*b*d*f+b*e^2)-c*(-3*d*e*f+e^3)))*arctanh((2*f*x+e)/(-4*d*f+e^2
)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)
```

3.13.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = \frac{2f(-Bce+bBf+Acf)x+Bcf^2x^2-\frac{2\left(Bf\left(-be^2+2bdf+aef\right)+Bc\left(e^3-3def\right)+Af\left(-ce^2+2cdf+bef-2af^2\right)\right)\arctan\left(\frac{e+2fx}{\sqrt{-e^2+4d\ldots}}\right.}{\sqrt{-e^2+4df}}}{2f^3}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

output
```
(2*f*(-(B*c*e) + b*B*f + A*c*f)*x + B*c*f^2*x^2 - (2*(B*f*(-(b*e^2) + 2*b*
d*f + a*e*f) + B*c*(e^3 - 3*d*e*f) + A*f*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a
*f^2))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*f*(
-(b*e) + a*f) + A*f*(-(c*e) + b*f) + B*c*(e^2 - d*f))*Log[d + x*(e + f*x)]
)/(2*f^3)
```

3.13.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}dx$$

$$\Big\downarrow \text{2159}$$

$$\int \left(\frac{-x\left(Bf(be-af)+Af(ce-bf)-Bc(e^2-df)\right)-Af(cd-af)+Bd(ce-bf)}{f^2(d+ex+fx^2)}-\frac{-Acf-bBf+Bce}{f^2}+\frac{Bc}{f}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af(2af^2-bef-2cdf+ce^2)+Bf(-aef-2bdf+be^2)-Bc(e^3-3def)\right)}{f^3\sqrt{e^2-4df}}-\frac{\log(d+ex+fx^2)\left(Af(ce-bf)-B(af^2-bef-cdf+ce^2)\right)}{2f^3}-\frac{x(-Acf-bBf+Bce)}{f^2}+\frac{Bcx^2}{2f}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x]
```

output
```
-(((B*c*e - b*B*f - A*c*f)*x)/f^2) + (B*c*x^2)/(2*f) - ((B*f*(b*e^2 - 2*b*
d*f - a*e*f) - B*c*(e^3 - 3*d*e*f) + A*f*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^
2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^3*Sqrt[e^2 - 4*d*f]) - ((A*
f*(c*e - b*f) - B*(c*e^2 - c*d*f - b*e*f + a*f^2))*Log[d + e*x + f*x^2])/(
2*f^3)
```

3.13.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2159
```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

3.13.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.03

method	result
default	$\frac{\frac{1}{2}Bc\,x^2 f+Acfx+Bbfx-Bcex}{f^2}+\frac{\frac{\left(Ab\,f^2-Acef+Ba\,f^2-Bbef-Bcdf+Bc\,e^2\right)\ln\left(f\,x^2+ex+d\right)}{2f}+\frac{2\left(Aa\,f^2-Acdf-Bbdf+Bcde-\frac{\left(Ab\,f^2-Ace\right.}{}\right.}{}}{f^2}$
risch	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/f^2*(1/2*B*c*x^2*f+A*c*f*x+B*b*f*x-B*c*e*x)+1/f^2*(1/2*(A*b*f^2-A*c*e*f+
B*a*f^2-B*b*e*f-B*c*d*f+B*c*e^2)/f*ln(f*x^2+e*x+d)+2*(A*a*f^2-A*c*d*f-B*b*
d*f+B*c*d*e-1/2*(A*b*f^2-A*c*e*f+B*a*f^2-B*b*e*f-B*c*d*f+B*c*e^2)*e/f)/(4*
d*f-e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)))
```

3.13.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 583, normalized size of antiderivative = 3.17

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx = \left[\frac{(Bce^2f^2-4\,Bcdf^3)x^2-(Bce^3-2\,Aaf^3+(2\,(Bb+Ac)d+(Ba+Ab)e)f^2-(3\,Bcde+(Bb+Ac)e^2)}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/2*((B*c*e^2*f^2 - 4*B*c*d*f^3)*x^2 - (B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A
*c)*d + (B*a + A*b)*e)*f^2 - (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(e^2 - 4
*d*f)*log((2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x +
e))/(f*x^2 + e*x + d)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (4*B*c*d*e +
 (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*B*c*d^2 + 4
*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b + A*c)*e^3)*
f)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4), 1/2*((B*c*e^2*f^2 - 4*B*c*d*
f^3)*x^2 + 2*(B*c*e^3 - 2*A*a*f^3 + (2*(B*b + A*c)*d + (B*a + A*b)*e)*f^2
- (3*B*c*d*e + (B*b + A*c)*e^2)*f)*sqrt(-e^2 + 4*d*f)*arctan(-sqrt(-e^2 +
4*d*f)*(2*f*x + e)/(e^2 - 4*d*f)) - 2*(B*c*e^3*f + 4*(B*b + A*c)*d*f^3 - (
4*B*c*d*e + (B*b + A*c)*e^2)*f^2)*x + (B*c*e^4 - 4*(B*a + A*b)*d*f^3 + (4*
B*c*d^2 + 4*(B*b + A*c)*d*e + (B*a + A*b)*e^2)*f^2 - (5*B*c*d*e^2 + (B*b +
 A*c)*e^3)*f)*log(f*x^2 + e*x + d))/(e^2*f^3 - 4*d*f^4)]
```

3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1260 vs. $2(175) = 350$.

Time = 6.33 (sec) , antiderivative size = 1260, normalized size of antiderivative = 6.85

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{d+ex+fx^2}\,dx = \frac{Bcx^2}{2f} + x\left(\frac{Ac}{f} + \frac{Bb}{f} - \frac{Bce}{f^2}\right)$$
$$+\left(-\frac{\sqrt{-4df+e^2}(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3)}{2f^3\cdot(4df-e^2)}\right.$$
$$\left.+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)\log\left(x+\frac{-Aaef^2+2Abdf^2-Acdef+2Badf^2-Bb\ldots}{\ldots}\right.$$
$$+\left(\frac{\sqrt{-4df+e^2}(-2Aaf^3+Abef^2+2Acdf^2-Ace^2f+Baef^2+2Bbdf^2-Bbe^2f-3Bcdef+Bce^3)}{2f^3\cdot(4df-e^2)}\right.$$
$$\left.+\frac{Abf^2-Acef+Baf^2-Bbef-Bcdf+Bce^2}{2f^3}\right)\log\left(x+\frac{-Aaef^2+2Abdf^2-Acdef+2Badf^2-Bb\ldots}{\ldots}\right.$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

output
```
B*c*x**2/(2*f) + x*(A*c/f + B*b/f - B*c*e/f**2) + (-sqrt(-4*d*f + e**2)*(-
2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d
*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*
b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3))*log(
x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 - B*b*d*e*f - 2
*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(-sqrt(-4*d*f + e**2)*(-2*A*a*f**3 + A
*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b*d*f**2 - B*b*e*
*2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f - e**2)) + (A*b*f**2 - A*c*e
*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/(2*f**3)) + e**2*f**2*(-sqrt
(-4*d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*
a*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*
d*f - e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e*
*2)/(2*f**3)))/(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a
*e*f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)) + (sqrt(-4*
d*f + e**2)*(-2*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*
f**2 + 2*B*b*d*f**2 - B*b*e**2*f - 3*B*c*d*e*f + B*c*e**3)/(2*f**3*(4*d*f
- e**2)) + (A*b*f**2 - A*c*e*f + B*a*f**2 - B*b*e*f - B*c*d*f + B*c*e**2)/
(2*f**3))*log(x + (-A*a*e*f**2 + 2*A*b*d*f**2 - A*c*d*e*f + 2*B*a*d*f**2 -
 B*b*d*e*f - 2*B*c*d**2*f + B*c*d*e**2 - 4*d*f**3*(sqrt(-4*d*f + e**2)*(-2
*A*a*f**3 + A*b*e*f**2 + 2*A*c*d*f**2 - A*c*e**2*f + B*a*e*f**2 + 2*B*b...
```

3.13.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)}{d+ex+fx^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.13.8 Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{d+ex+fx^2}\,dx = \frac{Bcfx^2-2\,Bcex+2\,Bbfx+2\,Acfx}{2\,f^2}$$
$$+\frac{(Bce^2-Bcdf-Bbef-Acef+Baf^2+Abf^2)\log\left(fx^2+ex+d\right)}{2\,f^3}$$
$$-\frac{(Bce^3-3\,Bcdef-Bbe^2f-Ace^2f+2\,Bbdf^2+2\,Acdf^2+Baef^2+Abef^2-2\,Aaf^3)\arctan\left(\frac{2\,fx+e}{\sqrt{-e^2+4\,d}}\right.}{\sqrt{-e^2+4\,df}\,f^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/2*(B*c*f*x^2 - 2*B*c*e*x + 2*B*b*f*x + 2*A*c*f*x)/f^2 + 1/2*(B*c*e^2 - B
*c*d*f - B*b*e*f - A*c*e*f + B*a*f^2 + A*b*f^2)*log(f*x^2 + e*x + d)/f^3 -
 (B*c*e^3 - 3*B*c*d*e*f - B*b*e^2*f - A*c*e^2*f + 2*B*b*d*f^2 + 2*A*c*d*f^
2 + B*a*e*f^2 + A*b*e*f^2 - 2*A*a*f^3)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*
f))/(sqrt(-e^2 + 4*d*f)*f^3)
```

3.13.9 Mupad [B] (verification not implemented)

Time = 13.66 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.48

$$\int \frac{(A+Bx)\,(a+bx+cx^2)}{d+ex+fx^2}\,dx = x\left(\frac{A\,c+B\,b}{f}-\frac{B\,c\,e}{f^2}\right)$$
$$-\frac{\ln\left(f\,x^2+e\,x+d\right)\,\left(B\,c\,e^4-4\,A\,b\,d\,f^3-4\,B\,a\,d\,f^3-A\,c\,e^3\,f-B\,b\,e^3\,f+A\,b\,e^2\,f^2+B\,a\,e^2\,f^2+4\,B\,c\right.}{2\,\left(4\,d\,f^4-e^2\,f^3\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{e}{\sqrt{4\,d\,f-e^2}}+\frac{2\,f\,x}{\sqrt{4\,d\,f-e^2}}\right)\,\left(B\,c\,e^3-2\,A\,a\,f^3+A\,b\,e\,f^2+2\,A\,c\,d\,f^2+B\,a\,e\,f^2+2\,B\,b\,d\,f^2-A\,c\,e^2\,f-\right.}{f^3\,\sqrt{4\,d\,f-e^2}}$$
$$+\frac{B\,c\,x^2}{2\,f}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2))/(d + e*x + f*x^2),x)
```

output
```
x*((A*c + B*b)/f - (B*c*e)/f^2) - (log(d + e*x + f*x^2)*(B*c*e^4 - 4*A*b*d
*f^3 - 4*B*a*d*f^3 - A*c*e^3*f - B*b*e^3*f + A*b*e^2*f^2 + B*a*e^2*f^2 + 4
*B*c*d^2*f^2 + 4*A*c*d*e*f^2 + 4*B*b*d*e*f^2 - 5*B*c*d*e^2*f))/(2*(4*d*f^4
 - e^2*f^3)) - (atan(e/(4*d*f - e^2)^(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*
(B*c*e^3 - 2*A*a*f^3 + A*b*e*f^2 + 2*A*c*d*f^2 + B*a*e*f^2 + 2*B*b*d*f^2 -
 A*c*e^2*f - B*b*e^2*f - 3*B*c*d*e*f))/(f^3*(4*d*f - e^2)^(1/2)) + (B*c*x^
2)/(2*f)
```

3.13. $\int \frac{(A+Bx)(a+bx+cx^2)}{d+ex+fx^2}\,dx$

3.14 $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2} \, dx$

3.14.1 Optimal result

Integrand size = 30, antiderivative size = 542

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2} \, dx$$
$$= \frac{\left(B(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+Af\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)\right)x}{f^4}$$
$$-\frac{\left(Acf(ce-2bf)-B\left(b^2f^2-2cf(be-af)+c^2(e^2-df)\right)\right)x^2}{2f^3}$$
$$-\frac{c(Bce-2bBf-Acf)x^3}{3f^2}+\frac{Bc^2x^4}{4f}$$
$$-\frac{(Af(c^2(e^4-4de^2f+2d^2f^2)-f^2(2abef-2a^2f^2-b^2(e^2-2df))+2cf(af(e^2-2df)-b(e^3-3def)))}{}$$
$$+\frac{(Af(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)+B(c^2(e^4-3de^2f+d^2f^2)-f^2(2abef-a^2f^2-b^2(e^2-df}{2f^5}$$

output
```
(B*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+A*f*(b^2*f^2-2*c*f*(-a*f+b*e
)+c^2*(-d*f+e^2)))*x/f^4-1/2*(A*c*f*(-2*b*f+c*e)-B*(b^2*f^2-2*c*f*(-a*f+b*
e)+c^2*(-d*f+e^2)))*x^2/f^3-1/3*c*(-A*c*f-2*B*b*f+B*c*e)*x^3/f^2+1/4*B*c^2
*x^4/f+1/2*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2
-3*d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^
2)-b*(-2*d*e*f+e^3))))*ln(f*x^2+e*x+d)/f^5-(A*f*(c^2*(2*d^2*f^2-4*d*e^2*f+
e^4)-f^2*(2*a*b*e*f-2*a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*
(-3*d*e*f+e^3)))-B*(c^2*(5*d^2*e*f^2-5*d*e^3*f+e^5)+f^2*(a^2*e*f^2-2*a*b*f
*(-2*d*f+e^2)+b^2*(-3*d*e*f+e^3))+2*c*f*(a*e*f*(-3*d*f+e^2)-b*(2*d^2*f^2-4
*d*e^2*f+e^4))))*arctanh((2*f*x+e)/(-4*d*f+e^2)^(1/2))/f^5/(-4*d*f+e^2)^(1
/2)
```

3.14.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 535, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$

$$= \frac{12f(-B(ce-bf)\left(f(-be+2af)+c(e^2-2df)\right)+Af(b^2f^2+2cf(-be+af)+c^2(e^2-df)))\,x+6f^2(Ac}{}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

output
```
(12*f*(-(B*(c*e - b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f))) + A*f*(b^2*
f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x + 6*f^2*(A*c*f*(-(c*e) +
2*b*f) + B*(b^2*f^2 + 2*c*f*(-(b*e) + a*f) + c^2*(e^2 - d*f)))*x^2 + 4*c*f
^3*(-(B*c*e) + 2*b*B*f + A*c*f)*x^3 + 3*B*c^2*f^4*x^4 - (12*(-(A*f*(c^2*(e
^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^2 + b^2*(e^2 - 2*d
*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f)))) + B*(c^2*(e^5 - 5*d
*e^3*f + 5*d^2*e*f^2) + f^2*(a^2*e*f^2 + 2*a*b*f*(-e^2 + 2*d*f) + b^2*(e^3
 - 3*d*e*f)) - 2*c*f*(-(a*e*f*(e^2 - 3*d*f)) + b*(e^4 - 4*d*e^2*f + 2*d^2*
f^2))))*ArcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + 6*(A*
f*(-(c*e) + b*f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*
d*e^2*f + d^2*f^2) + f^2*(-2*a*b*e*f + a^2*f^2 + b^2*(e^2 - d*f)) - 2*c*f*
(a*f*(-e^2 + d*f) + b*(e^3 - 2*d*e*f))))*Log[d + x*(e + f*x)])/(12*f^5)
```

3.14.3 Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {1344, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{1344}$$

$$\int \left(\frac{x\left(B\left(-f^2\left(-a^2f^2+2abef-\left(b^2\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)+\right.}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(Af\left(-f^2\left(-2a^2f^2+2abef-\left(b^2\left(e^2-2df\right)\right)\right)+2cf\left(af\left(e^2-2df\right)-b\left(e^3-3def\right)\right)+c^2\left(2d\right.\right.\right.}{}$$

$$\frac{\log\left(d+ex+fx^2\right)\left(B\left(-f^2\left(-a^2f^2+2abef-\left(b^2\left(e^2-df\right)\right)\right)+2cf\left(af\left(e^2-df\right)-b\left(e^3-2def\right)\right)+c^2\left(d^2f^2-3d\right.\right.\right.}{2f^5}$$

$$\frac{x\left(Af\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)+B(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)\right)}{f^4}-$$

$$\frac{x^2\left(Acf(ce-2bf)-B\left(-2cf(be-af)+b^2f^2+c^2\left(e^2-df\right)\right)\right)}{2f^3}-\frac{cx^3(-Acf-2bBf+Bce)}{3f^2}+$$

$$\frac{Bc^2x^4}{4f}$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x]
```

output
```
((B*(c*e - b*f)*(f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + A*f*(b^2*f^2 - 2*c*f
*(b*e - a*f) + c^2*(e^2 - d*f)))*x)/f^4 - ((A*c*f*(c*e - 2*b*f) - B*(b^2*f
^2 - 2*c*f*(b*e - a*f) + c^2*(e^2 - d*f)))*x^2)/(2*f^3) - (c*(B*c*e - 2*b*
B*f - A*c*f)*x^3)/(3*f^2) + (B*c^2*x^4)/(4*f) - ((A*f*(c^2*(e^4 - 4*d*e^2*
f + 2*d^2*f^2) - f^2*(2*a*b*e*f - 2*a^2*f^2 - b^2*(e^2 - 2*d*f)) + 2*c*f*(
a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))) - B*(c^2*(e^5 - 5*d*e^3*f + 5*d^2*
e*f^2) + f^2*(a^2*e*f^2 - 2*a*b*f*(e^2 - 2*d*f) + b^2*(e^3 - 3*d*e*f)) + 2
*c*f*(a*e*f*(e^2 - 3*d*f) - b*(e^4 - 4*d*e^2*f + 2*d^2*f^2))))*ArcTanh[(e
+ 2*f*x)/Sqrt[e^2 - 4*d*f]])/(f^5*Sqrt[e^2 - 4*d*f]) + ((A*f*(c*e - b*f)*(
f*(b*e - 2*a*f) - c*(e^2 - 2*d*f)) + B*(c^2*(e^4 - 3*d*e^2*f + d^2*f^2) -
f^2*(2*a*b*e*f - a^2*f^2 - b^2*(e^2 - d*f)) + 2*c*f*(a*f*(e^2 - d*f) - b*(
e^3 - 2*d*e*f))))*Log[d + e*x + f*x^2])/(2*f^5)
```

3.14.3.1 Defintions of rubi rules used

rule 1344
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x +
c*x^2)^p*(d + e*x + f*x^2)^q*(g + h*x), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && IGtQ[p, 0] && In
tegerQ[q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.14.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 800, normalized size of antiderivative = 1.48

method	result
default	$\frac{1}{4}B\,c^{2}x^{4}f^{3}+\frac{1}{3}A\,c^{2}f^{3}x^{3}+\frac{2}{3}Bbc\,f^{3}x^{3}-\frac{1}{3}B\,c^{2}e\,f^{2}x^{3}+Abc\,f^{3}x^{2}-\frac{1}{2}A\,c^{2}e\,f^{2}x^{2}+Bac\,f^{3}x^{2}+\frac{1}{2}B\,b^{2}f^{3}x^{2}-Bbce\,f^{2}x^{2}-\frac{1}{2}B\,c^{2}d\,f^{2}x^{2}+\frac{1}{2}$
risch	Expression too large to display

input
```
int((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/f^4*(1/4*B*c^2*x^4*f^3+1/3*A*c^2*f^3*x^3+2/3*B*b*c*f^3*x^3-1/3*B*c^2*e*f
^2*x^3+A*b*c*f^3*x^2-1/2*A*c^2*e*f^2*x^2+B*a*c*f^3*x^2+1/2*B*b^2*f^3*x^2-B
*b*c*e*f^2*x^2-1/2*B*c^2*d*f^2*x^2+1/2*B*c^2*e^2*f*x^2+2*A*a*c*f^3*x+A*b^2
*f^3*x-2*A*b*c*e*f^2*x-A*c^2*d*f^2*x+A*c^2*e^2*f*x+2*B*a*b*f^3*x-2*B*a*c*e
*f^2*x-B*b^2*e*f^2*x-2*B*b*c*d*f^2*x+2*B*b*c*e^2*f*x+2*B*c^2*d*e*f*x-B*c^2
*e^3*x)+1/f^4*(1/2*(2*A*a*b*f^4-2*A*a*c*e*f^3-A*b^2*e*f^3-2*A*b*c*d*f^3+2*
A*b*c*e^2*f^2+2*A*c^2*d*e*f^2-A*c^2*e^3*f+B*a^2*f^4-2*B*a*b*e*f^3-2*B*a*c*
d*f^3+2*B*a*c*e^2*f^2-B*b^2*d*f^3+B*b^2*e^2*f^2+4*B*b*c*d*e*f^2-2*B*b*c*e^
3*f+B*c^2*d^2*f^2-3*B*c^2*d*e^2*f+B*c^2*e^4)/f*ln(f*x^2+e*x+d)+2*(A*a^2*f^
4-2*A*a*c*d*f^3-A*b^2*d*f^3+2*A*b*c*d*e*f^2+A*c^2*d^2*f^2-A*c^2*d*e^2*f-2*
B*a*b*d*f^3+2*B*a*c*d*e*f^2+B*b^2*d*e*f^2+2*B*b*c*d^2*f^2-2*B*b*c*d*e^2*f-
2*B*c^2*d^2*e*f+B*c^2*d*e^3-1/2*(2*A*a*b*f^4-2*A*a*c*e*f^3-A*b^2*e*f^3-2*A
*b*c*d*f^3+2*A*b*c*e^2*f^2+2*A*c^2*d*e*f^2-A*c^2*e^3*f+B*a^2*f^4-2*B*a*b*e
*f^3-2*B*a*c*d*f^3+2*B*a*c*e^2*f^2-B*b^2*d*f^3+B*b^2*e^2*f^2+4*B*b*c*d*e*f
^2-2*B*b*c*e^3*f+B*c^2*d^2*f^2-3*B*c^2*d*e^2*f+B*c^2*e^4)*e/f)/(4*d*f-e^2)
^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)))
```

3.14.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 1837, normalized size of antiderivative = 3.39

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/12*(3*(B*c^2*e^2*f^4 - 4*B*c^2*d*f^5)*x^4 - 4*(B*c^2*e^3*f^3 + 4*(2*B*b
*c + A*c^2)*d*f^5 - (4*B*c^2*d*e + (2*B*b*c + A*c^2)*e^2)*f^4)*x^3 + 6*(B*
c^2*e^4*f^2 - 4*(B*b^2 + 2*(B*a + A*b)*c)*d*f^5 + (4*B*c^2*d^2 + 4*(2*B*b*
c + A*c^2)*d*e + (B*b^2 + 2*(B*a + A*b)*c)*e^2)*f^4 - (5*B*c^2*d*e^2 + (2*
B*b*c + A*c^2)*e^3)*f^3)*x^2 - 6*(B*c^2*e^5 - 2*A*a^2*f^5 + (2*(2*B*a*b +
A*b^2 + 2*A*a*c)*d + (B*a^2 + 2*A*a*b)*e)*f^4 - (2*(2*B*b*c + A*c^2)*d^2 +
 3*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2 + 2*A*a*c)*e^2)*f^3 +
(5*B*c^2*d^2*e + 4*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2 + 2*(B*a + A*b)*c)*e^3
)*f^2 - (5*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f)*sqrt(e^2 - 4*d*f)*log((
2*f^2*x^2 + 2*e*f*x + e^2 - 2*d*f - sqrt(e^2 - 4*d*f)*(2*f*x + e))/(f*x^2
+ e*x + d)) - 12*(B*c^2*e^5*f + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*f^5 - (4*(
2*B*b*c + A*c^2)*d^2 + 4*(B*b^2 + 2*(B*a + A*b)*c)*d*e + (2*B*a*b + A*b^2
+ 2*A*a*c)*e^2)*f^4 + (8*B*c^2*d^2*e + 5*(2*B*b*c + A*c^2)*d*e^2 + (B*b^2
+ 2*(B*a + A*b)*c)*e^3)*f^3 - (6*B*c^2*d*e^3 + (2*B*b*c + A*c^2)*e^4)*f^2)
*x + 6*(B*c^2*e^6 - 4*(B*a^2 + 2*A*a*b)*d*f^5 + (4*(B*b^2 + 2*(B*a + A*b)*
c)*d^2 + 4*(2*B*a*b + A*b^2 + 2*A*a*c)*d*e + (B*a^2 + 2*A*a*b)*e^2)*f^4 -
(4*B*c^2*d^3 + 8*(2*B*b*c + A*c^2)*d^2*e + 5*(B*b^2 + 2*(B*a + A*b)*c)*d*e
^2 + (2*B*a*b + A*b^2 + 2*A*a*c)*e^3)*f^3 + (13*B*c^2*d^2*e^2 + 6*(2*B*b*c
 + A*c^2)*d*e^3 + (B*b^2 + 2*(B*a + A*b)*c)*e^4)*f^2 - (7*B*c^2*d*e^4 + (2
*B*b*c + A*c^2)*e^5)*f)*log(f*x^2 + e*x + d))/(e^2*f^5 - 4*d*f^6), 1/12...
```

3.14.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4663 vs. $2(520) = 1040$.

Time = 82.39 (sec) , antiderivative size = 4663, normalized size of antiderivative = 8.60

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

output
```
B*c**2*x**4/(4*f) + x**3*(A*c**2/(3*f) + 2*B*b*c/(3*f) - B*c**2*e/(3*f**2)
) + x**2*(A*b*c/f - A*c**2*e/(2*f**2) + B*a*c/f + B*b**2/(2*f) - B*b*c*e/f
**2 - B*c**2*d/(2*f**2) + B*c**2*e**2/(2*f**3)) + x*(2*A*a*c/f + A*b**2/f
- 2*A*b*c*e/f**2 - A*c**2*d/f**2 + A*c**2*e**2/f**3 + 2*B*a*b/f - 2*B*a*c*
e/f**2 - B*b**2*e/f**2 - 2*B*b*c*d/f**2 + 2*B*b*c*e**2/f**3 + 2*B*c**2*d*e
/f**3 - B*c**2*e**3/f**4) + (-sqrt(-4*d*f + e**2)*(-2*A*a**2*f**5 + 2*A*a*
b*e*f**4 + 4*A*a*c*d*f**4 - 2*A*a*c*e**2*f**3 + 2*A*b**2*d*f**4 - A*b**2*e
**2*f**3 - 6*A*b*c*d*e*f**3 + 2*A*b*c*e**3*f**2 - 2*A*c**2*d**2*f**3 + 4*A
*c**2*d*e**2*f**2 - A*c**2*e**4*f + B*a**2*e*f**4 + 4*B*a*b*d*f**4 - 2*B*a
*b*e**2*f**3 - 6*B*a*c*d*e*f**3 + 2*B*a*c*e**3*f**2 - 3*B*b**2*d*e*f**3 +
B*b**2*e**3*f**2 - 4*B*b*c*d**2*f**3 + 8*B*b*c*d*e**2*f**2 - 2*B*b*c*e**4*
f + 5*B*c**2*d**2*e*f**2 - 5*B*c**2*d*e**3*f + B*c**2*e**5)/(2*f**5*(4*d*f
 - e**2)) + (2*A*a*b*f**4 - 2*A*a*c*e*f**3 - A*b**2*e*f**3 - 2*A*b*c*d*f**
3 + 2*A*b*c*e**2*f**2 + 2*A*c**2*d*e*f**2 - A*c**2*e**3*f + B*a**2*f**4 -
2*B*a*b*e*f**3 - 2*B*a*c*d*f**3 + 2*B*a*c*e**2*f**2 - B*b**2*d*f**3 + B*b*
*2*e**2*f**2 + 4*B*b*c*d*e*f**2 - 2*B*b*c*e**3*f + B*c**2*d**2*f**2 - 3*B*
c**2*d*e**2*f + B*c**2*e**4)/(2*f**5))*log(x + (-A*a**2*e*f**4 + 4*A*a*b*d
*f**4 - 2*A*a*c*d*e*f**3 - A*b**2*d*e*f**3 - 4*A*b*c*d**2*f**3 + 2*A*b*c*d
*e**2*f**2 + 3*A*c**2*d**2*e*f**2 - A*c**2*d*e**3*f + 2*B*a**2*d*f**4 - 2*
B*a*b*d*e*f**3 - 4*B*a*c*d**2*f**3 + 2*B*a*c*d*e**2*f**2 - 2*B*b**2*d**...
```

3.14.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.14.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 743, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)(a+bx+cx^2)^2}{d+ex+fx^2}\,dx$$
$$= \frac{3\,Bc^2f^3x^4 - 4\,Bc^2ef^2x^3 + 8\,Bbcf^3x^3 + 4\,Ac^2f^3x^3 + 6\,Bc^2e^2fx^2 - 6\,Bc^2df^2x^2 - 12\,Bbcef^2x^2 - 6\,Ac^2ef^2}{}$$
$$+ \frac{(Bc^2e^4 - 3\,Bc^2de^2f - 2\,Bbce^3f - Ac^2e^3f + Bc^2d^2f^2 + 4\,Bbcdef^2 + 2\,Ac^2def^2 + Bb^2e^2f^2 + 2\,Bace^2f}{}$$
$$- \frac{(Bc^2e^5 - 5\,Bc^2de^3f - 2\,Bbce^4f - Ac^2e^4f + 5\,Bc^2d^2ef^2 + 8\,Bbcde^2f^2 + 4\,Ac^2de^2f^2 + Bb^2e^3f^2 + 2\,Ba}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*B*c^2*f^3*x^4 - 4*B*c^2*e*f^2*x^3 + 8*B*b*c*f^3*x^3 + 4*A*c^2*f^3*
x^3 + 6*B*c^2*e^2*f*x^2 - 6*B*c^2*d*f^2*x^2 - 12*B*b*c*e*f^2*x^2 - 6*A*c^2
*e*f^2*x^2 + 6*B*b^2*f^3*x^2 + 12*B*a*c*f^3*x^2 + 12*A*b*c*f^3*x^2 - 12*B*
c^2*e^3*x + 24*B*c^2*d*e*f*x + 24*B*b*c*e^2*f*x + 12*A*c^2*e^2*f*x - 24*B*
b*c*d*f^2*x - 12*A*c^2*d*f^2*x - 12*B*b^2*e*f^2*x - 24*B*a*c*e*f^2*x - 24*
A*b*c*e*f^2*x + 24*B*a*b*f^3*x + 12*A*b^2*f^3*x + 24*A*a*c*f^3*x)/f^4 + 1/
2*(B*c^2*e^4 - 3*B*c^2*d*e^2*f - 2*B*b*c*e^3*f - A*c^2*e^3*f + B*c^2*d^2*f
^2 + 4*B*b*c*d*e*f^2 + 2*A*c^2*d*e*f^2 + B*b^2*e^2*f^2 + 2*B*a*c*e^2*f^2 +
 2*A*b*c*e^2*f^2 - B*b^2*d*f^3 - 2*B*a*c*d*f^3 - 2*A*b*c*d*f^3 - 2*B*a*b*e
*f^3 - A*b^2*e*f^3 - 2*A*a*c*e*f^3 + B*a^2*f^4 + 2*A*a*b*f^4)*log(f*x^2 +
e*x + d)/f^5 - (B*c^2*e^5 - 5*B*c^2*d*e^3*f - 2*B*b*c*e^4*f - A*c^2*e^4*f
+ 5*B*c^2*d^2*e*f^2 + 8*B*b*c*d*e^2*f^2 + 4*A*c^2*d*e^2*f^2 + B*b^2*e^3*f^
2 + 2*B*a*c*e^3*f^2 + 2*A*b*c*e^3*f^2 - 4*B*b*c*d^2*f^3 - 2*A*c^2*d^2*f^3
- 3*B*b^2*d*e*f^3 - 6*B*a*c*d*e*f^3 - 6*A*b*c*d*e*f^3 - 2*B*a*b*e^2*f^3 -
A*b^2*e^2*f^3 - 2*A*a*c*e^2*f^3 + 4*B*a*b*d*f^4 + 2*A*b^2*d*f^4 + 4*A*a*c*
d*f^4 + B*a^2*e*f^4 + 2*A*a*b*e*f^4 - 2*A*a^2*f^5)*arctan((2*f*x + e)/sqrt
(-e^2 + 4*d*f))/(sqrt(-e^2 + 4*d*f)*f^5)
```

3.14.9 Mupad [B] (verification not implemented)

Time = 15.37 (sec) , antiderivative size = 893, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx = x^3\left(\frac{A\,c^2+2\,B\,b\,c}{3\,f}-\frac{B\,c^2\,e}{3\,f^2}\right)+x\left(\frac{A\,b^2+2\,B\,a\,b+2\,A\,a\,c}{f}-\frac{d\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}+\frac{e\left(\frac{e\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{f}-\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{f}+\frac{B\,c^2\,d}{f^2}\right)}{f}\right)$$

$$-x^2\left(\frac{e\left(\frac{A\,c^2+2\,B\,b\,c}{f}-\frac{B\,c^2\,e}{f^2}\right)}{2\,f}-\frac{B\,b^2+2\,A\,c\,b+2\,B\,a\,c}{2\,f}+\frac{B\,c^2\,d}{2\,f^2}\right)$$

$$-\frac{\ln\left(f\,x^2+e\,x+d\right)\left(-4\,B\,a^2\,d\,f^5+B\,a^2\,e^2\,f^4+8\,B\,a\,b\,d\,e\,f^4-8\,A\,a\,b\,d\,f^5-2\,B\,a\,b\,e^3\,f^3+2\,A\,a\,b\,e^2\,f^{\cdots}\right.}{\cdots}$$

$$+\frac{B\,c^2\,x^4}{4\,f}$$

$$+\frac{\mathrm{atan}\left(\frac{e}{\sqrt{4\,d\,f-e^2}}+\frac{2\,f\,x}{\sqrt{4\,d\,f-e^2}}\right)\left(-B\,a^2\,e\,f^4+2\,A\,a^2\,f^5-4\,B\,a\,b\,d\,f^4+2\,B\,a\,b\,e^2\,f^3-2\,A\,a\,b\,e\,f^4+6\,B\,a\,c\cdots\right.}{\cdots}$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^2)/(d + e*x + f*x^2),x)
```

output
```
x^3*((A*c^2 + 2*B*b*c)/(3*f) - (B*c^2*e)/(3*f^2)) + x*((A*b^2 + 2*A*a*c +
2*B*a*b)/f - (d*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/f + (e*((e*((A*c^2
+ 2*B*b*c)/f - (B*c^2*e)/f^2))/f - (B*b^2 + 2*A*b*c + 2*B*a*c)/f + (B*c^2*
d)/f^2))/f) - x^2*((e*((A*c^2 + 2*B*b*c)/f - (B*c^2*e)/f^2))/(2*f) - (B*b^
2 + 2*A*b*c + 2*B*a*c)/(2*f) + (B*c^2*d)/(2*f^2)) - (log(d + e*x + f*x^2)*
(B*c^2*e^6 - 4*B*a^2*d*f^5 - A*c^2*e^5*f - A*b^2*e^3*f^3 + B*a^2*e^2*f^4 +
 4*B*b^2*d^2*f^4 + B*b^2*e^4*f^2 - 4*B*c^2*d^3*f^3 + 6*A*c^2*d*e^3*f^2 - 8
*A*c^2*d^2*e*f^3 - 5*B*b^2*d*e^2*f^3 - 8*A*a*b*d*f^5 - 2*B*b*c*e^5*f + 13*
B*c^2*d^2*e^2*f^2 + 2*A*a*b*e^2*f^4 - 2*A*a*c*e^3*f^3 + 8*A*b*c*d^2*f^4 -
2*B*a*b*e^3*f^3 + 8*B*a*c*d^2*f^4 + 2*A*b*c*e^4*f^2 + 2*B*a*c*e^4*f^2 + 4*
A*b^2*d*e*f^4 - 7*B*c^2*d*e^4*f - 10*A*b*c*d*e^2*f^3 - 10*B*a*c*d*e^2*f^3
+ 12*B*b*c*d*e^3*f^2 - 16*B*b*c*d^2*e*f^3 + 8*A*a*c*d*e*f^4 + 8*B*a*b*d*e*
f^4))/(2*(4*d*f^6 - e^2*f^5)) + (B*c^2*x^4)/(4*f) + (atan(e/(4*d*f - e^2)^
(1/2) + (2*f*x)/(4*d*f - e^2)^(1/2))*(2*A*a^2*f^5 - B*c^2*e^5 - 2*A*b^2*d*
f^4 - B*a^2*e*f^4 + A*c^2*e^4*f + A*b^2*e^2*f^3 + 2*A*c^2*d^2*f^3 - B*b^2*
e^3*f^2 - 4*A*c^2*d*e^2*f^2 - 5*B*c^2*d^2*e*f^2 - 2*A*a*b*e*f^4 - 4*A*a*c*
d*f^4 - 4*B*a*b*d*f^4 + 2*B*b*c*e^4*f + 2*A*a*c*e^2*f^3 + 2*B*a*b*e^2*f^3
- 2*A*b*c*e^3*f^2 - 2*B*a*c*e^3*f^2 + 4*B*b*c*d^2*f^3 + 3*B*b^2*d*e*f^3 +
5*B*c^2*d*e^3*f - 8*B*b*c*d*e^2*f^2 + 6*A*b*c*d*e*f^3 + 6*B*a*c*d*e*f^3))/
(f^5*(4*d*f - e^2)^(1/2))
```

3.14. $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^2}{d+ex+fx^2}\,dx$

3.15 $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

3.15.1 Optimal result

Integrand size = 30, antiderivative size = 406

$$\begin{aligned}\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx &= -\frac{(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf))\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\sqrt{b^2-4ac}\,(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))}\\
&\quad+\frac{(B(cde-2bdf+aef)-A(ce^2-2cdf-bef+2af^2))\operatorname{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)}{\sqrt{e^2-4df}\,(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))}\\
&\quad+\frac{(Bcd-Ace+Abf-aBf)\log(a+bx+cx^2)}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))}\\
&\quad-\frac{(Bcd-Ace+Abf-aBf)\log(d+ex+fx^2)}{2(c^2d^2+f(b^2d-abe+a^2f)-c(bde-a(e^2-2df)))}\end{aligned}$$

output
```
1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*
d)-c*(b*d*e-a*(-2*d*f+e^2)))-1/2*(A*b*f-A*c*e-B*a*f+B*c*d)*ln(f*x^2+e*x+d)
/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))-(A*b^2*f+2*c*(-A
*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))/(-4*a*c+b^2)
^(1/2)+(B*(a*e*f-2*b*d*f+c*d*e)-A*(2*a*f^2-b*e*f-2*c*d*f+c*e^2))*arctanh((
2*f*x+e)/(-4*d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2
*d*f+e^2)))/(-4*d*f+e^2)^(1/2)
```

3.15.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$$
$$= \frac{\frac{2(Ab^2f+2c(Acd+aBe-aAf)-b(Bcd+Ace+aBf))\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}} - \frac{2(B(cde-2bdf+aef)+A(-ce^2+2cdf+bef-2af^2))\arctan\left(\frac{e+2f}{\sqrt{-e^2+}}\right.}{\sqrt{-e^2+4df}}}{2\left(c^2d^2-bcde+f\left(b^2d-a\right.\right.}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

output
```
((2*(A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Ar
cTan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] - (2*(B*(c*d*e -
2*b*d*f + a*e*f) + A*(-(c*e^2) + 2*c*d*f + b*e*f - 2*a*f^2))*ArcTan[(e + 2
*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] + (B*c*d - A*c*e + A*b*f - a
*B*f)*Log[a + x*(b + c*x)] + (-(B*c*d) + A*c*e - A*b*f + a*B*f)*Log[d + x*
(e + f*x)])/(2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 -
 2*d*f)))
```

3.15.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1355, 25, 1142, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{1355}$$
$$\frac{\int \frac{aB(ce-bf)+A(fb^2+c^2d-c(be+af))+c(Bcd-Ace+Abf-aBf)x}{cx^2+bx+a}dx}{f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2}+\frac{\int -\frac{Af(be-af)-Ac(e^2-df)+B(cde-bdf)+f(Bcd-Ace+Abf-aBf)x}{fx^2+ex+d}dx}{f(a^2f-abe+b^2d)+ac(e^2-2df)-bcde+c^2d^2}$$
$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aB(ce-bf)+A\left(fb^2+c^2d-c(be+af)\right)+c(Bcd-Ace+Abf-aBf)x}{cx^2+bx+a}dx}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}-\frac{\int \frac{Bd(ce-bf)-A\left(ce^2-bfe+af^2-cdf\right)+f(Bcd-Ace+Abf-aBf)x}{fx^2+ex+d}dx}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$

$\big\downarrow$ 1142

$$\frac{\frac{1}{2}\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)\int \frac{1}{cx^2+bx+a}dx+\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{}{cx}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$
$$\frac{\frac{1}{2}\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)\int \frac{1}{fx^2+ex+d}dx+\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{e+2fx}{fx^2+ex+}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$

$\big\downarrow$ 1083

$$\frac{\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{b+2cx}{cx^2+bx+a}dx-\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)\int \frac{}{b^2-}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$
$$\frac{\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{e+2fx}{fx^2+ex+d}dx-\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)\int \frac{1}{e^2-(e+2fx}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$

$\big\downarrow$ 219

$$\frac{\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{b+2cx}{cx^2+bx+a}dx-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}-$$
$$\frac{\frac{1}{2}(-aBf+Abf-Ace+Bcd)\int \frac{e+2fx}{fx^2+ex+d}dx-\frac{\text{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$

$\big\downarrow$ 1103

$$\frac{\frac{1}{2}\log\left(a+bx+cx^2\right)(-aBf+Abf-Ace+Bcd)-\frac{\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)}{\sqrt{b^2-4ac}}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$
$$\frac{\frac{1}{2}\log\left(d+ex+fx^2\right)(-aBf+Abf-Ace+Bcd)-\frac{\text{arctanh}\left(\frac{e+2fx}{\sqrt{e^2-4df}}\right)\left(B(aef-2bdf+cde)-A\left(2af^2-bef-2cdf+ce^2\right)\right)}{\sqrt{e^2-4df}}}{f\left(a^2f-abe+b^2d\right)+ac\left(e^2-2df\right)-bcde+c^2d^2}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x]
```

3.15. $\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$

output
```
(-(((A*b^2*f + 2*c*(A*c*d + a*B*e - a*A*f) - b*(B*c*d + A*c*e + a*B*f))*Ar
cTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/Sqrt[b^2 - 4*a*c]) + ((B*c*d - A*c*e
 + A*b*f - a*B*f)*Log[a + b*x + c*x^2])/2)/(c^2*d^2 - b*c*d*e + f*(b^2*d -
 a*b*e + a^2*f) + a*c*(e^2 - 2*d*f)) - (-(((B*(c*d*e - 2*b*d*f + a*e*f) -
A*(c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2))*ArcTanh[(e + 2*f*x)/Sqrt[e^2 - 4*d*
f]])/Sqrt[e^2 - 4*d*f]) + ((B*c*d - A*c*e + A*b*f - a*B*f)*Log[d + e*x + f
*x^2])/2)/(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*
f))
```

3.15.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1355
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*
(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = Simplify[c^2*d^2 - b*c*d*e +
a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2]}, Simp[1/q   Int[Simp[g*
c^2*d - g*b*c*e + a*h*c*e + g*b^2*f - a*b*h*f - a*g*c*f + c*(h*c*d - g*c*e
+ g*b*f - a*h*f)*x, x]/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[Simp[(-h)
*c*d*e + g*c*e^2 + b*h*d*f - g*c*d*f - g*b*e*f + a*g*f^2 - f*(h*c*d - g*c*e
 + g*b*f - a*h*f)*x, x]/(d + e*x + f*x^2), x], x] /; NeQ[q, 0]] /; FreeQ[{a
, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0]
```

3.15.4 Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 384, normalized size of antiderivative = 0.95

method	result
default	$\frac{\frac{\left(-Ab\,f^2+Acef+Ba\,f^2-Bcdf\right)\ln\left(f\,x^2+ex+d\right)}{2f}+\frac{2\left(Aa\,f^2-Abef-Acdf+Ac\,e^2+Bbdf-Bcde-\frac{\left(-Ab\,f^2+Acef+Ba\,f^2-Bcdf\right)e}{2f}\right)\arctan\left(\frac{2fx+}{\sqrt{4df-}}\right.}{\sqrt{4df-e^2}}}{a^2f^2-abef-2acdf+ac\,e^2+b^2df-bcde+c^2d^2}$
risch	Expression too large to display

input
```
int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(a^2*f^2-a*b*e*f-2*a*c*d*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*(1/2*(-A*b*f
^2+A*c*e*f+B*a*f^2-B*c*d*f)/f*ln(f*x^2+e*x+d)+2*(A*a*f^2-A*b*e*f-A*c*d*f+A
*c*e^2+B*b*d*f-B*c*d*e-1/2*(-A*b*f^2+A*c*e*f+B*a*f^2-B*c*d*f)*e/f)/(4*d*f-
e^2)^(1/2)*arctan((2*f*x+e)/(4*d*f-e^2)^(1/2)))+1/(a^2*f^2-a*b*e*f-2*a*c*d
*f+a*c*e^2+b^2*d*f-b*c*d*e+c^2*d^2)*(1/2*(A*b*c*f-A*c^2*e-B*a*c*f+B*c^2*d)
/c*ln(c*x^2+b*x+a)+2*(-A*a*c*f+A*b^2*f-A*b*c*e+A*c^2*d-B*a*b*f+B*a*c*e-1/2
*(A*b*c*f-A*c^2*e-B*a*c*f+B*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x+b)
/(4*a*c-b^2)^(1/2)))
```

3.15.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.15.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.15.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx$$
$$= \frac{(Bcd-Ace-Baf+Abf)\log(cx^2+bx+a)}{2\left(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2\right)} - \frac{(Bcd-Ace-Baf+Abf)\log(fx^2+ex+d)}{2\left(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2\right)} - \frac{\left(Bbcd-2\,Ac^2d-2\,Bace+Abce+Babf-Ab^2f+2\,Aacf\right)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{\left(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2\right)\sqrt{-b^2+4\,ac}} - \frac{\left(Bcde-Ace^2-2\,Bbdf+2\,Acdf+Baef+Abef-2\,Aaf^2\right)\arctan\left(\frac{2\,fx+e}{\sqrt{-e^2+4\,df}}\right)}{\left(c^2d^2-bcde+ace^2+b^2df-2\,acdf-abef+a^2f^2\right)\sqrt{-e^2+4\,df}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/2*(B*c*d - A*c*e - B*a*f + A*b*f)*log(c*x^2 + b*x + a)/(c^2*d^2 - b*c*d*
e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2) - 1/2*(B*c*d - A*c*
e - B*a*f + A*b*f)*log(f*x^2 + e*x + d)/(c^2*d^2 - b*c*d*e + a*c*e^2 + b^2
*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2) - (B*b*c*d - 2*A*c^2*d - 2*B*a*c*e +
 A*b*c*e + B*a*b*f - A*b^2*f + 2*A*a*c*f)*arctan((2*c*x + b)/sqrt(-b^2 + 4
*a*c))/((c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2
*f^2)*sqrt(-b^2 + 4*a*c)) - (B*c*d*e - A*c*e^2 - 2*B*b*d*f + 2*A*c*d*f + B
*a*e*f + A*b*e*f - 2*A*a*f^2)*arctan((2*f*x + e)/sqrt(-e^2 + 4*d*f))/((c^2
*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*e*f + a^2*f^2)*sqrt(-
e^2 + 4*d*f))
```

3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)(d+ex+fx^2)}\,dx = \text{Hanged}$$

input
```
int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)),x)
```

output
```
\text{Hanged}
```

3.16 $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

3.16.1 Optimal result

Integrand size = 30, antiderivative size = 1075

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx =$$

$$-\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(2c^2d+b^2f-c(be+2af)\right)+c(Ab^2f+2c(Acd+aBe-aAf)-}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(a+bx+cx^2)}$$

$$-\frac{(b^5(Bd-Ae)f^2-2b^4f(Bcde-A(ce^2-cdf+af^2))-4c^2(A(c^3d^3-3a^3f^3-a^2cf(e^2-7df)+ac^2d(3}{}$$

$$+\frac{(B(c^2de(e^2-3df)-2cdf(be^2-2bdf-aef)+f^2(b^2de-4abdf+a^2ef))-A(c^2(e^4-4de^2f+2d^2f^2)}{\sqrt{e^2-4df}\left(c^2d^2+f\left(b^2d-abe+a^2f\right)\right.}$$

$$+\frac{(A(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)-B(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))\log(a+}{2\left(c^2d^2+f\left(b^2d-abe+a^2f\right)-c\left(bde-a\left(e^2-2df\right)\right)\right)^2}$$

$$-\frac{(A(ce-bf)\left(f(be-2af)-c(e^2-2df)\right)-B(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))\log(d+}{2\left(c^2d^2+f\left(b^2d-abe+a^2f\right)-c\left(bde-a\left(e^2-2df\right)\right)\right)^2}$$

output
```
(-A*c*(2*a*c*e-b*(a*f+c*d))-(A*b-B*a)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))-c*(A*b
^2*f+2*c*(-A*a*f+A*c*d+B*a*e)-b*(A*c*e+B*a*f+B*c*d))*x)/(-4*a*c+b^2)/((-a*
f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)-(b^5*(-A*e+B*d)*f^2-2*b^4*f*
(B*c*d*e-A*(a*f^2-c*d*f+c*e^2))-4*c^2*(A*(c^3*d^3-3*a^3*f^3-a^2*c*f*(-7*d*
f+e^2)+a*c^2*d*(-5*d*f+3*e^2))-a*B*e*(c^2*d^2-3*a^2*f^2-a*c*(-2*d*f+e^2)))
-4*b^2*c*(B*c^2*d^2*e+A*f*(2*c^2*d^2+3*a^2*f^2+3*a*c*(-d*f+e^2)))+2*b*c*(B
*(c^3*d^3+3*a^3*f^3+a*c^2*d*(-7*d*f+e^2)+3*a^2*c*f*(d*f+e^2))+A*c*e*(3*c^2
*d^2+3*a^2*f^2+a*c*(2*d*f+3*e^2)))-b^3*(A*c*e*(-4*a*f^2-2*c*d*f+c*e^2)+B*(
4*a*c*d*f^2+a^2*f^3-c^2*d*(5*d*f+e^2))))*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1
/2))/(-4*a*c+b^2)^(3/2)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+
e^2)))^2+1/2*(A*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*
f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))*ln(c*x^2+b*x+a)/(c^2*d^2+f*(a
^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+e^2)))^2-1/2*(A*(-b*f+c*e)*(f*(-2*a*f
+b*e)-c*(-2*d*f+e^2))-B*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f
+e^2)))*ln(f*x^2+e*x+d)/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-a*(-2*d*f+
e^2)))^2+(B*(c^2*d*e*(-3*d*f+e^2)-2*c*d*f*(-a*e*f-2*b*d*f+b*e^2)+f^2*(a^2*
e*f-4*a*b*d*f+b^2*d*e))-A*(c^2*(2*d^2*f^2-4*d*e^2*f+e^4)-f^2*(2*a*b*e*f-2*
a^2*f^2-b^2*(-2*d*f+e^2))+2*c*f*(a*f*(-2*d*f+e^2)-b*(-3*d*e*f+e^3))))*arct
anh((2*f*x+e)/(-4*d*f+e^2)^(1/2))/(c^2*d^2+f*(a^2*f-a*b*e+b^2*d)-c*(b*d*e-
a*(-2*d*f+e^2)))^2/(-4*d*f+e^2)^(1/2)
```

3.16.2 Mathematica [A] (verified)

Time = 3.87 (sec) , antiderivative size = 952, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$$

$$= \frac{-\frac{2(c^2d^2-bcde+f(b^2d-abe+a^2f)+ac(e^2-2df))(A(b^3f+b^2c(-e+fx)+bc(-3af+c(d-ex))+2c^2(cdx+a(e-fx)))+B(2a^2cf-bc^2dx-a(b^2f+2\cdots}{(b^2-4ac)(a+x(b+cx))}}{\cdots}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

output
```
((-2*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*(
A*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x +
a*(e - f*x))) + B*(2*a^2*c*f - b*c^2*d*x - a*(b^2*f + 2*c^2*(d - e*x) + b*
c*(-e + f*x)))))/((b^2 - 4*a*c)*(a + x*(b + c*x))) - (2*(b^5*(B*d - A*e)*f
^2 + 2*b^4*f*(-(B*c*d*e) + a*A*f^2 + A*c*(e^2 - d*f)) - 4*b^2*(B*c^3*d^2*e
 + A*c*f*(2*c^2*d^2 + 3*a^2*f^2 + 3*a*c*(e^2 - d*f))) + 2*b*c*(B*(c^3*d^3
+ 3*a^3*f^3 + a*c^2*d*(e^2 - 7*d*f) + 3*a^2*c*f*(e^2 + d*f)) + A*c*e*(3*c^
2*d^2 + 3*a^2*f^2 + a*c*(3*e^2 + 2*d*f))) + 4*c^2*(a*B*e*(c^2*d^2 - 3*a^2*
f^2 - a*c*(e^2 - 2*d*f)) + A*(-(c^3*d^3) + 3*a^3*f^3 + a^2*c*f*(e^2 - 7*d*
f) + a*c^2*d*(-3*e^2 + 5*d*f))) + b^3*(A*c*e*(-(c*e^2) + 2*c*d*f + 4*a*f^2
) + B*(-4*a*c*d*f^2 - a^2*f^3 + c^2*d*(e^2 + 5*d*f))))*ArcTan[(b + 2*c*x)/
Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (2*(B*(c^2*d*e*(-e^2 + 3*d*f)
- 2*c*d*f*(-(b*e^2) + 2*b*d*f + a*e*f) + f^2*(-(b^2*d*e) + 4*a*b*d*f - a^2
*e*f)) + A*(c^2*(e^4 - 4*d*e^2*f + 2*d^2*f^2) + f^2*(-2*a*b*e*f + 2*a^2*f^
2 + b^2*(e^2 - 2*d*f)) + 2*c*f*(a*f*(e^2 - 2*d*f) - b*(e^3 - 3*d*e*f))))*A
rcTan[(e + 2*f*x)/Sqrt[-e^2 + 4*d*f]])/Sqrt[-e^2 + 4*d*f] - (A*(c*e - b*f)
*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(-(
b^2*d) + a^2*f) + c^2*d*(-e^2 + d*f)))*Log[a + x*(b + c*x)] + (A*(c*e - b*
f)*(f*(-(b*e) + 2*a*f) + c*(e^2 - 2*d*f)) + B*(2*c*d*f*(b*e - a*f) + f^2*(
-(b^2*d) + a^2*f) + c^2*d*(-e^2 + d*f)))*Log[d + x*(e + f*x)])/(2*(c^2*...
```

3.16.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2 (d+ex+fx^2)} \, dx$$

$$\Big\downarrow \; 1349$$

$$-\frac{\int -\frac{(Bdf-Aef)b^3-(Bcde-A(ce^2+af^2-2cdf))b^2+c(Bd(cd-3af)+Ae(cd+4af))b-cf(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aBe-aAf))x^2+\ldots}{(cx^2+bx\ldots}}{(b^2-4ac)((cd-a\ldots} \frac{(Ab-aB)(-c(2af+be)+b^2f+2c^2d)+cx(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f)+Ac(2\ldots}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{(Bd-Ae)fb^3-(-aAf^2+Bcde-Ac(e^2-2df))b^2+c(Bd(cd-3af)+Ae(cd+4af))b-cf(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aBe-aAf))x^2+2c(}{(cx^2+bx+a)}}{(b^2-4ac)((cd-af)^2}$$

$$\frac{(Ab-aB)\left(-c(2af+be)+b^2f+2c^2d\right)+cx\left(-b(aBf+Ace+Bcd)+2c(-aAf+aBe+Acd)+Ab^2f\right)+Ac(2}{(b^2-4ac)(a+bx+cx^2)((cd-af)^2-(bd-ae)(ce-bf))}$$

$\downarrow$ 2141

$$\frac{\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(cx^2+bx+a)}$$

$\downarrow$ 27

$$\frac{\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(cx^2+bx+a)}$$

$\downarrow$ 25

$$\frac{\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(cx^2+bx+a)}$$

$\downarrow$ 25

$$\frac{\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))(cx^2+bx+a)}$$

$\downarrow$ 25

3.16. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

3.16. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

$$\Big\downarrow \; 25$$

3.16. $\displaystyle\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\ldots\right.\right.}{\ldots}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\ldots\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\ldots\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\ldots\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

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$$\Big\downarrow 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

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$$\Big\downarrow 25$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{\left(b^2-4ac\right)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\left(cx^2+bx+a\right)}$$

$$\Big\downarrow 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

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$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

$$\int \frac{(Bd-Ae)f^2b^5-2f\left(-aAf^2+Bcde-Ac\left(e^2-df\right)\right)b^4-\left(Ace\left(ce^2-3af^2-2cdf\right)+B\left(a^2f^3+3acdf^2-c^2d\left(e^2+2df\right)\right)\right)b^3-2c\left(Bcde(cd-2af)+Af\left(2c^2d^2+5a^2f^2+5ac\right.\right.}{}$$

$$\frac{Ac(2ace-b(cd+af))+(Ab-aB)\left(fb^2+2c^2d-c(be+2af)\right)+c\left(Afb^2-(Bcd+Ace+aBf)b+2c(Acd+aB\right.}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)(cx^2+bx+a)}$$

$$\Big\downarrow \; 25$$

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input
```
Int[(A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x]
```

output
```
$Aborted
```

3.16. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

3.16.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 2141
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*((d_) + (e_.)*(x_) + (f_.)*(x
_)^2)), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Co
eff[Px, x, 2], q = c^2*d^2 - b*c*d*e + a*c*e^2 + b^2*d*f - 2*a*c*d*f - a*b*
e*f + a^2*f^2}, Simp[1/q   Int[(A*c^2*d - a*c*C*d - A*b*c*e + a*B*c*e + A*b
^2*f - a*b*B*f - a*A*c*f + a^2*C*f + c*(B*c*d - b*C*d - A*c*e + a*C*e + A*b
*f - a*B*f)*x)/(a + b*x + c*x^2), x], x] + Simp[1/q   Int[(c*C*d^2 - B*c*d*
e + A*c*e^2 + b*B*d*f - A*c*d*f - a*C*d*f - A*b*e*f + a*A*f^2 - f*(B*c*d -
b*C*d - A*c*e + a*C*e + A*b*f - a*B*f)*x)/(d + e*x + f*x^2), x], x] /; NeQ[
q, 0]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.16.4 Maple [F(-1)]

Timed out.

hanged

input
```
int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

output
```
int((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x)
```

3.16.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.16.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)**2/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.16.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.16.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3236 vs. $2(1061) = 2122$.

Time = 0.33 (sec) , antiderivative size = 3236, normalized size of antiderivative = 3.01

$$\int \frac{A+Bx}{\left(a+bx+cx^2\right)^2\left(d+ex+fx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
1/2*(B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*d^2*f - 2*B*b*c*d*e*f + 2*A*c^2*d*e*f
 + 2*A*b*c*e^2*f + B*b^2*d*f^2 + 2*B*a*c*d*f^2 - 2*A*b*c*d*f^2 - A*b^2*e*f
^2 - 2*A*a*c*e*f^2 - B*a^2*f^3 + 2*A*a*b*f^3)*log(c*x^2 + b*x + a)/(c^4*d^
4 - 2*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + 2*a*c^3*d^2*e^2 - 2*a*b*c^2*d*e^3 +
a^2*c^2*e^4 + 2*b^2*c^2*d^3*f - 4*a*c^3*d^3*f - 2*b^3*c*d^2*e*f + 2*a*b*c^
2*d^2*e*f + 4*a*b^2*c*d*e^2*f - 4*a^2*c^2*d*e^2*f - 2*a^2*b*c*e^3*f + b^4*
d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c^2*d^2*f^2 - 2*a*b^3*d*e*f^2 + 2*a^2*
b*c*d*e*f^2 + a^2*b^2*e^2*f^2 + 2*a^3*c*e^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*
c*d*f^3 - 2*a^3*b*e*f^3 + a^4*f^4) - 1/2*(B*c^2*d*e^2 - A*c^2*e^3 - B*c^2*
d^2*f - 2*B*b*c*d*e*f + 2*A*c^2*d*e*f + 2*A*b*c*e^2*f + B*b^2*d*f^2 + 2*B*
a*c*d*f^2 - 2*A*b*c*d*f^2 - A*b^2*e*f^2 - 2*A*a*c*e*f^2 - B*a^2*f^3 + 2*A*
a*b*f^3)*log(f*x^2 + e*x + d)/(c^4*d^4 - 2*b*c^3*d^3*e + b^2*c^2*d^2*e^2 +
 2*a*c^3*d^2*e^2 - 2*a*b*c^2*d*e^3 + a^2*c^2*e^4 + 2*b^2*c^2*d^3*f - 4*a*c
^3*d^3*f - 2*b^3*c*d^2*e*f + 2*a*b*c^2*d^2*e*f + 4*a*b^2*c*d*e^2*f - 4*a^2
*c^2*d*e^2*f - 2*a^2*b*c*e^3*f + b^4*d^2*f^2 - 4*a*b^2*c*d^2*f^2 + 6*a^2*c
^2*d^2*f^2 - 2*a*b^3*d*e*f^2 + 2*a^2*b*c*d*e*f^2 + a^2*b^2*e^2*f^2 + 2*a^3
*c*e^2*f^2 + 2*a^2*b^2*d*f^3 - 4*a^3*c*d*f^3 - 2*a^3*b*e*f^3 + a^4*f^4) +
(2*B*b*c^4*d^3 - 4*A*c^5*d^3 - 4*B*b^2*c^3*d^2*e + 4*B*a*c^4*d^2*e + 6*A*b
*c^4*d^2*e + B*b^3*c^2*d*e^2 + 2*B*a*b*c^3*d*e^2 - 12*A*a*c^4*d*e^2 - A*b^
3*c^2*e^3 - 4*B*a^2*c^3*e^3 + 6*A*a*b*c^3*e^3 + 5*B*b^3*c^2*d^2*f - 14*...
```

3.16.9 Mupad [B] (verification not implemented)

Time = 50.27 (sec) , antiderivative size = 118429, normalized size of antiderivative = 110.17

$$\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
int((A + B*x)/((a + b*x + c*x^2)^2*(d + e*x + f*x^2)),x)
```

output
```
symsum(log((x*(4*A^3*b^3*c^4*f^6 + 16*B^3*a^3*c^4*f^6 - 3*B^3*a^2*b^2*c^3*
f^6 + 4*B^3*a^2*c^5*e^2*f^4 + B^3*b^2*c^5*d^2*f^4 - 16*A^3*a*b*c^5*f^6 + 1
6*A^3*a*c^6*e*f^5 + 20*A^2*B*a^2*c^5*f^6 - 3*A^2*B*b^4*c^3*f^6 + 4*A^2*B*c
^7*d^2*f^4 - 16*B^3*a^2*c^5*d*f^5 - 4*A^3*b^2*c^5*e*f^5 + 6*B^3*a*b^2*c^4*
d*f^5 - 4*B^3*a^2*b*c^4*e*f^5 + A^2*B*b^2*c^5*e^2*f^4 - 24*A^2*B*a*c^6*d*f
^5 + 6*A*B^2*a*b^3*c^3*f^6 - 28*A*B^2*a^2*b*c^4*f^6 + 8*A^2*B*a*b^2*c^4*f^
6 - 4*A*B^2*b*c^6*d^2*f^4 + 8*A*B^2*a^2*c^5*e*f^5 - 6*A*B^2*b^3*c^4*d*f^5
+ 8*A^2*B*b^2*c^5*d*f^5 + 2*A^2*B*b^3*c^4*e*f^5 - 4*B^3*a*b*c^5*d*e*f^4 -
4*A*B^2*a*b*c^5*e^2*f^4 + 2*A*B^2*a*b^2*c^4*e*f^5 + 2*A*B^2*b^2*c^5*d*e*f^
4 + 16*A*B^2*a*b*c^5*d*f^5 - 12*A^2*B*a*b*c^5*e*f^5 + 8*A*B^2*a*c^6*d*e*f^
4 - 4*A^2*B*b*c^6*d*e*f^4))/(16*a^2*c^6*d^4 + a^4*b^4*f^4 + 16*a^4*c^4*e^4
 + b^4*c^4*d^4 + 16*a^6*c^2*f^4 + b^8*d^2*f^2 - 8*a*b^2*c^5*d^4 - 8*a^5*b^
2*c*f^4 + 2*a^2*b^6*d*f^3 - 2*a^3*b^5*e*f^3 - 64*a^3*c^5*d^3*f - 64*a^5*c^
3*d*f^3 - 2*b^5*c^3*d^3*e + 2*b^6*c^2*d^3*f + a^2*b^4*c^2*e^4 - 8*a^3*b^2*
c^3*e^4 + 32*a^3*c^5*d^2*e^2 + a^2*b^6*e^2*f^2 + 96*a^4*c^4*d^2*f^2 + b^6*
c^2*d^2*e^2 + 32*a^5*c^3*e^2*f^2 - 2*a*b^7*d*e*f^2 - 2*b^7*c*d^2*e*f + 54*
a^2*b^4*c^2*d^2*f^2 - 112*a^3*b^2*c^3*d^2*f^2 + 16*a*b^3*c^4*d^3*e - 2*a*b
^5*c^2*d*e^3 - 32*a^2*b*c^5*d^3*e - 32*a^3*b*c^4*d*e^3 - 20*a*b^4*c^3*d^3*
f - 12*a*b^6*c*d^2*f^2 - 20*a^3*b^4*c*d*f^3 - 2*a^2*b^5*c*e^3*f - 32*a^4*b
*c^3*e^3*f + 16*a^4*b^3*c*e*f^3 - 32*a^5*b*c^2*e*f^3 - 64*a^4*c^4*d*e^2...
```

3.16. $\int \frac{A+Bx}{(a+bx+cx^2)^2(d+ex+fx^2)}\,dx$

3.17 $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

3.17.1 Optimal result

Integrand size = 34, antiderivative size = 140

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)\,d^2(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2 d^2(a+bx+cx^2)} - \frac{6c(2cg-bh)\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}d^2}$$

output
```
1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^2+3/2*(-b*h
+2*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d^2/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctan
h((2*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d^2
```

3.17.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{\frac{(b^2-4ac)(-bg+2ah-2cgx+bhx)}{(a+x(b+cx))^2} + \frac{3(2cg-bh)(b+2cx)}{a+x(b+cx)} - \frac{12c(-2cg+bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2 d^2}$$

input
```
Integrate[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

output
```
(((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 +
(3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^
2*d^2)
```

3.17.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1329, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}dx$$

$$\Big\downarrow \text{1329}$$

$$\frac{\int \frac{g+hx}{(cx^2+bx+a)^3}dx}{d^2}$$

$$\Big\downarrow \text{1159}$$

$$\frac{-\frac{3(2cg-bh)\int \frac{1}{\left(cx^2+bx+a\right)^2}dx}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d^2}$$

$$\Big\downarrow \text{1086}$$

$$\frac{-\frac{3(2cg-bh)\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d^2}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{3(2cg-bh)\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{3(2cg-bh)\left(\frac{4c\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{\left(b^2-4ac\right)^{3/2}}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d^2}$$

input
```
Int[(g + h*x)/((a + b*x + c*x^2)*(a*d + b*d*x + c*d*x^2)^2),x]
```

output
```
(-1/2*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2)
- (3*(2*c*g - b*h)*(-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*
c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 -
4*a*c)))/d^2
```

3.17.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1329
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(c/f)^p   Int[(g + h
*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p
, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c
/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*
x + c*x^2])
```

3.17.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{bg-2ah+(-bh+2cg)x}{2\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{2}}+\frac{3(-bh+2cg)\left(\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2\left(4ac-b^{2}\right)}}{d^{2}}$
risch	$\frac{-\frac{3c^{2}(bh-2cg)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{9bc(bh-2cg)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5abch-10a\,c^{2}g+b^{3}h-2b^{2}cg\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{8a^{2}ch+a\,b^{2}h-10abcg+b^{3}g}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}d^{2}}-\frac{3c\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2\right.\right.}{}$

input
```
int((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d^2*(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3/2*(-b*
h+2*c*g)/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^
(3/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(132) = 264$.

Time = 0.35 (sec) , antiderivative size = 1150, normalized size of antiderivative = 8.21

$$\int \frac{g+hx}{(a+bx+cx^2)\left(ad+bdx+cdx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="fricas
")
```

output
```
[1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^
3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2
*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2
*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*
h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^
2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20
*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d^2*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^
2*b^3*c^3 - 64*a^3*b*c^4)*d^2*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 3
2*a^3*b^2*c^3 - 128*a^4*c^4)*d^2*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^
3*c^2 - 64*a^4*b*c^3)*d^2*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 6
4*a^5*c^3)*d^2), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h
)*x^3 + 9*(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(
2*a^2*c^2*g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2
*h)*x^3 + (2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b
*c^2*g - a*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*
x + b)/(b^2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a
^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5
+ a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c...
```

3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 709 vs. $2(133) = 266$.

Time = 1.09 (sec) , antiderivative size = 709, normalized size of antiderivative = 5.06

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$$

$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}}{6bc^2h-12c^3g}\right.}{d^2}$$

$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)}}}{6bc^2h-12c^3g}\right.}{d^2}$$

$$+\frac{-8a^2ch-ab^2h+10abcg-b^3g+x^3(-6bc^2h+12c^3g)+x^2(-9b^2ch}{32a^4c^2d^2-16a^3b^2cd^2+2a^2b^4d^2+x^4\cdot(32a^2c^4d^2-16ab^2c^3d^2+2b^4c^2d^2)+x^3\cdot(64a^2bc^3d^2-32ab^3c^2d^2}$$

input
```
integrate((h*x+g)/(c*x**2+b*x+a)/(c*d*x**2+b*d*x+a*d)**2,x)
```

output
```
3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(
-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c -
b**2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h -
 2*c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h -
 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d**2 - 3*c*sqrt(-1/(4*a*c - b**2)**
5)*(b*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h -
2*c*g) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*
a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4
*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12
*c**3*g))/d**2 + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*
b*c**2*h + 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h
+ 20*a*c**2*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d**2 - 16*a**3*b**2*
c*d**2 + 2*a**2*b**4*d**2 + x**4*(32*a**2*c**4*d**2 - 16*a*b**2*c**3*d**2
+ 2*b**4*c**2*d**2) + x**3*(64*a**2*b*c**3*d**2 - 32*a*b**3*c**2*d**2 + 4*
b**5*c*d**2) + x**2*(64*a**3*c**3*d**2 - 12*a*b**4*c*d**2 + 2*b**6*d**2) +
 x*(64*a**3*b*c**2*d**2 - 32*a**2*b**3*c*d**2 + 4*a*b**5*d**2))
```

3.17.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.17.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.56

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{6\,(2\,c^2g-bch)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4d^2-8\,ab^2cd^2+16\,a^2c^2d^2)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,abchx-b^3g+10\,abcg-}{2\,(b^4d^2-8\,ab^2cd^2+16\,a^2c^2d^2)(cx^2+bx+a)^2}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="giac")
```

output
```
6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d^2 - 8*a
*b^2*c*d^2 + 16*a^2*c^2*d^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b
*c^2*h*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x -
 2*b^3*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^
4*d^2 - 8*a*b^2*c*d^2 + 16*a^2*c^2*d^2)*(c*x^2 + b*x + a)^2)
```

3.17.9 Mupad [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.82

$$\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx = \frac{6\,c\,\mathrm{atan}\left(\frac{d^2\,\left(\frac{6\,c^2\,x\,(b\,h-2\,c\,g)}{d^2\,{\left(4\,a\,c-b^2\right)}^{5/2}}+\frac{3\,c\,(b\,h-2\,c\,g)\,\left(16\,a^2\,b\,c^2\,d^2-8\,a\,b^3\,c\,d^2+b^5\,d^2\right)}{d^4\,{\left(4\,a\,c-b^2\right)}^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2\,g-3\,b\,c\,h}\right)\,(b\,h-2\,c\,g)}{d^2\,{\left(4\,a\,c-b^2\right)}^{5/2}} - \frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,\left(b^2+5\,a\,c\right)\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,c^2\,x^3\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c\,x^2\,(b\,h-2\,c\,g)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^2\,\left(b^2\,d^2+2\,a\,c\,d^2\right)+a^2\,d^2+c^2\,d^2\,x^4+2\,a\,b\,d^2\,x+2\,b\,c\,d^2\,x^3}$$

input
```
int((g + h*x)/((a*d + b*d*x + c*d*x^2)^2*(a + b*x + c*x^2)),x)
```

output
```
(6*c*atan((d^2*((6*c^2*x*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) + (3*c*(
b*h - 2*c*g)*(b^5*d^2 + 16*a^2*b*c^2*d^2 - 8*a*b^3*c*d^2))/(d^4*(4*a*c - b
^2)^(5/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))
/(6*c^2*g - 3*b*c*h))*(b*h - 2*c*g))/(d^2*(4*a*c - b^2)^(5/2)) - ((b^3*g +
 a*b^2*h + 8*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x
*(5*a*c + b^2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*
(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/
(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^2*(b^2*d^2 + 2*a*c*d^2) + a^2*d^2 +
 c^2*d^2*x^4 + 2*a*b*d^2*x + 2*b*c*d^2*x^3)
```

3.17. $\int \frac{g+hx}{(a+bx+cx^2)(ad+bdx+cdx^2)^2}\,dx$

3.18 $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

3.18.1 Optimal result

Integrand size = 34, antiderivative size = 140

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = -\frac{bg-2ah+(2cg-bh)x}{2(b^2-4ac)\,d\,(a+bx+cx^2)^2} + \frac{3(2cg-bh)(b+2cx)}{2(b^2-4ac)^2\,d\,(a+bx+cx^2)} - \frac{6c(2cg-bh)\operatorname{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}\,d}$$

output
```
1/2*(-b*g+2*a*h-(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^2+3/2*(-b*h+2
*c*g)*(2*c*x+b)/(-4*a*c+b^2)^2/d/(c*x^2+b*x+a)-6*c*(-b*h+2*c*g)*arctanh((2
*c*x+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)/d
```

3.18.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.94

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \frac{\frac{(b^2-4ac)(-bg+2ah-2cgx+bhx)}{(a+x(b+cx))^2} + \frac{3(2cg-bh)(b+2cx)}{a+x(b+cx)} - \frac{12c(-2cg+bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)^2\,d}$$

input
```
Integrate[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

output
```
(((b^2 - 4*a*c)*(-(b*g) + 2*a*h - 2*c*g*x + b*h*x))/(a + x*(b + c*x))^2 +
(3*(2*c*g - b*h)*(b + 2*c*x))/(a + x*(b + c*x)) - (12*c*(-2*c*g + b*h)*Arc
Tan[(b + 2*c*x)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*a*c)^
2*d)
```

3.18.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1329, 1159, 1086, 1083, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(a+bx+cx^2)^2\,(ad+bdx+cdx^2)}dx$$

$$\Big\downarrow \text{1329}$$

$$\frac{\int \frac{g+hx}{(cx^2+bx+a)^3}dx}{d}$$

$$\Big\downarrow \text{1159}$$

$$\frac{-\frac{3(2cg-bh)\int \frac{1}{\left(cx^2+bx+a\right)^2}dx}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d}$$

$$\Big\downarrow \text{1086}$$

$$\frac{-\frac{3(2cg-bh)\left(-\frac{2c\int \frac{1}{cx^2+bx+a}dx}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d}$$

$$\Big\downarrow \text{1083}$$

$$\frac{-\frac{3(2cg-bh)\left(\frac{4c\int \frac{1}{b^2-(b+2cx)^2-4ac}d(b+2cx)}{b^2-4ac}-\frac{b+2cx}{\left(b^2-4ac\right)\left(a+bx+cx^2\right)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{-\frac{3(2cg-bh)\left(\frac{4c\,\mathrm{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{b+2cx}{(b^2-4ac)(a+bx+cx^2)}\right)}{2(b^2-4ac)}-\frac{-2ah+x(2cg-bh)+bg}{2(b^2-4ac)(a+bx+cx^2)^2}}{d}$$

input
```
Int[(g + h*x)/((a + b*x + c*x^2)^2*(a*d + b*d*x + c*d*x^2)),x]
```

output
```
(-1/2*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2)^2)
- (3*(2*c*g - b*h)*(-((b + 2*c*x)/((b^2 - 4*a*c)*(a + b*x + c*x^2))) + (4*
c*ArcTanh[(b + 2*c*x)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3/2)))/(2*(b^2 -
4*a*c)))/d
```

3.18.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1086
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && ILtQ[p, -1]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1329
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(c/f)^p   Int[(g + h
*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p
, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c
/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*
x + c*x^2])
```

3.18.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{bg-2ah+(-bh+2cg)x}{2\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)^{2}}+\frac{3(-bh+2cg)\left(\frac{2cx+b}{\left(4ac-b^{2}\right)\left(c\,x^{2}+bx+a\right)}+\frac{4c\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^{2}}}\right)}{\left(4ac-b^{2}\right)^{\frac{3}{2}}}\right)}{2\left(4ac-b^{2}\right)}}{d}$
risch	$\frac{-\frac{3c^{2}(bh-2cg)x^{3}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{9bc(bh-2cg)x^{2}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(5abch-10a\,c^{2}g+b^{3}h-2b^{2}cg\right)x}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{8a^{2}ch+a\,b^{2}h-10abcg+b^{3}g}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{2}+bx+a\right)^{2}d}-\frac{3c\ln\left(\left(32a^{2}c^{3}-16a\,b^{2}c^{2}+2\right.\right.}{}$

input
```
int((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(4*a*c-b^2)/(c*x^2+b*x+a)^2+3/2*(-b*h+
2*c*g)/(4*a*c-b^2)*((2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)+4*c/(4*a*c-b^2)^(3
/2)*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))))
```

3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 555 vs. $2(132) = 264$.

Time = 0.34 (sec) , antiderivative size = 1130, normalized size of antiderivative = 8.07

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="fricas
")
```

output
```
[1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*(2*(b^
3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 6*(2*a^2*c^2*g - a^2
*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 + (2*(b^2
*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a*b^2*c*
h)*x)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^2 + 2*b*c*x + b^2 - 2*a*c + sqrt(b^2
- 4*a*c)*(2*c*x + b))/(c*x^2 + b*x + a)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^
2)*g - (a*b^4 + 4*a^2*b^2*c - 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20
*a^2*c^3)*g - (b^5 + a*b^3*c - 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^
3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*d*x^4 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*
b^3*c^3 - 64*a^3*b*c^4)*d*x^3 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^
3*b^2*c^3 - 128*a^4*c^4)*d*x^2 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2
- 64*a^4*b*c^3)*d*x + (a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^
3)*d), 1/2*(6*(2*(b^2*c^3 - 4*a*c^4)*g - (b^3*c^2 - 4*a*b*c^3)*h)*x^3 + 9*
(2*(b^3*c^2 - 4*a*b*c^3)*g - (b^4*c - 4*a*b^2*c^2)*h)*x^2 - 12*(2*a^2*c^2*
g - a^2*b*c*h + (2*c^4*g - b*c^3*h)*x^4 + 2*(2*b*c^3*g - b^2*c^2*h)*x^3 +
(2*(b^2*c^2 + 2*a*c^3)*g - (b^3*c + 2*a*b*c^2)*h)*x^2 + 2*(2*a*b*c^2*g - a
*b^2*c*h)*x)*sqrt(-b^2 + 4*a*c)*arctan(-sqrt(-b^2 + 4*a*c)*(2*c*x + b)/(b^
2 - 4*a*c)) - (b^5 - 14*a*b^3*c + 40*a^2*b*c^2)*g - (a*b^4 + 4*a^2*b^2*c -
 32*a^3*c^2)*h + 2*(2*(b^4*c + a*b^2*c^2 - 20*a^2*c^3)*g - (b^5 + a*b^3*c
- 20*a^2*b*c^2)*h)*x)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^...
```

3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(128) = 256$.

Time = 1.08 (sec) , antiderivative size = 680, normalized size of antiderivative = 4.86

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$$
$$= \frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}}{6bc^2h-12c^3g}\right.}{d}$$
$$-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)\log\left(x+\frac{192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(bh-2cg)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)}}}{6bc^2h-12c^3g}\right.}{d}$$
$$+\frac{-8a^2ch-ab^2h+10abcg-b^3g+x^3(-6bc^2h+12c^3g)+x^2(-9b^2ch+18bc}{32a^4c^2d-16a^3b^2cd+2a^2b^4d+x^4\cdot(32a^2c^4d-16ab^2c^3d+2b^4c^2d)+x^3\cdot(64a^2bc^3d-32ab^3c^2d+4b^5cd}$$

input
```
integrate((h*x+g)/(c*x**2+b*x+a)**2/(c*d*x**2+b*d*x+a*d),x)
```

output
```
3*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g)*log(x + (-192*a**3*c**4*sqrt(
-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c -
b**2)**5)*(b*h - 2*c*g) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h -
 2*c*g) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h -
 6*b*c**2*g)/(6*b*c**2*h - 12*c**3*g))/d - 3*c*sqrt(-1/(4*a*c - b**2)**5)*
(b*h - 2*c*g)*log(x + (192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c
*g) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) + 36*a*b
**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(b*h - 2*c*g) - 3*b**6*c*sqrt(-1/(4*a*
c - b**2)**5)*(b*h - 2*c*g) + 3*b**2*c*h - 6*b*c**2*g)/(6*b*c**2*h - 12*c*
*3*g))/d + (-8*a**2*c*h - a*b**2*h + 10*a*b*c*g - b**3*g + x**3*(-6*b*c**2
*h + 12*c**3*g) + x**2*(-9*b**2*c*h + 18*b*c**2*g) + x*(-10*a*b*c*h + 20*a
*c**2*g - 2*b**3*h + 4*b**2*c*g))/(32*a**4*c**2*d - 16*a**3*b**2*c*d + 2*a
**2*b**4*d + x**4*(32*a**2*c**4*d - 16*a*b**2*c**3*d + 2*b**4*c**2*d) + x*
*3*(64*a**2*b*c**3*d - 32*a*b**3*c**2*d + 4*b**5*c*d) + x**2*(64*a**3*c**3
*d - 12*a*b**4*c*d + 2*b**6*d) + x*(64*a**3*b*c**2*d - 32*a**2*b**3*c*d +
4*a*b**5*d))
```

3.18.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="maxima
")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.18.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.48

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \frac{6\,(2\,c^2g-bch)\arctan\left(\frac{2\,cx+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4d-8\,ab^2cd+16\,a^2c^2d)\sqrt{-b^2+4\,ac}} + \frac{12\,c^3gx^3-6\,bc^2hx^3+18\,bc^2gx^2-9\,b^2chx^2+4\,b^2cgx+20\,ac^2gx-2\,b^3hx-10\,abchx-b^3g+10\,abcg-}{2\,(b^4d-8\,ab^2cd+16\,a^2c^2d)(cx^2+bx+a)^2}$$

input
```
integrate((h*x+g)/(c*x^2+b*x+a)^2/(c*d*x^2+b*d*x+a*d),x, algorithm="giac")
```

output
```
6*(2*c^2*g - b*c*h)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((b^4*d - 8*a*b
^2*c*d + 16*a^2*c^2*d)*sqrt(-b^2 + 4*a*c)) + 1/2*(12*c^3*g*x^3 - 6*b*c^2*h
*x^3 + 18*b*c^2*g*x^2 - 9*b^2*c*h*x^2 + 4*b^2*c*g*x + 20*a*c^2*g*x - 2*b^3
*h*x - 10*a*b*c*h*x - b^3*g + 10*a*b*c*g - a*b^2*h - 8*a^2*c*h)/((b^4*d -
8*a*b^2*c*d + 16*a^2*c^2*d)*(c*x^2 + b*x + a)^2)
```

3.18.9 Mupad [B] (verification not implemented)

Time = 12.76 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.68

$$\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx = \frac{6\,c\,\mathrm{atan}\left(\frac{d\left(\frac{6\,c^2\,x\,(b\,h-2\,c\,g)}{d\,\left(4\,a\,c-b^2\right)^{5/2}}+\frac{3\,c\,(b\,h-2\,c\,g)\,\left(16\,d\,a^2\,b\,c^2-8\,d\,a\,b^3\,c+d\,b^5\right)}{d^2\,\left(4\,a\,c-b^2\right)^{5/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}{6\,c^2\,g-3\,b\,c\,h}\right)\,(b\,h-2\,c\,g)}{d\,(4\,a\,c-b^2)^{5/2}} - \frac{\frac{8\,c\,h\,a^2+h\,a\,b^2-10\,c\,g\,a\,b+g\,b^3}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x\,(b^2+5\,a\,c)\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{3\,c^2\,x^3\,(b\,h-2\,c\,g)}{16\,a^2\,c^2-8\,a\,b^2\,c+b^4}+\frac{9\,b\,c\,x^2\,(b\,h-2\,c\,g)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{a^2\,d+x^2\,\left(d\,b^2+2\,a\,c\,d\right)+c^2\,d\,x^4+2\,b\,c\,d\,x^3+2\,a\,b\,d\,x}$$

input
```
int((g + h*x)/((a*d + b*d*x + c*d*x^2)*(a + b*x + c*x^2)^2),x)
```

output
```
(6*c*atan((d*((6*c^2*x*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) + (3*c*(b*h
- 2*c*g)*(b^5*d - 8*a*b^3*c*d + 16*a^2*b*c^2*d))/(d^2*(4*a*c - b^2)^(5/2)*
(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))/(6*c^2*g
- 3*b*c*h))*(b*h - 2*c*g))/(d*(4*a*c - b^2)^(5/2)) - ((b^3*g + a*b^2*h + 8
*a^2*c*h - 10*a*b*c*g)/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*a*c + b^
2)*(b*h - 2*c*g))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (3*c^2*x^3*(b*h - 2*c*g
))/(b^4 + 16*a^2*c^2 - 8*a*b^2*c) + (9*b*c*x^2*(b*h - 2*c*g))/(2*(b^4 + 16
*a^2*c^2 - 8*a*b^2*c)))/(a^2*d + x^2*(b^2*d + 2*a*c*d) + c^2*d*x^4 + 2*b*c
*d*x^3 + 2*a*b*d*x)
```

3.18. $\int \frac{g+hx}{(a+bx+cx^2)^2(ad+bdx+cdx^2)}\,dx$

3.19 $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.19.1 Optimal result

Integrand size = 32, antiderivative size = 617

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$= \frac{B\sqrt{a+bx+cx^2}}{f} - \frac{(2Bce-bBf-2Acf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$

$$+ \frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)(Af(ce-bf)+B(f(be-af)-c(e^2-df)))\right)\text{ar}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{}}}$$

$$- \frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e+\sqrt{e^2-4df}\right)(Af(ce-bf)+B(f(be-af)-c(e^2-df)))\right)\text{ar}}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{}}}$$

output
```
-1/2*(-2*A*c*f-B*b*f+2*B*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^
(1/2))/f^2/c^(1/2)+B*(c*x^2+b*x+a)^(1/2)/f+1/2*arctanh(1/4*(4*a*f+2*x*(b*f
-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)
^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*
(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-a*f+b*e)-c*(-d
*f+e^2)))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*
c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(
4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)
/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^
(1/2))^(1/2))*(2*f*(A*f*(-a*f+c*d)-B*d*(-b*f+c*e))-(A*f*(-b*f+c*e)+B*(f*(-
a*f+b*e)-c*(-d*f+e^2)))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(
1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.19.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.28 (sec) , antiderivative size = 1115, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{Bf\sqrt{a+x(b+cx)}+\frac{(-2Bce+bBf+2Acf)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right)}{\sqrt{c}}-\operatorname{RootSum}\Big[c^2d-bce+b^2f+2\sqrt{a}ce\#1}{}$$

input
```
Integrate[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(B*f*Sqrt[a + x*(b + c*x)] + ((-2*B*c*e + b*B*f + 2*A*c*f)*ArcTanh[(Sqrt[c
]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])])/Sqrt[c] - RootSum[c^2*d - b*c*e
+ b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*
a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(B*c^2*d*e*Log[x]) + b*B*c*e^2*
Log[x] + A*c^2*d*f*Log[x] - b^2*B*e*f*Log[x] - A*b*c*e*f*Log[x] + A*b^2*f^
2*Log[x] + a*b*B*f^2*Log[x] - a*A*c*f^2*Log[x] + B*c^2*d*e*Log[-Sqrt[a] +
Sqrt[a + b*x + c*x^2] - x*#1] - b*B*c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*
x^2] - x*#1] - A*c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^
2*B*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + A*b*c*e*f*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1] - A*b^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*
x + c*x^2] - x*#1] - a*b*B*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
] + a*A*c*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*B*c
*e^2*Log[x]*#1 + 2*Sqrt[a]*B*c*d*f*Log[x]*#1 + 2*Sqrt[a]*b*B*e*f*Log[x]*#1
 + 2*Sqrt[a]*A*c*e*f*Log[x]*#1 - 2*Sqrt[a]*A*b*f^2*Log[x]*#1 - 2*a^(3/2)*B
*f^2*Log[x]*#1 + 2*Sqrt[a]*B*c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
x*#1]*#1 - 2*Sqrt[a]*B*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*
#1 - 2*Sqrt[a]*b*B*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 - 2
*Sqrt[a]*A*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*Sqrt[
a]*A*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + 2*a^(3/2)*B*f
^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + B*c*d*e*Log[x]*#1^...
```

3.19.3 Rubi [A] (warning: unable to verify)

Time = 1.48 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.281$, Rules used = {1352, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \ 1352$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{(2Bce-bBf-2Acf)x^2-(2Abf-B(2cd+be-2af))x+bBd-2aAf}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{(2Bce-bBf-2Acf)x^2-(2Abf-B(2cd+be-2af))x+bBd-2aAf}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2f}$$

$\Big\downarrow$ 2143

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\frac{\int \frac{2\left(Af(cd-af)-\frac{1}{2}B(2cde-2bdf)+\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{(-2Acf-bBf+2Bce)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{2f}$$

$\Big\downarrow$ 27

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\frac{2\int \frac{Af(cd-af)-B(cde-bdf)+\left(Af(ce-bf)-B\left(ce^2-bfe+af^2-cdf\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{(-2Acf-bBf+2Bce)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{2f}$$

$\Big\downarrow$ 1092

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\frac{2\int \frac{Af(cd-af)-B(cde-bdf)+\left(Af(ce-bf)-B\left(ce^2-bfe+af^2-cdf\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{2(-2Acf-bBf+2Bce)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}}{2f}$$

$\Big\downarrow$ 219

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\frac{2\int \frac{Af(cd-af)-B(cde-bdf)+\left(Af(ce-bf)-B\left(ce^2-bfe+af^2-cdf\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-2Acf-bBf+2Bce)}{\sqrt{c}f}}{2f}$$

$\Big\downarrow$ 1365

$$\frac{B\sqrt{a+bx+cx^2}}{f}-\frac{\frac{2\left(\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(e-\sqrt{e^2-4df}\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2f(Af(cd-af)-Bd(ce-b\right.}{}\right.}{f}}{}$$

$\Big\downarrow$ 1154

3.19. $\displaystyle \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\frac{B\sqrt{a+bx+cx^2}}{f}-$$

$$2\left(\frac{2\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\int\frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{\right.\right.}}{}}}{\sqrt{e^2-4df}}\right.$$

$$\downarrow 219$$

$$\frac{B\sqrt{a+bx+cx^2}}{f}-$$

$$2\left(\frac{\left(2f(Af(cd-af)-Bd(ce-bf))-\left(\sqrt{e^2-4df}+e\right)\left(Bf(be-af)+Af(ce-bf)-Bc\left(e^2-df\right)\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.$$

input
```
Int[((A + B*x)*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(B*Sqrt[a + b*x + c*x^2])/f - (((2*B*c*e - b*B*f - 2*A*c*f)*ArcTanh[(b + 2
*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f) + (2*(-(((2*f*(A*f*(
c*d - a*f) - B*d*(c*e - b*f)) - (e - Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) +
 A*f*(c*e - b*f) - B*c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((2*f*(A*f*(c*d - a*f) - B*d*(c*e - b*
f)) - (e + Sqrt[e^2 - 4*d*f])*(B*f*(b*e - a*f) + A*f*(c*e - b*f) - B*c*(e^
2 - d*f)))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sq
rt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (
c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 -
 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*
d*f]])))/f)/(2*f)
```

3.19. $\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1352
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))
Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*
(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1
))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*
d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.19.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1175 vs. $2(558) = 1116$.

Time = 0.99 (sec) , antiderivative size = 1176, normalized size of antiderivative = 1.91

method	result	size
risch	Expression too large to display	1176
default	Expression too large to display	1595

input
```
int((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
B*(c*x^2+b*x+a)^(1/2)/f+1/2/f*(1/f*(2*A*c*f+B*b*f-2*B*c*e)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*(2*A*b*f^2*(-4*d*f+e^2)^(1/2)-2*A
*c*e*f*(-4*d*f+e^2)^(1/2)+4*A*a*f^3-2*A*b*e*f^2-4*A*c*d*f^2+2*A*c*e^2*f+2*
B*a*f^2*(-4*d*f+e^2)^(1/2)-2*B*b*e*f*(-4*d*f+e^2)^(1/2)-2*B*c*d*f*(-4*d*f+
e^2)^(1/2)+2*B*c*e^2*(-4*d*f+e^2)^(1/2)-2*B*a*e*f^2-4*B*b*d*f^2+2*B*b*e^2*
f+6*B*c*d*e*f-2*B*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)
^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1
/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*
f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(2*A*b*f^2*(-4*d*f+e^2)^(1/2)-2*
A*c*e*f*(-4*d*f+e^2)^(1/2)-4*A*a*f^3+2*A*b*e*f^2+4*A*c*d*f^2-2*A*c*e^2*f+2
*B*a*f^2*(-4*d*f+e^2)^(1/2)-2*B*b*e*f*(-4*d*f+e^2)^(1/2)-2*B*c*d*f*(-4*d*f
+e^2)^(1/2)+2*B*c*e^2*(-4*d*f+e^2)^(1/2)+2*B*a*e*f^2+4*B*b*d*f^2-2*B*b*e^2
*f-6*B*c*d*e*f+2*B*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^
2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f...
```

3.19.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.19.6 Sympy [F]

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral((A + B*x)*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.19.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.19.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.19.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{(A+B\,x)\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.20 $\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.20.1 Optimal result

Integrand size = 32, antiderivative size = 1092

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx =$$

$$-\frac{(2Acf(4ce-5bf)-B(b^2f^2-2cf(5be-4af)+8c^2(e^2-df))+2cf(2Bce-bBf-2Acf)x)\sqrt{a+bx+}}{8cf^3}$$

$$+\frac{B(a+bx+cx^2)^{3/2}}{3f}$$

$$+\frac{(2Acf(3b^2f^2-12cf(be-af)+8c^2(e^2-df))-B(b^3f^3+6bcf^2(be-2af)-24c^2f(be^2-bdf-aef)+1}{16c^{3/2}f^4}$$

$$-\frac{(2cf(Bd(ce-bf)(ce^2-2cdf-bef+2af^2)+Af(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))-c(}{}$$

$$+\frac{(2f(Bd(ce-bf)(ce^2-2cdf-bef+2af^2)+Af(2cdf(be-af)-f^2(b^2d-a^2f)-c^2d(e^2-df)))-(e+}{}$$

output
```
1/3*B*(c*x^2+b*x+a)^(3/2)/f+1/16*(2*A*c*f*(3*b^2*f^2-12*c*f*(-a*f+b*e)+8*c
^2*(-d*f+e^2))-B*(b^3*f^3+6*b*c*f^2*(-2*a*f+b*e)-24*c^2*f*(-a*e*f-b*d*f+b*
e^2)+16*c^3*(-2*d*e*f+e^3)))*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(3/2)/f^4-1/8*(2*A*c*f*(-5*b*f+4*c*e)-B*(b^2*f^2-2*c*f*(-4*a*f+5*b
*e)+8*c^2*(-d*f+e^2))+2*c*f*(-2*A*c*f-B*b*f+2*B*c*e)*x)*(c*x^2+b*x+a)^(1/2
)/c/f^3-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4
*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2
-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*c*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e
*f-2*c*d*f+c*e^2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e
^2)))-c*(A*f*(-b*f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*
d*e^2*f+e^4)-f^2*(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-
b*(-2*d*e*f+e^3))))*(e-(-4*d*f+e^2)^(1/2)))/c/f^4*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*a
rctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2
))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(
-4*d*f+e^2)^(1/2))^(1/2))*(2*f*(B*d*(-b*f+c*e)*(2*a*f^2-b*e*f-2*c*d*f+c*e^
2)+A*f*(2*c*d*f*(-a*f+b*e)-f^2*(-a^2*f+b^2*d)-c^2*d*(-d*f+e^2)))-(A*f*(-b*
f+c*e)*(f*(-2*a*f+b*e)-c*(-2*d*f+e^2))+B*(c^2*(d^2*f^2-3*d*e^2*f+e^4)-f^2*
(2*a*b*e*f-a^2*f^2-b^2*(-d*f+e^2))+2*c*f*(a*f*(-d*f+e^2)-b*(-2*d*e*f+e^3))
))*(e+(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*...
```

3.20.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.53 (sec) , antiderivative size = 3516, normalized size of antiderivative = 3.22

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*f*Sqrt[a + x*(b + c*x)]*(6*A*c*f*(-4*c*e + 5*b*f + 2*c*f*x) + B*(
3*b^2*f^2 + 2*c*f*(-15*b*e + 16*a*f + 7*b*f*x) + 4*c^2*(6*e^2 - 6*d*f - 3*
e*f*x + 2*f^2*x^2))) + 3*(8*A*c^2*(2*c*d + 3*b*e)*f^2 + B*(16*c^3*e^3 + 6*
b^2*c*e*f^2 + 24*c^2*(b*d + a*e)*f^2 + b^3*f^3))*ArcTanh[(Sqrt[c]*x)/(Sqrt
[a] - Sqrt[a + x*(b + c*x)])] + 6*c*f*(2*B*(8*c^2*d*e + 6*b*c*e^2 + 3*a*b*
f^2) + A*(8*c^2*e^2 + 3*b^2*f^2 + 12*a*c*f^2))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[
a] + Sqrt[a + x*(b + c*x)])] + 24*c^(3/2)*RootSum[c^2*d - b*c*e + b^2*f +
2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 -
 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (B*c^3*d*e^3*Log[x] - b*B*c^2*e^4*Log[x] -
2*B*c^3*d^2*e*f*Log[x] + b*B*c^2*d*e^2*f*Log[x] - A*c^3*d*e^2*f*Log[x] + 2
*b^2*B*c*e^3*f*Log[x] + A*b*c^2*e^3*f*Log[x] + b*B*c^2*d^2*f^2*Log[x] + A*
c^3*d^2*f^2*Log[x] - 3*b^2*B*c*d*e*f^2*Log[x] + 2*a*B*c^2*d*e*f^2*Log[x] -
 b^3*B*e^2*f^2*Log[x] - 2*A*b^2*c*e^2*f^2*Log[x] - 2*a*b*B*c*e^2*f^2*Log[x
] + b^3*B*d*f^3*Log[x] + A*b^2*c*d*f^3*Log[x] - 2*a*A*c^2*d*f^3*Log[x] + A
*b^3*e*f^3*Log[x] + 2*a*b^2*B*e*f^3*Log[x] + 2*a*A*b*c*e*f^3*Log[x] - 2*a*
A*b^2*f^4*Log[x] - a^2*b*B*f^4*Log[x] + a^2*A*c*f^4*Log[x] - B*c^3*d*e^3*L
og[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b*B*c^2*e^4*Log[-Sqrt[a] + S
qrt[a + b*x + c*x^2] - x*#1] + 2*B*c^3*d^2*e*f*Log[-Sqrt[a] + Sqrt[a + b*x
 + c*x^2] - x*#1] - b*B*c^2*d*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1] + A*c^3*d*e^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2...
```

3.20.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{1352}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{\int \frac{3\sqrt{cx^2+bx+a}\left((2Bce-bBf-2Acf)x^2-(2Abf-B(2cd+be-2af))x+bBd-2aAf\right)}{2(fx^2+ex+d)}dx}{3f}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{\int \frac{\sqrt{cx^2+bx+a}\left((2Bce-bBf-2Acf)x^2-(2Abf-B(2cd+be-2af))x+bBd-2aAf\right)}{fx^2+ex+d}dx}{2f}$$

$$\Big\downarrow \text{2138}$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$\frac{\sqrt{a+bx+cx^2}\left(-B\left(-2cf(5be-4af)+b^2f^2+8c^2\left(e^2-df\right)\right)+2cfx(-2Acf-bBf+2Bce)+2Acf(4ce-5bf)\right)}{4cf^2}-\frac{\int -\frac{Bdf^2b^3-10cdf(Be-Af)b^2-4cd\left(2Ace\cdots\right.}{\cdots}}{\cdots}$$

$\downarrow$ 27

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$\frac{\int \frac{Bdf^2b^3-10cdf(Be-Af)b^2-4cd\left(-5aBf^2+2Acef-2Bc\left(e^2-df\right)\right)b-\left(2Acf\left(8\left(e^2-df\right)c^2-12f(be-af)c+3b^2f^2\right)-B\left(16\left(e^3-2def\right)c^3-24f\left(be^2-afe-bdf\right)c^2+6\cdots\right.\right.}{\cdots}}{\cdots}$$

$\downarrow$ 2143

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$\frac{\frac{\int \frac{16c\left(Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$\downarrow$ 27

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$\frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$-\frac{\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$\downarrow$ 25

3.20. $\displaystyle\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}\cdots}{f}\cdots}{\cdots}$$

$\big\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}\cdots}{f}\cdots}{\cdots}$$

$\big\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}\cdots}{f}\cdots}{\cdots}$$

$\big\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}\cdots}{f}\cdots}{\cdots}$$

$\big\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}\cdots}{f}\cdots}{\cdots}$$

$\big\downarrow$ 25

3.20. $\displaystyle\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right.\right.\right.\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}\cdots$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}\cdots$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right.\right.\right.\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}\cdots$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}\cdots$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - \frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right.\right.\right.\cdots}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}\cdots$$

$\downarrow$ 25

3.20. $\displaystyle\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - $$
$$\frac{\dfrac{16c\int -\dfrac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - $$
$$-\frac{\dfrac{16c\int -\dfrac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - $$
$$\frac{\dfrac{16c\int -\dfrac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - $$
$$-\frac{\dfrac{16c\int -\dfrac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f} - $$
$$\frac{\dfrac{16c\int -\dfrac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

3.20. $\displaystyle\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{\frac{16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+\cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-\frac{-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\cdots\right.\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}}{\cdots}$$

$$\Big\downarrow \; 25$$

3.20. $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+ \cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)} \over f$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$-\frac{16c\int -\frac{Bd(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)-Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)-\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2 \cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}}{f}$$

$\downarrow$ 25

$$\frac{B\left(a+bx+cx^2\right)^{3/2}}{3f}-$$

$$16c\int -\frac{Bd(ce-bf)\left(ce^2-bfe+2af^2-2cdf\right)+Af\left(-d\left(e^2-df\right)c^2+2df(be-af)c-f^2\left(b^2d-a^2f\right)\right)+\left(Af(ce-bf)\left(f(be-2af)-c\left(e^2-2df\right)\right)+B\left(\left(e^4-3dfe^2+d^2f^2\right)c^2+ \cdots\right.\right.}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)} \over f$$

input
```
Int[((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
$Aborted
```

3.20.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

3.20. $\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

rule 1352
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))
Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*
(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1
))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*
d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.20. $\int \frac{(A+Bx)(a+bx+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. 2(1027) = 2054.

Time = 1.08 (sec) , antiderivative size = 2278, normalized size of antiderivative = 2.09

method	result	size
risch	Expression too large to display	2278
default	Expression too large to display	2908

input
```
int((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/24/c*(8*B*c^2*f^2*x^2+12*A*c^2*f^2*x+14*B*b*c*f^2*x-12*B*c^2*e*f*x+30*A*
b*c*f^2-24*A*c^2*e*f+32*B*a*c*f^2+3*B*b^2*f^2-30*B*b*c*e*f-24*B*c^2*d*f+24
*B*c^2*e^2)*(c*x^2+b*x+a)^(1/2)/f^3+1/16/f^3/c*(1/f*(24*A*a*c^2*f^3+6*A*b^
2*c*f^3-24*A*b*c^2*e*f^2-16*A*c^3*d*f^2+16*A*c^3*e^2*f+12*B*a*b*c*f^3-24*B
*a*c^2*e*f^2-B*b^3*f^3-6*B*b^2*c*e*f^2-24*B*b*c^2*d*f^2+24*B*b*c^2*e^2*f+3
2*B*c^3*d*e*f-16*B*c^3*e^3)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^
(1/2)-8/f^2*c*(-2*A*a*c*e*f^3*(-4*d*f+e^2)^(1/2)-2*A*f^3*b*c*d*(-4*d*f+e^2
)^(1/2)+2*A*b*c*e^2*f^2*(-4*d*f+e^2)^(1/2)+2*A*c^2*d*e*f^2*(-4*d*f+e^2)^(1
/2)-2*B*a*b*e*f^3*(-4*d*f+e^2)^(1/2)-2*B*a*c*d*f^3*(-4*d*f+e^2)^(1/2)+2*B*
a*c*e^2*f^2*(-4*d*f+e^2)^(1/2)-2*B*b*c*e^3*f*(-4*d*f+e^2)^(1/2)-3*B*c^2*d*
e^2*f*(-4*d*f+e^2)^(1/2)+6*A*b*c*d*e*f^3+6*B*a*c*d*e*f^3-8*B*b*c*d*e^2*f^2
-B*b^2*e^3*f^2+A*c^2*e^4*f-B*a^2*e*f^4-2*A*b^2*d*f^4+A*b^2*e^2*f^3+2*A*c^2
*d^2*f^3+4*B*b*c*d*e*f^2*(-4*d*f+e^2)^(1/2)+2*A*a^2*f^5-B*c^2*e^5-4*A*a*c*
d*f^4+2*A*a*c*e^2*f^3-2*A*b*c*e^3*f^2-4*A*c^2*d*e^2*f^2-4*B*a*b*d*f^4+2*B*
a*b*e^2*f^3-2*B*a*c*e^3*f^2+3*B*b^2*d*e*f^3+4*B*b*c*d^2*f^3+2*B*b*c*e^4*f-
5*B*c^2*d^2*e*f^2+5*B*c^2*d*e^3*f-2*A*a*b*e*f^4+B*a^2*f^4*(-4*d*f+e^2)^(1/
2)+B*c^2*e^4*(-4*d*f+e^2)^(1/2)+2*A*a*b*f^4*(-4*d*f+e^2)^(1/2)-A*f^3*b^2*e
*(-4*d*f+e^2)^(1/2)-A*f*c^2*e^3*(-4*d*f+e^2)^(1/2)-B*b^2*d*f^3*(-4*d*f+e^2
)^(1/2)+B*f^2*b^2*e^2*(-4*d*f+e^2)^(1/2)+B*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2))
/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)...
```

3.20.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.20.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.20.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.20.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)*(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.20.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(a+bx+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(A+B\,x)\,\left(c\,x^2+b\,x+a\right)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

output
```
int(((A + B*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.21 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$

3.21.1 Optimal result

Integrand size = 32, antiderivative size = 416

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf+\sqrt{b^2-4ac}(ce-bf)}} + \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)\operatorname{arctanh}\left(\frac{4cd-\left(b+\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b+\sqrt{b^2-4ac}\right)f\right)x}{2\sqrt{2}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}\sqrt{d+ex+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-bce+b^2f-2acf-\sqrt{b^2-4ac}(ce-bf)}}$$

output
```
1/2*arctanh(1/4*(4*c*d-e*(b+(-4*a*c+b^2)^(1/2))+2*x*(c*e-f*(b+(-4*a*c+b^2)
^(1/2))))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c
*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(2*A*c-B*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-
4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f-(-b*f+c*e)*(-4*a*c+b^2)^(1/2
))^(1/2)+1/2*arctanh(1/4*(4*c*d+2*x*(c*e-f*(b-(-4*a*c+b^2)^(1/2)))-e*(b-(-
4*a*c+b^2)^(1/2)))*2^(1/2)/(f*x^2+e*x+d)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*
f+(-b*f+c*e)*(-4*a*c+b^2)^(1/2))^(1/2))*(B*b-2*A*c-B*(-4*a*c+b^2)^(1/2))*2
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-b*c*e+b^2*f-2*a*c*f+(-b*f+c*e)*(-4*a*c+
b^2)^(1/2))^(1/2)
```

3.21.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 278, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = -\text{RootSum}\Big[cd^2-bde+ae^2+2bd\sqrt{f}\#1 - 4ae\sqrt{f}\#1-2cd\#1^2+be\#1^2+4af\#1^2-2b\sqrt{f}\#1^3 + c\#1^4\&, \frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)-Ae\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)+2A\sqrt{f}\,\text{lo}}{bd\sqrt{f}-2ae\sqrt{f}-2cd\#1+be\#1+4a}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-RootSum[c*d^2 - b*d*e + a*e^2 + 2*b*d*Sqrt[f]*#1 - 4*a*e*Sqrt[f]*#1 - 2*c
*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*#1^3 + c*#1^4 & , (B*d*Log[-
(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt[f]*x) + Sqrt[d
+ e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2
] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1]*#1^2)/(b*d*S
qrt[f] - 2*a*e*Sqrt[f] - 2*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*#1^2 +
 2*c*#1^3) & ]
```

3.21.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.094, Rules used = {1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

$$\Big\downarrow \text{1365}$$

$$-\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\int \frac{1}{\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{fx^2+ex+d}}dx}{\sqrt{b^2-4ac}} - \frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\left(b+2cx+\sqrt{b^2-4ac}\right)\sqrt{fx^2+ex+d}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{2\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\int \frac{1}{4\left(4dc^2-2\left(b-\sqrt{b^2-4ac}\right)ec+\left(b-\sqrt{b^2-4ac}\right)^2f\right)-\frac{\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b-\sqrt{b^2-4ac}\right)f\right)x\right)^2}{fx^2+ex+d}}d\frac{4cd}{}}{\sqrt{b^2-4ac}}$$

$$\frac{2\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{4\left(4dc^2-2\left(b+\sqrt{b^2-4ac}\right)ec+\left(b+\sqrt{b^2-4ac}\right)^2f\right)-\frac{\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e+2\left(ce-\left(b+\sqrt{b^2-4ac}\right)f\right)x\right)^2}{fx^2+ex+d}}d\frac{4cd-}{}}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\operatorname{arctanh}\left(\frac{2x\left(ce-f\left(b-\sqrt{b^2-4ac}\right)\right)-e\left(b-\sqrt{b^2-4ac}\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\operatorname{arctanh}\left(\frac{2x\left(ce-f\left(\sqrt{b^2-4ac}+b\right)\right)-e\left(\sqrt{b^2-4ac}+b\right)+4cd}{2\sqrt{2}\sqrt{d+ex+fx^2}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}(ce-bf)-2acf+b^2f-bce+2c^2d}}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

output
```
((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(4*c*d - (b - Sqrt[b^2 - 4*a*
c])*e + 2*(c*e - (b - Sqrt[b^2 - 4*a*c])*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b
*c*e + b^2*f - 2*a*c*f + Sqrt[b^2 - 4*a*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x
^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f +
 Sqrt[b^2 - 4*a*c]*(c*e - b*f)]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*Ar
cTanh[(4*c*d - (b + Sqrt[b^2 - 4*a*c])*e + 2*(c*e - (b + Sqrt[b^2 - 4*a*c]
)*f)*x)/(2*Sqrt[2]*Sqrt[2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a
*c]*(c*e - b*f)]*Sqrt[d + e*x + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[
2*c^2*d - b*c*e + b^2*f - 2*a*c*f - Sqrt[b^2 - 4*a*c]*(c*e - b*f)])
```

3.21. $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$

3.21.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 804 vs. $2(370) = 740$.

Time = 1.01 (sec) , antiderivative size = 805, normalized size of antiderivative = 1.94

method	result
default	$-\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\ln\left(\frac{-\frac{-\sqrt{-4ac+b^2}\,bf+ce\sqrt{-4ac+b^2}+2acf-b^2f+bce-2c^2d}{c^2}-\frac{\left(f\sqrt{-4ac+b^2}+bf-ce\right)\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}{c}+\sqrt{\cdots}}{\cdots}\right.}{\sqrt{-4ac+b\cdots}}$

input
```
int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-2*A*c+B*(-4*a*c+b^2)^(1/2)+B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)
^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*
ln((-(-(-4*a*c+b^2)^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2
*c^2*d)/c^2-(f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))
/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+
b*c*e-2*c^2*d)/c^2)^(1/2)*(4*f*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-4*(f*(-4
*a*c+b^2)^(1/2)+b*f-c*e)/c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^
2)^(1/2)*b*f+c*e*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2
))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))-(2*A*c+B*(-4*a*c+b^2)^(1/2)-B*b)/(-4*
a*c+b^2)^(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)^(1/2)+2*a*c*
f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c
+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2-(-f*(-4*a*c+b^2)^(1/2)+b*f-c*
e)/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-2*((-4*a*c+b^2)^(1/2)*b*f-c*e
*(-4*a*c+b^2)^(1/2)+2*a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2)*(4*f*(x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))^2-4*(-f*(-4*a*c+b^2)^(1/2)+b*f-c*e)/c*(x-1/2/c*(-
b+(-4*a*c+b^2)^(1/2)))-2*((-4*a*c+b^2)^(1/2)*b*f-c*e*(-4*a*c+b^2)^(1/2)+2*
a*c*f-b^2*f+b*c*e-2*c^2*d)/c^2)^(1/2))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
```

3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22103 vs. $2(369) = 738$.

Time = 277.46 (sec) , antiderivative size = 22103, normalized size of antiderivative = 53.13

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.21.6 Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x)/((a + b*x + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.21.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.21.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,
0]%%%}+%%
```

3.21.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+B\,x}{(c\,x^2+b\,x+a)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((a + b*x + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.22 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

3.22.1 Optimal result

Integrand size = 29, antiderivative size = 780

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$
$$= \frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)}\sqrt{-Ace+B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-}\right.}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2d^2}}$$
$$-\frac{\sqrt{-Ace+B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)}\sqrt{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e\right.}}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2}}$$

output
```
-1/2*arctanh(1/2*e^(1/2)*(a*(A*c*e-B*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f
+e^2))^(1/2)))-c*x*(B*a*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1
/2))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2)/(-A*c*e+B*(c*d-a*f-(c^2
*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(B*a*e+A*(c*d-a*f+(c^2*d^2+a^
2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(-A*c*e+B*(c*d-a*f-(c^2*d^2+a^2*f^2
+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*(B*a*e+A*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-
2*d*f+e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)/e^(1/2)/(c^2*d^2+a^2*f^2
+a*c*(-2*d*f+e^2))^(1/2)+1/2*arctanh(1/2*e^(1/2)*(-c*x*(B*a*e+A*(c*d-a*f-(
c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))+a*(A*c*e-B*(c*d-a*f+(c^2*d^2+a^2
*f^2+a*c*(-2*d*f+e^2))^(1/2))))*2^(1/2)/a^(1/2)/c^(1/2)/(f*x^2+e*x+d)^(1/2
)/(B*a*e+A*(c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)/(-A*c
*e+B*(c*d-a*f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2))*(B*a*e+A*(
c*d-a*f-(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*(-A*c*e+B*(c*d-a*
f+(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)))^(1/2)*2^(1/2)/a^(1/2)/c^(1/2)
/e^(1/2)/(c^2*d^2+a^2*f^2+a*c*(-2*d*f+e^2))^(1/2)
```

3.22.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.28

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \frac{1}{2}\text{RootSum}\Bigg[cd^2+ae^2-4ae\sqrt{f}\#1-2cd\#1^2+4af\#1^2 + c\#1^4\&, \frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)-Ae\log\left(-\sqrt{f}x+\sqrt{d+ex+fx^2}-\#1\right)+2A\sqrt{f}\,\text{lo}}{ae\sqrt{f}+cd\#1-2af}$$

input
```
Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

output
```
RootSum[c*d^2 + a*e^2 - 4*a*e*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 + c*#1^
4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] - #1] - A*e*Log[-(Sqrt
[f]*x) + Sqrt[d + e*x + f*x^2] - #1] + 2*A*Sqrt[f]*Log[-(Sqrt[f]*x) + Sqrt
[d + e*x + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqrt[d + e*x + f*x^2] -
#1]*#1^2)/(a*e*Sqrt[f] + c*d*#1 - 2*a*f*#1 - c*#1^3) & ]/2
```

3.22.3 Rubi [A] (verified)

Time = 2.36 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {1369, 25, 1363, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}dx$$

$$\Big\downarrow \text{1369}$$

$$\frac{\int -\frac{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(Ace-B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(cx^2+a)\sqrt{fx^2+ex+d}}dx}{2\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}-\frac{\int -\frac{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(cx^2+a)\sqrt{fx^2+ex+d}}dx}{2\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(cx^2+a)\sqrt{fx^2+ex+d}}dx}{2\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}-\frac{\int \frac{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)+\left(Ace-B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac(e^2-2df)}\right)\right)x}{(cx^2+a)\sqrt{fx^2+ex+d}}dx}{2\sqrt{a^2f^2+ac\left(e^2-2df\right)+c^2d^2}}$$

$$\Big\downarrow \text{1363}$$

$$\frac{a\left(aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\left(Ace-B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\int}{}$$

$$\frac{a\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\left(aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\int}{}$$

$$\Big\downarrow \text{221}$$

$$\frac{\left(Ace-B\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\sqrt{aBe+A\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\text{arc}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\sqrt{B\left(cd-a\right.}}$$

$$\frac{\sqrt{aBe+A\left(cd-af-\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)}\left(Ace-B\left(cd-af+\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\right)\right)\text{arc}}{\sqrt{2}\sqrt{a}\sqrt{c}\sqrt{e}\sqrt{c^2d^2+a^2f^2+ac\left(e^2-2df\right)}\sqrt{B\left(cd-a\right.}}$$

input
```
Int[(A + B*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]),x]
```

output
```
-((Sqrt[a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
)]*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]))*A
rcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e
^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e
^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[a*B*e + A*(c*d - a*f - Sq
rt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[-(A*c*e) + B*(c*d - a*f +
 Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2])])/(S
qrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)]
*Sqrt[-(A*c*e) + B*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)
])])) + ((A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f
)]))*Sqrt[a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f
)])]*ArcTanh[(Sqrt[e]*(a*(A*c*e - B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 +
a*c*(e^2 - 2*d*f)])) - c*(a*B*e + A*(c*d - a*f + Sqrt[c^2*d^2 + a^2*f^2 +
a*c*(e^2 - 2*d*f)]))*x))/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[-(A*c*e) + B*(c*d -
 a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[a*B*e + A*(c*d -
 a*f + Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)])]*Sqrt[d + e*x + f*x^2]
)])/(Sqrt[2]*Sqrt[a]*Sqrt[c]*Sqrt[e]*Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2
*d*f)]*Sqrt[-(A*c*e) + B*(c*d - a*f - Sqrt[c^2*d^2 + a^2*f^2 + a*c*(e^2 -
2*d*f)])]))
```

3.22. $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$

3.22.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

3.22.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.54

method	result
default	$-\frac{\left(-Ac+B\sqrt{-ac}\right)\ln\left(\frac{-\frac{2\left(e\sqrt{-ac}+fa-cd\right)}{c}+\frac{\left(-2\sqrt{-ac}\,f+ce\right)\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{e\sqrt{-ac}+fa-cd}{c}}\sqrt{f\left(x+\frac{\sqrt{-ac}}{c}\right)^{2}+\frac{\left(-2\sqrt{-ac}\,f+ce\right)\left(x+\cdots\right.}{c}\cdots}}{x+\frac{\sqrt{-ac}}{c}}\cdots\right.}{2\sqrt{-ac}\,c\sqrt{-\frac{e\sqrt{-ac}+fa-cd}{c}}}\cdots$

input
```
int((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(e*(-a*c)^(1/2)+f*a-c*d)/c)^(1
/2)*ln((-2*(e*(-a*c)^(1/2)+f*a-c*d)/c+1/c*(-2*(-a*c)^(1/2)*f+c*e)*(x+(-a*c
)^(1/2)/c)+2*(-(e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/2)*(f*(x+(-a*c)^(1/2)/c)^2+1
/c*(-2*(-a*c)^(1/2)*f+c*e)*(x+(-a*c)^(1/2)/c)-(e*(-a*c)^(1/2)+f*a-c*d)/c)^
(1/2))/(x+(-a*c)^(1/2)/c))-1/2*(A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(-e*(
-a*c)^(1/2)+f*a-c*d)/c)^(1/2)*ln((-2*(-e*(-a*c)^(1/2)+f*a-c*d)/c+(2*(-a*c)
^(1/2)*f+c*e)/c*(x-(-a*c)^(1/2)/c)+2*(-(-e*(-a*c)^(1/2)+f*a-c*d)/c)^(1/2)*
(f*(x-(-a*c)^(1/2)/c)^2+(2*(-a*c)^(1/2)*f+c*e)/c*(x-(-a*c)^(1/2)/c)-(-e*(-
a*c)^(1/2)+f*a-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))
```

3.22.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6861 vs. 2(703) = 1406.

Time = 28.21 (sec) , antiderivative size = 6861, normalized size of antiderivative = 8.80

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.22.6 Sympy [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx = \int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+a)/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Integral((A + B*x)/((a + c*x**2)*sqrt(d + e*x + f*x**2)), x)
```

3.22.7 Maxima [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx=\int \frac{Bx+A}{(cx^2+a)\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + e*x + d)), x)
```

3.22.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+e*x+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Valuesym2poly/r2sym(con
st gen &
```

3.22.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+ex+fx^2}}\,dx=\int \frac{A+B\,x}{(c\,x^2+a)\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((a + c*x^2)*(d + e*x + f*x^2)^(1/2)), x)
```

3.23 $\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$

3.23.1 Optimal result

Integrand size = 29, antiderivative size = 302

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \frac{\left(bB-2Ac-B\sqrt{b^2-4ac}\right)\operatorname{arctanh}\left(\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b-\sqrt{b^2-4ac}\right)f}} + \frac{\left(2Ac-B\left(b+\sqrt{b^2-4ac}\right)\right)\operatorname{arctanh}\left(\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)fx}{\sqrt{2}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}\sqrt{d+fx^2}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{2c^2d-2acf+b\left(b+\sqrt{b^2-4ac}\right)f}}$$

output
```
1/2*arctanh(1/2*(2*c*d-f*x*(b-(-4*a*c+b^2)^(1/2)))*2^(1/2)/(f*x^2+d)^(1/2)
/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(B*b-2*A*c-B*(-4*a*c+
b^2)^(1/2))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*c*d-f*x*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)
/(f*x^2+d)^(1/2)/(2*c^2*d-2*a*c*f+b*f*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*A*
c-B*(b+(-4*a*c+b^2)^(1/2)))*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c^2*d-2*a*c*f+b*
f*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

3.23.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx$$
$$= -\text{RootSum}\Big[cd^2+2bd\sqrt{f}\#1-2cd\#1^2+4af\#1^2-2b\sqrt{f}\#1^3$$
$$+c\#1^4\&,\frac{Bd\log\left(-\sqrt{f}x+\sqrt{d+fx^2}-\#1\right)+2A\sqrt{f}\log\left(-\sqrt{f}x+\sqrt{d+fx^2}-\#1\right)\#1-B\log\left(-\sqrt{f}\right.}{bd\sqrt{f}-2cd\#1+4af\#1-3b\sqrt{f}\#1^2+2c\#1^3}$$

input
```
Integrate[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

output
```
-RootSum[c*d^2 + 2*b*d*Sqrt[f]*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*b*Sqrt[f]*
#1^3 + c*#1^4 & , (B*d*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1] + 2*A*Sqrt
[f]*Log[-(Sqrt[f]*x) + Sqrt[d + f*x^2] - #1]*#1 - B*Log[-(Sqrt[f]*x) + Sqr
t[d + f*x^2] - #1]*#1^2)/(b*d*Sqrt[f] - 2*c*d*#1 + 4*a*f*#1 - 3*b*Sqrt[f]*
#1^2 + 2*c*#1^3) & ]
```

3.23.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{d+fx^2}\,(a+bx+cx^2)}\,dx$$
$$\Big\downarrow \text{1367}$$
$$-\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\int \frac{1}{\left(b+2cx-\sqrt{b^2-4ac}\right)\sqrt{fx^2+d}}dx}{\sqrt{b^2-4ac}}-\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{\left(b+2cx+\sqrt{b^2-4ac}\right)\sqrt{fx^2+d}}dx}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 488$$

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\int \frac{1}{4dc^2+\left(b-\sqrt{b^2-4ac}\right)^2 f-\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)fx\right)^2}{fx^2+d}}d\frac{2cd-\left(b-\sqrt{b^2-4ac}\right)fx}{\sqrt{fx^2+d}}}{\sqrt{b^2-4ac}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\int \frac{1}{4dc^2+\left(b+\sqrt{b^2-4ac}\right)^2 f-\frac{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)fx\right)^2}{fx^2+d}}d\frac{2cd-\left(b+\sqrt{b^2-4ac}\right)fx}{\sqrt{fx^2+d}}}{\sqrt{b^2-4ac}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(-B\sqrt{b^2-4ac}-2Ac+bB\right)\operatorname{arctanh}\left(\frac{2cd-fx\left(b-\sqrt{b^2-4ac}\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(b-\sqrt{b^2-4ac}\right)-2acf+2c^2d}}+\frac{\left(2Ac-B\left(\sqrt{b^2-4ac}+b\right)\right)\operatorname{arctanh}\left(\frac{2cd-fx\left(\sqrt{b^2-4ac}+b\right)}{\sqrt{2}\sqrt{d+fx^2}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{bf\left(\sqrt{b^2-4ac}+b\right)-2acf+2c^2d}}$$

input
```
Int[(A + B*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]),x]
```

output
```
((b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*ArcTanh[(2*c*d - (b - Sqrt[b^2 - 4*a*
c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b - Sqrt[b^2 - 4*a*c])*f]*Sq
rt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d - 2*a*c*f + b*(b
- Sqrt[b^2 - 4*a*c])*f]) + ((2*A*c - B*(b + Sqrt[b^2 - 4*a*c]))*ArcTanh[(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*f*x)/(Sqrt[2]*Sqrt[2*c^2*d - 2*a*c*f + b*(b
 + Sqrt[b^2 - 4*a*c])*f]*Sqrt[d + f*x^2])])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqr
t[2*c^2*d - 2*a*c*f + b*(b + Sqrt[b^2 - 4*a*c])*f])
```

3.23.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 638 vs. $2(266) = 532$.

Time = 0.77 (sec) , antiderivative size = 639, normalized size of antiderivative = 2.12

method	result
default	$-\frac{\left(-2Ac+B\sqrt{-4ac+b^2}+Bb\right)\ln\left(-\frac{-\sqrt{-4ac+b^2}\,bf+2acf-b^2f-2c^2d}{c^2}-\frac{f\left(b+\sqrt{-4ac+b^2}\right)\left(x+\frac{b+\sqrt{-4ac+b^2}}{2c}\right)}{c}+\sqrt{-\frac{2\left(-\sqrt{-4ac+b^2}\,bf+2a\cdots\right.}{c^2}\cdots}\cdots\right.}{\sqrt{-4ac+b^2}\,c\sqrt{-\frac{2\left(-\sqrt{-4ac+b^2}\,b\cdots\right.}{\cdots}}\cdots}$

input
```
int((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-2*A*c+B*(-4*a*c+b^2)^(1/2)+B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*(-(-4*a*c+b^2)
^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*ln((-(-(-4*a*c+b^2)^(1/2)*b*f
+2*a*c*f-b^2*f-2*c^2*d)/c^2-f*(b+(-4*a*c+b^2)^(1/2))/c*(x+1/2*(b+(-4*a*c+b
^2)^(1/2))/c)+1/2*(-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)
^(1/2)*(4*f*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2-4*f*(b+(-4*a*c+b^2)^(1/2))/
c*(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c)-2*(-(-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*
f-2*c^2*d)/c^2)^(1/2))/(x+1/2*(b+(-4*a*c+b^2)^(1/2))/c))-(2*A*c+B*(-4*a*c+
b^2)^(1/2)-B*b)/(-4*a*c+b^2)^(1/2)/c/(-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b
^2*f-2*c^2*d)/c^2)^(1/2)*ln((-((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*
d)/c^2-f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))+1/2*(-
2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2)*(4*f*(x-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))^2-4*f*(b-(-4*a*c+b^2)^(1/2))/c*(x-1/2/c*(-b+(-4*a
*c+b^2)^(1/2)))-2*((-4*a*c+b^2)^(1/2)*b*f+2*a*c*f-b^2*f-2*c^2*d)/c^2)^(1/2
))/(x-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))
```

3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8977 vs. $2(263) = 526$.

Time = 43.72 (sec) , antiderivative size = 8977, normalized size of antiderivative = 29.73

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.23.6 Sympy [F]

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+Bx}{\sqrt{d+fx^2}\,(a+bx+cx^2)}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x+a)/(f*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(d + f*x**2)*(a + b*x + c*x**2)), x)
```

3.23.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.23.8 Giac [F(-2)]

Exception generated.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((B*x+A)/(c*x^2+b*x+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{poly1[%%%{-4,[3,2,0]%%%}+%%%{16,[1,3,1]%%%},%%%{4,[4,2,
0]%%%}+%%
```

3.23.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+bx+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{f\,x^2+d}\,\left(c\,x^2+b\,x+a\right)}\,dx$$

input
```
int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)),x)
```

output
```
int((A + B*x)/((d + f*x^2)^(1/2)*(a + b*x + c*x^2)), x)
```

3.24 $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

3.24.1 Optimal result

Integrand size = 26, antiderivative size = 101

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \frac{A\arctan\left(\frac{\sqrt{cd-af}\,x}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}} - \frac{B\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

output
```
A*arctan(x*(-a*f+c*d)^(1/2)/a^(1/2)/(f*x^2+d)^(1/2))/a^(1/2)/(-a*f+c*d)^(1
/2)-B*arctanh(c^(1/2)*(f*x^2+d)^(1/2)/(-a*f+c*d)^(1/2))/c^(1/2)/(-a*f+c*d)
^(1/2)
```

3.24.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 349 vs. 2(101) = 202.

Time = 1.45 (sec) , antiderivative size = 349, normalized size of antiderivative = 3.46

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$
$$= \frac{\sqrt{a}B\left(\left(\sqrt{a}\sqrt{f}+\sqrt{-cd+af}\right)\sqrt{-cd+2af-2\sqrt{a}\sqrt{f}\sqrt{-cd+af}}\arctan\left(\frac{\sqrt{c}\left(\sqrt{f}x-\sqrt{d+fx^2}\right)}{\sqrt{-cd+2af-2\sqrt{a}\sqrt{f}\sqrt{-cd+af}}}\right)+(\cdots}{\cdots}$$

input
```
Integrate[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

output
```
(Sqrt[a]*B*((Sqrt[a]*Sqrt[f] + Sqrt[-(c*d) + a*f])*Sqrt[-(c*d) + 2*a*f - 2
*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d +
 f*x^2]))/Sqrt[-(c*d) + 2*a*f - 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]] + (
-(Sqrt[a]*Sqrt[f]) + Sqrt[-(c*d) + a*f])*Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*S
qrt[f]*Sqrt[-(c*d) + a*f]]*ArcTan[(Sqrt[c]*(Sqrt[f]*x - Sqrt[d + f*x^2]))/
Sqrt[-(c*d) + 2*a*f + 2*Sqrt[a]*Sqrt[f]*Sqrt[-(c*d) + a*f]]]) + A*c^(3/2)*
d*ArcTanh[(a*Sqrt[f] + c*x*(Sqrt[f]*x - Sqrt[d + f*x^2]))/(Sqrt[a]*Sqrt[-(
c*d) + a*f])])/(Sqrt[a]*c^(3/2)*d*Sqrt[-(c*d) + a*f])
```

3.24.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1343, 291, 218, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}dx$$

$$\Big\downarrow \text{1343}$$

$$A\int \frac{1}{(cx^2+a)\sqrt{fx^2+d}}dx+B\int \frac{x}{(cx^2+a)\sqrt{fx^2+d}}dx$$

$$\Big\downarrow \text{291}$$

$$A\int \frac{1}{a-\frac{(af-cd)x^2}{fx^2+d}}d\frac{x}{\sqrt{fx^2+d}}+B\int \frac{x}{(cx^2+a)\sqrt{fx^2+d}}dx$$

$$\Big\downarrow \text{218}$$

$$B\int \frac{x}{(cx^2+a)\sqrt{fx^2+d}}dx+\frac{A\arctan\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}B\int \frac{1}{(cx^2+a)\sqrt{fx^2+d}}dx^2+\frac{A\arctan\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}$$

$$\Big\downarrow \text{73}$$

$$\frac{B\int \frac{1}{\frac{cx^4}{f}+a-\frac{cd}{f}}d\sqrt{fx^2+d}}{f}+\frac{A\arctan\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}$$

$$\downarrow 221$$

$$\frac{A\arctan\left(\frac{x\sqrt{cd-af}}{\sqrt{a}\sqrt{d+fx^2}}\right)}{\sqrt{a}\sqrt{cd-af}}-\frac{B\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+fx^2}}{\sqrt{cd-af}}\right)}{\sqrt{c}\sqrt{cd-af}}$$

input
```
Int[(A + B*x)/((a + c*x^2)*Sqrt[d + f*x^2]),x]
```

output
```
(A*ArcTan[(Sqrt[c*d - a*f]*x)/(Sqrt[a]*Sqrt[d + f*x^2])])/(Sqrt[a]*Sqrt[c*
d - a*f]) - (B*ArcTanh[(Sqrt[c]*Sqrt[d + f*x^2])/Sqrt[c*d - a*f]])/(Sqrt[c
]*Sqrt[c*d - a*f])
```

3.24.3.1 Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 1343
```
Int[((g_) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q
_), x_Symbol] :> Simp[g   Int[(a + c*x^2)^p*(d + f*x^2)^q, x], x] + Simp[h
  Int[x*(a + c*x^2)^p*(d + f*x^2)^q, x], x] /; FreeQ[{a, c, d, f, g, h, p,
q}, x]
```

3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(81) = 162$.

Time = 0.58 (sec) , antiderivative size = 337, normalized size of antiderivative = 3.34

method	result
default	$-\frac{(-Ac+B\sqrt{-ac})\ln\left(\frac{-\frac{2(fa-cd)}{c}-\frac{2f\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}+2\sqrt{-\frac{fa-cd}{c}}\sqrt{f\left(x+\frac{\sqrt{-ac}}{c}\right)^2-\frac{2f\sqrt{-ac}\left(x+\frac{\sqrt{-ac}}{c}\right)}{c}-\frac{fa-cd}{c}}}{x+\frac{\sqrt{-ac}}{c}}\right)}{2\sqrt{-ac}\,c\sqrt{-\frac{fa-cd}{c}}}-\frac{(Ac+B\sqrt{}}{}$

input
```
int((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(a*f-c*d)/c)^(1/2)*ln((-2*(a*f
-c*d)/c-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)+2*(-(a*f-c*d)/c)^(1/2)*(f*(x
+(-a*c)^(1/2)/c)^2-2*f*(-a*c)^(1/2)/c*(x+(-a*c)^(1/2)/c)-(a*f-c*d)/c)^(1/2
))/(x+(-a*c)^(1/2)/c))-1/2*(A*c+B*(-a*c)^(1/2))/(-a*c)^(1/2)/c/(-(a*f-c*d)
/c)^(1/2)*ln((-2*(a*f-c*d)/c+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1/2)/c)+2*(-(a*
f-c*d)/c)^(1/2)*(f*(x-(-a*c)^(1/2)/c)^2+2*f*(-a*c)^(1/2)/c*(x-(-a*c)^(1/2)
/c)-(a*f-c*d)/c)^(1/2))/(x-(-a*c)^(1/2)/c))
```

3.24.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(81) = 162$.

Time = 0.37 (sec) , antiderivative size = 1515, normalized size of antiderivative = 15.00

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2
- 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*
c)*f*x + (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a
^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x
^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d
^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c*f)) + sqrt(-A^2*B^2/(a*
c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c^2 + A^2*c^3)*d^2 - (B^2*a^
2*c + A^2*a*c^2)*d*f))/x) + 1/4*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*c*f
)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^2*c
*f))*log(((A*B^3*a + A^3*B*c)*f*x - (A^2*B*c^2*d - A^2*B*a*c*f + (B*a*c^3*
d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*
d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a - A^2*c + 2*(a*c^2*d - a^2*
c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))/(a*c^2*d - a^
2*c*f)) + sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2))*((B^2*a*c
^2 + A^2*c^3)*d^2 - (B^2*a^2*c + A^2*a*c^2)*d*f))/x) - 1/4*sqrt((B^2*a - A
^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^
3*c*f^2)))/(a*c^2*d - a^2*c*f))*log(((A*B^3*a + A^3*B*c)*f*x + (A^2*B*c^2*
d - A^2*B*a*c*f - (B*a*c^3*d^2 - 2*B*a^2*c^2*d*f + B*a^3*c*f^2)*sqrt(-A^2*
B^2/(a*c^3*d^2 - 2*a^2*c^2*d*f + a^3*c*f^2)))*sqrt(f*x^2 + d)*sqrt((B^2*a
- A^2*c - 2*(a*c^2*d - a^2*c*f)*sqrt(-A^2*B^2/(a*c^3*d^2 - 2*a^2*c^2*d*...
```

3.24.6 Sympy [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx = \int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+a)/(f*x**2+d)**(1/2),x)
```

output
```
Integral((A + B*x)/((a + c*x**2)*sqrt(d + f*x**2)), x)
```

3.24.7 Maxima [F]

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx=\int \frac{Bx+A}{(cx^2+a)\sqrt{fx^2+d}}\,dx$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + a)*sqrt(f*x^2 + d)), x)
```

3.24.8 Giac [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(c*x^2+a)/(f*x^2+d)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$$
$$=\begin{cases} \frac{B\,\mathrm{atan}\left(\frac{c\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}}+\frac{A\,\mathrm{atan}\left(\frac{x\,\sqrt{c\,d-a\,f}}{\sqrt{a}\,\sqrt{f\,x^2+d}}\right)}{\sqrt{-a\,(a\,f-c\,d)}} & \text{if }\ 0<c\,d-a\,f \\ \frac{A\,\ln\left(\frac{\sqrt{a\,\left(f\,x^2+d\right)}+x\,\sqrt{a\,f-c\,d}}{\sqrt{a\,\left(f\,x^2+d\right)}-x\,\sqrt{a\,f-c\,d}}\right)}{2\,\sqrt{a\,(a\,f-c\,d)}}+\frac{B\,\mathrm{atan}\left(\frac{c\sqrt{f\,x^2+d}}{\sqrt{a\,c\,f-c^2\,d}}\right)}{\sqrt{a\,c\,f-c^2\,d}} & \text{if }\ c\,d-a\,f<0 \\ \int \frac{A+B\,x}{(c\,x^2+a)\,\sqrt{f\,x^2+d}}\,dx & \text{if }\ c\,d-a\,f\notin\mathbb{R}\vee a\,f=c\,d \end{cases}$$

input
```
int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)),x)
```

output
```
piecewise(0 < - a*f + c*d, (B*atan((c*(d + f*x^2)^(1/2))/(- c^2*d + a*c*f)
^(1/2)))/(- c^2*d + a*c*f)^(1/2) + (A*atan((x*(- a*f + c*d)^(1/2))/(a^(1/2
)*(d + f*x^2)^(1/2))))/(-a*(a*f - c*d))^(1/2), - a*f + c*d < 0, (A*log(((a
*(d + f*x^2))^(1/2) + x*(a*f - c*d)^(1/2))/((a*(d + f*x^2))^(1/2) - x*(a*f
 - c*d)^(1/2))))/(2*(a*(a*f - c*d))^(1/2)) + (B*atan((c*(d + f*x^2)^(1/2))
/(- c^2*d + a*c*f)^(1/2)))/(- c^2*d + a*c*f)^(1/2), ~in(- a*f + c*d, 'real
') | a*f == c*d, int((A + B*x)/((a + c*x^2)*(d + f*x^2)^(1/2)), x))
```

3.24. $\int \frac{A+Bx}{(a+cx^2)\sqrt{d+fx^2}}\,dx$

3.25 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx$

3.25.1 Optimal result

Integrand size = 30, antiderivative size = 139

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \frac{1}{2}\sqrt{-\frac{13}{5}+\sqrt{10}}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})\,x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{1}{2}\sqrt{\frac{13}{5}+\sqrt{10}}\,\text{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})\,x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

output
```
1/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+1
0^(1/2))^(1/2))*(-65+25*10^(1/2))^(1/2)+1/10*arctanh(1/2*(x*(1-4*10^(1/2))
+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(65+25*10^(1/2))
^(1/2)
```

3.25.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.07

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\frac{1}{2}\text{RootSum}\Big[5+20\#1+8\#1^2-8\#1^3 +2\#1^4\&, \frac{-7\log(x)+7\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)+2\log(x)\#1-2\log\left(-1+\sqrt{1+3x-2x^2}-\right.}{5+4\#1-6\#1^2+2\#1^3}$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

output
```
-1/2*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-7*Log[x] + 7*Log[-
1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 2*Log[x]*#1 - 2*Log[-1 + Sqrt[1 + 3*x
- 2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x
*#1]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ]
```

3.25.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1365, 27, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)\sqrt{-2x^2+3x+1}}dx$$

$$\Big\downarrow \text{1365}$$

$$\frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{1}{10}\left(5+4\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx$$

$$\Big\downarrow \text{1154}$$

$$-\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1+\sqrt{10}\right)}d\left(-\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{\left(5-4\sqrt{10}\right)\arctan\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\left(5+4\sqrt{10}\right)\operatorname{arctanh}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{\sqrt{10}-1}}-\frac{\left(5-4\sqrt{10}\right)\arctan\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}$$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x - 2*x^2]),x]
```

output
```
-1/10*((5 - 4*Sqrt[10])*ArcTan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*
Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/Sqrt[1 + Sqrt[10]] + ((5 + 4*S
qrt[10])*ArcTanh[(3*(4 + Sqrt[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt
[10]]*Sqrt[1 + 3*x - 2*x^2])])/(10*Sqrt[-1 + Sqrt[10]])
```

3.25.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.25.4 Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\,\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\,\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{20\sqrt{1+\sqrt{10}}}+\frac{\left(8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{-1+\sqrt{\cdots}}}\right.}{\cdots}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+100\operatorname{RootOf}\left(400\_Z^{4}-520\_Z^{2}-81\right)^{2}-130\right)\ln\left(\frac{129200x\operatorname{RootOf}\left(400\_Z^{4}-520\_Z^{2}-81\right)^{4}\operatorname{RootOf}\left(\_Z^{2}+100\cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/20*(-8+10^(1/2))*10^(1/2)/(1+10^(1/2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1
/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/
3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1
/2))+1/20*(8+10^(1/2))*10^(1/2)/(-1+10^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*
10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18
*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/
2))^(1/2))
```

3.25.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(99) = 198.

Time = 0.29 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.22

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x+(4\sqrt{5}x-7\sqrt{2}x)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+}}{x}\right. + \frac{1}{10}\sqrt{5}\sqrt{5\sqrt{5}\sqrt{2}+13}\log\left(\frac{9\sqrt{5}\sqrt{2}x-(4\sqrt{5}x-7\sqrt{2}x)\sqrt{5\sqrt{5}\sqrt{2}+13}-18x+18\sqrt{-2x^2+3x+}}{x}\right. + \frac{1}{10}\sqrt{5}\sqrt{-5\sqrt{5}\sqrt{2}+13}\log\left(-\frac{9\sqrt{5}\sqrt{2}x+(4\sqrt{5}x+7\sqrt{2}x)\sqrt{-5\sqrt{5}\sqrt{2}+13}+18x-18\sqrt{-2x^2+}}{x}\right. - \frac{1}{10}\sqrt{5}\sqrt{-5\sqrt{5}\sqrt{2}+13}\log\left(-\frac{9\sqrt{5}\sqrt{2}x-(4\sqrt{5}x+7\sqrt{2}x)\sqrt{-5\sqrt{5}\sqrt{2}+13}+18x-18\sqrt{-2x^2+}}{x}\right.$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

output
```
-1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*sqrt(5)*sqrt(2)*x + (4*s
qrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 13) - 18*x + 18*sqrt(-2*x
^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(5*sqrt(5)*sqrt(2) + 13)*log((9*
sqrt(5)*sqrt(2)*x - (4*sqrt(5)*x - 7*sqrt(2)*x)*sqrt(5*sqrt(5)*sqrt(2) + 1
3) - 18*x + 18*sqrt(-2*x^2 + 3*x + 1) - 18)/x) + 1/10*sqrt(5)*sqrt(-5*sqrt
(5)*sqrt(2) + 13)*log(-(9*sqrt(5)*sqrt(2)*x + (4*sqrt(5)*x + 7*sqrt(2)*x)*
sqrt(-5*sqrt(5)*sqrt(2) + 13) + 18*x - 18*sqrt(-2*x^2 + 3*x + 1) + 18)/x)
- 1/10*sqrt(5)*sqrt(-5*sqrt(5)*sqrt(2) + 13)*log(-(9*sqrt(5)*sqrt(2)*x - (
4*sqrt(5)*x + 7*sqrt(2)*x)*sqrt(-5*sqrt(5)*sqrt(2) + 13) + 18*x - 18*sqrt(
-2*x^2 + 3*x + 1) + 18)/x)
```

3.25.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\int \frac{x}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx - \int \frac{2}{3x^2\sqrt{-2x^2+3x+1}-4x\sqrt{-2x^2+3x+1}-2\sqrt{-2x^2+3x+1}}\,dx$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(1/2),x)
```

output
```
-Integral(x/(3*x**2*sqrt(-2*x**2 + 3*x + 1) - 4*x*sqrt(-2*x**2 + 3*x + 1)
- 2*sqrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(-2*x**2 + 3*x +
 1) - 4*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.60

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = -\frac{1}{20}\sqrt{10}\left(\frac{\sqrt{10}\arcsin\left(\frac{8\sqrt{17}\sqrt{10}x}{17|6x+2\sqrt{10}-4|}+\frac{2\sqrt{17}x}{17|6x+2\sqrt{10}-4|}-\frac{6\sqrt{17}\sqrt{10}}{17|6x+2\sqrt{10}-4|}+\frac{24\sqrt{17}}{17|6x+2\sqrt{10}-4|}\right)}{\sqrt{\sqrt{10}+1}}-\frac{\sqrt{10}\log\left(-\right.}{}\right.$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

output
```
-1/20*sqrt(10)*(sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(
10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(
10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/
sqrt(sqrt(10) + 1) - sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x +
1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2
*sqrt(10) - 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1)
- 8*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(
17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(
10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/sqrt(sqrt(10) + 1) -
8*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*
x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x
 - 2*sqrt(10) - 4) + 1/18)/sqrt(sqrt(10) - 1))
```

3.25.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16965 vs. 2(99) = 198.

Time = 255.51 (sec) , antiderivative size = 16965, normalized size of antiderivative = 122.05

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

output
```
1/5*sqrt(25*sqrt(10) - 65)*(arctan(25440019409633258254215013/362376882997
347899477595900837238918963 20*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*s
qrt(10) - 65))^15 - 6111149415804811055946029/1288451139546125864809229869
643516156313 6*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^1
4 - 9585786192235661610867711 77/144950753198939159791038360334895567585 28*
(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^13 - 3257049408
4284092271734368324 61/579803012795756639164153441339582270341120*(sqrt(34)
 + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) - 65))^12 + 973696292113400209 4
58145664321/362376882997347899477595900837238918963 20*(sqrt(34) + 2*sqrt(1
0) + sqrt(2) + sqrt(25*sqrt(10) - 65))^11 - 655458488897784251494816231968
7/156703516971826118693014443605292505497 60*(sqrt(34) + 2*sqrt(10) + sqrt(
2) + sqrt(25*sqrt(10) - 65))^10 + 140497661411428951431238119562151 3/72475
376599469579895519180167447783792640*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sq
rt(25*sqrt(10) - 65))^9 - 463735929740301197982553262464586 59/579803012795
756639164153441339582270341120*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*
sqrt(10) - 65))^8 + 298897656655623559051647178006829 89/120792294332449299
825865300279079639654 40*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10
) - 65))^7 - 258711594916326381414900981895662852 19/579803012795756639164 1
534413395822703411 20*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(25*sqrt(10) -
 65))^6 + 654417817646644718606382912450321517 1/24158458866489859965173...
```

3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x-2x^2}}\,dx = \int \frac{x+2}{\sqrt{-2\,x^2+3\,x+1}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

input
```
int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x - 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

3.26 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx$

3.26.1 Optimal result

Integrand size = 30, antiderivative size = 166

$$\begin{aligned}\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = &-\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}\\&-\frac{9}{2}\sqrt{\frac{1}{5}\left(-3+\sqrt{10}\right)}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})\,x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)\\&+\frac{9}{2}\sqrt{\frac{1}{5}\left(3+\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})\,x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)\end{aligned}$$

output
```
-2/17*(15+14*x)/(-2*x^2+3*x+1)^(1/2)-9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4
*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(1+10^(1/2))^(1/2))*(-15+5*10^(1/2))^(1/2
)+9/10*arctanh(1/2*(x*(1-4*10^(1/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(
-1+10^(1/2))^(1/2))*(15+5*10^(1/2))^(1/2)
```

3.26.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$
$$-\frac{2(15+14x)}{17\sqrt{1+3x-2x^2}}+\frac{9}{2}\text{RootSum}\Big[5+20\#1+8\#1^2-8\#1^3$$
$$+2\#1^4\&,\frac{3\log(x)-3\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)-2\log(x)\#1+2\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right.}{5+4\#1-6\#1^2+2\#1^3}$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

output
```
(-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) + (9*RootSum[5 + 20*#1 + 8*#1^
2 - 8*#1^3 + 2*#1^4 & , (3*Log[x] - 3*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#
1] - 2*Log[x]*#1 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1]*#1)/(5 + 4*#1
- 6*#1^2 + 2*#1^3) & ])/2
```

3.26.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {1349, 27, 1365, 27, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)(-2x^2+3x+1)^{3/2}}dx$$
$$\Big\downarrow \text{1349}$$
$$\frac{2}{17}\int \frac{153x}{2(-3x^2+4x+2)\sqrt{-2x^2+3x+1}}dx-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$
$$\Big\downarrow \text{27}$$
$$9\int \frac{x}{(-3x^2+4x+2)\sqrt{-2x^2+3x+1}}dx-\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$
$$\Big\downarrow \text{1365}$$

$$9\left(\frac{1}{5}\left(5-\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{1}{5}\left(5+\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}\right.$$
$$\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

$$\Big\downarrow \text{27}$$

$$9\left(\frac{1}{10}\left(5-\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{1}{10}\left(5+\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}\right.$$
$$\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

$$\Big\downarrow \text{1154}$$

$$9\left(-\frac{1}{5}\left(5+\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{1}{5}\left(5-\sqrt{10}\right)\right.$$
$$\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

$$\Big\downarrow \text{217}$$

$$9\left(-\frac{1}{5}\left(5+\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{\left(5-\sqrt{10}\right)\text{ar}}{}\right.$$
$$\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

$$\Big\downarrow \text{219}$$

$$9\left(\frac{\left(5+\sqrt{10}\right)\text{arctanh}\left(\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{\sqrt{10}-1}}-\frac{\left(5-\sqrt{10}\right)\arctan\left(\frac{\left(1+4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}\right)-$$
$$\frac{2(14x+15)}{17\sqrt{-2x^2+3x+1}}$$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(3/2)),x]
```

3.26. $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx$

output
```
(-2*(15 + 14*x))/(17*Sqrt[1 + 3*x - 2*x^2]) + 9*(-1/10*((5 - Sqrt[10])*Arc
Tan[(3*(4 - Sqrt[10]) + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 +
 3*x - 2*x^2])])/Sqrt[1 + Sqrt[10]] + ((5 + Sqrt[10])*ArcTanh[(3*(4 + Sqrt
[10]) + (1 - 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])]
)/(10*Sqrt[-1 + Sqrt[10]]))
```

3.26.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.26.4 Maple [A] (verified)

Time = 2.40 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.17

method	result
risch	$-\frac{2(15+14x)}{17\sqrt{-2x^2+3x+1}}-\frac{9\left(-2+\sqrt{10}\right)\sqrt{10}\arctan\left(\frac{-1-\sqrt{10}+\frac{9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{1+\sqrt{10}}\sqrt{-18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{1}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)-1-\sqrt{10}}}\right)}{20\sqrt{1+\sqrt{10}}}+\frac{9\left(2+\sqrt{10}\right.}{}$
default	$-\frac{\left(8+\sqrt{10}\right)\sqrt{10}\left(\frac{1}{3\left(-\frac{1}{9}+\frac{\sqrt{10}}{9}\right)\sqrt{-2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)-\frac{1}{9}+\frac{\sqrt{10}}{9}}}-\frac{\left(\frac{1}{3}-\right.}{3\left(-\frac{1}{9}+\frac{\sqrt{10}}{9}\right)\left(\frac{8}{9}-\frac{8\sqrt{10}}{9}-\left(\frac{1}{3}-\frac{4\sqrt{10}}{3}\right)^2\right)\sqrt{-}}\right.}{}$
trager	$\frac{2(15+14x)\sqrt{-2x^2+3x+1}}{17(2x^2-3x-1)}-18\,\mathrm{RootOf}\left(6400\_Z^4-480\_Z^2-1\right)\ln\left(-\frac{8595200x\,\mathrm{RootOf}\left(6400\_Z^4-480\_Z^2-\right.}{}\right.$

input
```
int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/17*(15+14*x)/(-2*x^2+3*x+1)^(1/2)-9/20*(-2+10^(1/2))*10^(1/2)/(1+10^(1/
2))^(1/2)*arctan(9/2*(-2/9-2/9*10^(1/2)+(1/3+4/3*10^(1/2))*(x-2/3+1/3*10^(
1/2)))/(1+10^(1/2))^(1/2)/(-18*(x-2/3+1/3*10^(1/2))^2+9*(1/3+4/3*10^(1/2))
*(x-2/3+1/3*10^(1/2))-1-10^(1/2))^(1/2))+9/20*(2+10^(1/2))*10^(1/2)/(-1+10
^(1/2))^(1/2)*arctanh(9/2*(-2/9+2/9*10^(1/2)+(1/3-4/3*10^(1/2))*(x-2/3-1/3
*10^(1/2)))/(-1+10^(1/2))^(1/2)/(-18*(x-2/3-1/3*10^(1/2))^2+9*(1/3-4/3*10^
(1/2))*(x-2/3-1/3*10^(1/2))-1+10^(1/2))^(1/2))
```

3.26.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.17

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$

$$-\frac{17\sqrt{5}(2x^2-3x-1)\sqrt{-81\sqrt{10}+243}\log\left(-\frac{45\sqrt{10}x+\left(3\sqrt{10}\sqrt{5}x+10\sqrt{5}x\right)\sqrt{-81\sqrt{10}+243}+90x-90\sqrt{-2x^2+3x+1}+90}{x}\right.}{}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

output
```
-1/170*(17*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(-81*sqrt(10) + 243)*log(-(45*sqr
t(10)*x + (3*sqrt(10)*sqrt(5)*x + 10*sqrt(5)*x)*sqrt(-81*sqrt(10) + 243) +
 90*x - 90*sqrt(-2*x^2 + 3*x + 1) + 90)/x) - 17*sqrt(5)*(2*x^2 - 3*x - 1)*
sqrt(-81*sqrt(10) + 243)*log(-(45*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x + 10*
sqrt(5)*x)*sqrt(-81*sqrt(10) + 243) + 90*x - 90*sqrt(-2*x^2 + 3*x + 1) + 9
0)/x) + 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(10) + 3)*log(9*(5*sqrt(10)
*x + (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(sqrt(10) + 3) - 10*x + 10*
sqrt(-2*x^2 + 3*x + 1) - 10)/x) - 153*sqrt(5)*(2*x^2 - 3*x - 1)*sqrt(sqrt(
10) + 3)*log(9*(5*sqrt(10)*x - (3*sqrt(10)*sqrt(5)*x - 10*sqrt(5)*x)*sqrt(
sqrt(10) + 3) - 10*x + 10*sqrt(-2*x^2 + 3*x + 1) - 10)/x) + 600*x^2 - 20*s
qrt(-2*x^2 + 3*x + 1)*(14*x + 15) - 900*x - 300)/(2*x^2 - 3*x - 1)
```

3.26.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx =$$
$$-\int \frac{x}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2}$$
$$-\int \frac{2}{-6x^4\sqrt{-2x^2+3x+1}+17x^3\sqrt{-2x^2+3x+1}-5x^2\sqrt{-2x^2+3x+1}-10x\sqrt{-2x^2+3x+1}-2\sqrt{-2}$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(3/2),x)
```

output
```
-Integral(x/(-6*x**4*sqrt(-2*x**2 + 3*x + 1) + 17*x**3*sqrt(-2*x**2 + 3*x
+ 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*sqrt(-2*x**2 + 3*x + 1) - 2*s
qrt(-2*x**2 + 3*x + 1)), x) - Integral(2/(-6*x**4*sqrt(-2*x**2 + 3*x + 1)
+ 17*x**3*sqrt(-2*x**2 + 3*x + 1) - 5*x**2*sqrt(-2*x**2 + 3*x + 1) - 10*x*
sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(118) = 236$.

Time = 0.29 (sec) , antiderivative size = 678, normalized size of antiderivative = 4.08

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = \frac{1}{340}\sqrt{10}\left(\frac{124\sqrt{10}x}{\sqrt{10}\sqrt{-2x^2+3x+1}+\sqrt{-2x^2+3x+1}} - \frac{}{\sqrt{10}\sqrt{-2}}\right.$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

output
```
1/340*sqrt(10)*(124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*
x^2 + 3*x + 1)) - 124*sqrt(10)*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-
2*x^2 + 3*x + 1)) + 153*sqrt(10)*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x +
 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(1
7)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10
) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) + 1)) - 128*x/(sqrt(1
0)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 128*x/(sqrt(10)*sqrt
(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 1224*arcsin(8/17*sqrt(17)*s
qrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10)
 - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/
abs(6*x + 2*sqrt(10) - 4))/(sqrt(10)*sqrt(sqrt(10) + 1) + sqrt(sqrt(10) +
1)) + 153*sqrt(10)*log(-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqr
t(10) - 1)/abs(6*x - 2*sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) -
 4) - 2/9/abs(6*x - 2*sqrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 42*sqrt
(10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) + 42*sqrt(
10)/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) + 1224*log(
-2/9*sqrt(10) + 2/3*sqrt(-2*x^2 + 3*x + 1)*sqrt(sqrt(10) - 1)/abs(6*x - 2*
sqrt(10) - 4) + 2/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) - 2/9/abs(6*x - 2*s
qrt(10) - 4) + 1/18)/(sqrt(10) - 1)^(3/2) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*
x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 312/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1)...
```

3.26.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 16200 vs. $2(118) = 236$.

Time $= 199.98$ (sec) , antiderivative size $= 16200$, normalized size of antiderivative $= 97.59$

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

output
```
9/5*sqrt(5*sqrt(10) - 15)*(arctan(8071500681781594274179/61050658134324300
77384327396413440*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15)
)^15 + 10484728956613061562911/19536210602983776247629847668523008*(sqrt(3
4) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^14 - 392953500112821193
344779/610506581343243007738432739641344*(sqrt(34) + 2*sqrt(10) + sqrt(2)
+ sqrt(5*sqrt(10) - 15))^13 - 232778738969433338870776481/9768105301491888
1238149238342615040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 1
5))^12 + 387456520966533311136353209/3052532906716215038692163698206720*(s
qrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^11 + 5089330466552
6024512781737621/97681053014918881238149238342615040*(sqrt(34) + 2*sqrt(10
) + sqrt(2) + sqrt(5*sqrt(10) - 15))^10 - 53907284436630715798578564023/61
05065813432430077384327396413440*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5
*sqrt(10) - 15))^9 - 3975705644447121719005999176019/976810530149188812381
49238342615040*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^8
 + 40923441491788207049248439479/678340645936936675264925266268160*(sqrt(3
4) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(10) - 15))^7 - 1647091282014574981
57439549336639/97681053014918881238149238342615040*(sqrt(34) + 2*sqrt(10)
+ sqrt(2) + sqrt(5*sqrt(10) - 15))^6 + 38683931995792936810617108953/11776
74732529403950112717476160*(sqrt(34) + 2*sqrt(10) + sqrt(2) + sqrt(5*sqrt(
10) - 15))^5 - 4655039540820226689943605414024731/976810530149188812381...
```

3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{3/2}}\,dx = \int \frac{x+2}{(-2\,x^2+3\,x+1)^{3/2}\,(-3\,x^2+4\,x+2)}\,dx$$

input
```
int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x - 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

3.27 $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

3.27.1 Optimal result

Integrand size = 30, antiderivative size = 193

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = -\frac{2(15+14x)}{51(1+3x-2x^2)^{3/2}} - \frac{2(291+4814x)}{867\sqrt{1+3x-2x^2}} + \frac{9}{2}\sqrt{\frac{1}{5}\left(-53+17\sqrt{10}\right)}\arctan\left(\frac{3(4-\sqrt{10})+(1+4\sqrt{10})x}{2\sqrt{1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right) + \frac{9}{2}\sqrt{\frac{1}{5}\left(53+17\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(1-4\sqrt{10})x}{2\sqrt{-1+\sqrt{10}}\sqrt{1+3x-2x^2}}\right)$$

output
```
-2/51*(15+14*x)/(-2*x^2+3*x+1)^(3/2)-2/867*(291+4814*x)/(-2*x^2+3*x+1)^(1/
2)+9/10*arctan(1/2*(12-3*10^(1/2)+x*(1+4*10^(1/2)))/(-2*x^2+3*x+1)^(1/2)/(
1+10^(1/2))^(1/2))*(-265+85*10^(1/2))^(1/2)+9/10*arctanh(1/2*(x*(1-4*10^(1
/2))+12+3*10^(1/2))/(-2*x^2+3*x+1)^(1/2)/(-1+10^(1/2))^(1/2))*(265+85*10^(
1/2))^(1/2)
```

3.27.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.95

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = -\frac{2(546+5925x+13860x^2-9628x^3)}{867(1+3x-2x^2)^{3/2}} - \frac{9}{2}\text{RootSum}\Big[5+20\#1+8\#1^2-8\#1^3+2\#1^4\&, \frac{-13\log(x)+13\log\left(-1+\sqrt{1+3x-2x^2}-x\#1\right)+6\log(}{}$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

output
```
(-2*(546 + 5925*x + 13860*x^2 - 9628*x^3))/(867*(1 + 3*x - 2*x^2)^(3/2)) -
 (9*RootSum[5 + 20*#1 + 8*#1^2 - 8*#1^3 + 2*#1^4 & , (-13*Log[x] + 13*Log[
-1 + Sqrt[1 + 3*x - 2*x^2] - x*#1] + 6*Log[x]*#1 - 6*Log[-1 + Sqrt[1 + 3*x
 - 2*x^2] - x*#1]*#1 - 2*Log[x]*#1^2 + 2*Log[-1 + Sqrt[1 + 3*x - 2*x^2] -
x*#1]*#1^2)/(5 + 4*#1 - 6*#1^2 + 2*#1^3) & ])/2
```

3.27.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1349, 27, 2135, 27, 1365, 27, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)(-2x^2+3x+1)^{5/2}}dx$$

$$\Big\downarrow \text{1349}$$

$$\frac{2}{51}\int -\frac{-168x^2-235x+112}{2(-3x^2+4x+2)(-2x^2+3x+1)^{3/2}}dx - \frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{51}\int \frac{-168x^2-235x+112}{(-3x^2+4x+2)(-2x^2+3x+1)^{3/2}}dx - \frac{2(14x+15)}{51(-2x^2+3x+1)^{3/2}}$$

$$\Big\downarrow \text{2135}$$

$$\frac{1}{51}\left(-\frac{2}{17}\int -\frac{7803(3x+2)}{2\left(-3x^2+4x+2\right)\sqrt{-2x^2+3x+1}}dx-\frac{2(4814x+291)}{17\sqrt{-2x^2+3x+1}}\right)-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{51}\left(459\int \frac{3x+2}{\left(-3x^2+4x+2\right)\sqrt{-2x^2+3x+1}}dx-\frac{2(4814x+291)}{17\sqrt{-2x^2+3x+1}}\right)-\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 1365

$$\frac{1}{51}\left(459\left(\frac{3}{5}\left(5-2\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{3}{5}\left(5+2\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2}} \right.\right.$$
$$\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 27

$$\frac{1}{51}\left(459\left(\frac{3}{10}\left(5-2\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{-2x^2+3x+1}}dx+\frac{3}{10}\left(5+2\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\sqrt{-2x^2}} \right.\right.$$
$$\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 1154

$$\frac{1}{51}\left(459\left(-\frac{3}{5}\left(5+2\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{3}{5}\left(5\right.\right.\right.$$
$$\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 217

$$\frac{1}{51}\left(459\left(-\frac{3}{5}\left(5+2\sqrt{10}\right)\int \frac{1}{-\frac{\left(\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{-2x^2+3x+1}-4\left(1-\sqrt{10}\right)}d\left(-\frac{\left(1-4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{-2x^2+3x+1}}\right)-\frac{3\left(5\right.}{}\right.\right.$$
$$\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

$\Big\downarrow$ 219

3.27. $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

$$\frac{1}{51}\left(459\left(\frac{3(5+2\sqrt{10})\operatorname{arctanh}\left(\frac{(1-4\sqrt{10})x+3(4+\sqrt{10})}{2\sqrt{\sqrt{10}-1}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{\sqrt{10}-1}}-\frac{3(5-2\sqrt{10})\arctan\left(\frac{(1+4\sqrt{10})x+3(4-\sqrt{10})}{2\sqrt{1+\sqrt{10}}\sqrt{-2x^2+3x+1}}\right)}{10\sqrt{1+\sqrt{10}}}\right)-\frac{2(4}{17\sqrt{}}\right.$$

$$\frac{2(14x+15)}{51\left(-2x^2+3x+1\right)^{3/2}}$$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x - 2*x^2)^(5/2)),x]
```

output
```
(-2*(15 + 14*x))/(51*(1 + 3*x - 2*x^2)^(3/2)) + ((-2*(291 + 4814*x))/(17*S
qrt[1 + 3*x - 2*x^2]) + 459*((-3*(5 - 2*Sqrt[10])*ArcTan[(3*(4 - Sqrt[10])
 + (1 + 4*Sqrt[10])*x)/(2*Sqrt[1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/(10*
Sqrt[1 + Sqrt[10]]) + (3*(5 + 2*Sqrt[10])*ArcTanh[(3*(4 + Sqrt[10]) + (1 -
 4*Sqrt[10])*x)/(2*Sqrt[-1 + Sqrt[10]]*Sqrt[1 + 3*x - 2*x^2])])/(10*Sqrt[-
1 + Sqrt[10]])))/51
```

3.27.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.27. $\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.27.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.27 (sec) , antiderivative size = 484, normalized size of antiderivative = 2.51

method	result
trager	$\frac{2(9628x^3-13860x^2-5925x-546)\sqrt{-2x^2+3x+1}}{867(2x^2-3x-1)^2} - 18\,\mathrm{RootOf}\left(6400\_Z^4 - 8480\_Z^2 - 81\right)\ln\left(-\frac{-2105600x\,\mathrm{Roo}}{}\right.$
default	Expression too large to display

input
```
int((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/867*(9628*x^3-13860*x^2-5925*x-546)/(2*x^2-3*x-1)^2*(-2*x^2+3*x+1)^(1/2)
-18*RootOf(6400*_Z^4-8480*_Z^2-81)*ln(-(-2105600*x*RootOf(6400*_Z^4-8480*_
Z^2-81)^5+5362400*RootOf(6400*_Z^4-8480*_Z^2-81)^3*x+74880*(-2*x^2+3*x+1)^
(1/2)*RootOf(6400*_Z^4-8480*_Z^2-81)^2+473760*RootOf(6400*_Z^4-8480*_Z^2-8
1)^3-3406349*RootOf(6400*_Z^4-8480*_Z^2-81)*x-99945*(-2*x^2+3*x+1)^(1/2)-6
32106*RootOf(6400*_Z^4-8480*_Z^2-81))/(80*x*RootOf(6400*_Z^4-8480*_Z^2-81)
^2-87*x-34))+9/10*RootOf(_Z^2+400*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)*ln
(-(2105600*x*RootOf(6400*_Z^4-8480*_Z^2-81)^4*RootOf(_Z^2+400*RootOf(6400*
_Z^4-8480*_Z^2-81)^2-530)-217440*RootOf(6400*_Z^4-8480*_Z^2-81)^2*RootOf(_
Z^2+400*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)*x+473760*RootOf(6400*_Z^4-84
80*_Z^2-81)^2*RootOf(_Z^2+400*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)-149760
0*(-2*x^2+3*x+1)^(1/2)*RootOf(6400*_Z^4-8480*_Z^2-81)^2-2187*RootOf(_Z^2+4
00*RootOf(6400*_Z^4-8480*_Z^2-81)^2-530)*x+4374*RootOf(_Z^2+400*RootOf(640
0*_Z^4-8480*_Z^2-81)^2-530)-14580*(-2*x^2+3*x+1)^(1/2))/(80*x*RootOf(6400*
_Z^4-8480*_Z^2-81)^2-19*x+34))
```

3.27.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(137) = 274$.

Time = 0.31 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.27

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx =$$

$$-\frac{43680\,x^4 - 131040\,x^3 - 867\,\sqrt{5}(4\,x^4 - 12\,x^3 + 5\,x^2 + 6\,x + 1)\sqrt{-1377\,\sqrt{10} + 4293}\log\left(-\frac{405\,\sqrt{10}x+\left(13\,\sqrt{10}\right.}{}\right.}{}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

output
```
-1/8670*(43680*x^4 - 131040*x^3 - 867*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*
x + 1)*sqrt(-1377*sqrt(10) + 4293)*log(-(405*sqrt(10)*x + (13*sqrt(10)*sqr
t(5)*x + 40*sqrt(5)*x)*sqrt(-1377*sqrt(10) + 4293) + 810*x - 810*sqrt(-2*x
^2 + 3*x + 1) + 810)/x) + 867*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*s
qrt(-1377*sqrt(10) + 4293)*log(-(405*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x +
 40*sqrt(5)*x)*sqrt(-1377*sqrt(10) + 4293) + 810*x - 810*sqrt(-2*x^2 + 3*x
 + 1) + 810)/x) - 7803*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*
sqrt(10) + 53)*log(9*(45*sqrt(10)*x + (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*
x)*sqrt(17*sqrt(10) + 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 78
03*sqrt(5)*(4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)*sqrt(17*sqrt(10) + 53)*log(9
*(45*sqrt(10)*x - (13*sqrt(10)*sqrt(5)*x - 40*sqrt(5)*x)*sqrt(17*sqrt(10)
+ 53) - 90*x + 90*sqrt(-2*x^2 + 3*x + 1) - 90)/x) + 54600*x^2 - 20*(9628*x
^3 - 13860*x^2 - 5925*x - 546)*sqrt(-2*x^2 + 3*x + 1) + 65520*x + 10920)/(
4*x^4 - 12*x^3 + 5*x^2 + 6*x + 1)
```

3.27.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx =$$
$$-\int \frac{x}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2 \cdots}$$
$$-\int \frac{2}{12x^6\sqrt{-2x^2+3x+1}-52x^5\sqrt{-2x^2+3x+1}+55x^4\sqrt{-2x^2+3x+1}+22x^3\sqrt{-2x^2+3x+1}-31x^2 \cdots}$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(-2*x**2+3*x+1)**(5/2),x)
```

output
```
-Integral(x/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*x**5*sqrt(-2*x**2 + 3*x
+ 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*sqrt(-2*x**2 + 3*x + 1) -
 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x**2 + 3*x + 1) - 2*sqrt(-
2*x**2 + 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(-2*x**2 + 3*x + 1) - 52*
x**5*sqrt(-2*x**2 + 3*x + 1) + 55*x**4*sqrt(-2*x**2 + 3*x + 1) + 22*x**3*s
qrt(-2*x**2 + 3*x + 1) - 31*x**2*sqrt(-2*x**2 + 3*x + 1) - 16*x*sqrt(-2*x*
*2 + 3*x + 1) - 2*sqrt(-2*x**2 + 3*x + 1)), x)
```

3.27.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. $2(137) = 274$.

Time = 0.32 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.61

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

output
```
1/17340*sqrt(10)*(2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) + (-2
*x^2 + 3*x + 1)^(3/2)) - 2108*sqrt(10)*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2
) - (-2*x^2 + 3*x + 1)^(3/2)) - 56916*sqrt(10)*x/(2*sqrt(10)*sqrt(-2*x^2 +
 3*x + 1) + 11*sqrt(-2*x^2 + 3*x + 1)) + 56916*sqrt(10)*x/(2*sqrt(10)*sqrt
(-2*x^2 + 3*x + 1) - 11*sqrt(-2*x^2 + 3*x + 1)) + 1984*sqrt(10)*x/(sqrt(10
)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-2*x^2 + 3*x + 1)) - 1984*sqrt(10)*x/(sqrt
(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2 + 3*x + 1)) - 70227*sqrt(10)*arc
sin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10) - 4) + 2/17*sqrt(17)*x/a
bs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/abs(6*x + 2*sqrt(10) - 4
) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*sqrt(10)*sqrt(sqrt(10) +
1) + 11*sqrt(sqrt(10) + 1)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) +
(-2*x^2 + 3*x + 1)^(3/2)) - 2176*x/(sqrt(10)*(-2*x^2 + 3*x + 1)^(3/2) - (-
2*x^2 + 3*x + 1)^(3/2)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + 11*
sqrt(-2*x^2 + 3*x + 1)) + 58752*x/(2*sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - 11*
sqrt(-2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) + sqrt(-
2*x^2 + 3*x + 1)) - 2048*x/(sqrt(10)*sqrt(-2*x^2 + 3*x + 1) - sqrt(-2*x^2
+ 3*x + 1)) + 561816*arcsin(8/17*sqrt(17)*sqrt(10)*x/abs(6*x + 2*sqrt(10)
- 4) + 2/17*sqrt(17)*x/abs(6*x + 2*sqrt(10) - 4) - 6/17*sqrt(17)*sqrt(10)/
abs(6*x + 2*sqrt(10) - 4) + 24/17*sqrt(17)/abs(6*x + 2*sqrt(10) - 4))/(2*s
qrt(10)*sqrt(sqrt(10) + 1) + 11*sqrt(sqrt(10) + 1)) - 714*sqrt(10)/(sqr...
```

3.27.8 Giac [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(-2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x-2x^2)^{5/2}}\,dx=\int \frac{x+2}{\left(-2\,x^2+3\,x+1\right)^{5/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

input
```
int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x - 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

3.28 $\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$

3.28.1 Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = -\frac{1}{2}\sqrt{1+\frac{7\sqrt{\frac{2}{5}}}{5}}\operatorname{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right) + \frac{1}{2}\sqrt{1-\frac{7\sqrt{\frac{2}{5}}}{5}}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)$$

output
```
1/10*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55
+17*10^(1/2))^(1/2))*(25-7*10^(1/2))^(1/2)-1/10*arctanh(1/2*(x*(17-4*10^(1
/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2))*(25+7*10^(
1/2))^(1/2)
```

3.28.2 Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.72

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= -\frac{1}{5}\sqrt{25+7\sqrt{10}}\text{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)$$
$$+\frac{1}{5}\sqrt{25-7\sqrt{10}}\text{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

output
```
-1/5*(Sqrt[25 + 7*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*
x^2])/(1 + 2*x)]) + (Sqrt[25 - 7*Sqrt[10]]*ArcTanh[(Sqrt[1 + Sqrt[2/5]]*Sq
rt[1 + 3*x + 2*x^2])/(1 + 2*x)])/5
```

3.28.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)\sqrt{2x^2+3x+1}}\,dx$$
$$\Big\downarrow \text{1365}$$
$$\frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx+\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx$$
$$\Big\downarrow \text{27}$$

$$\frac{1}{10}\left(5-4\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx+\frac{1}{10}\left(5+4\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx$$

$$\Big\downarrow \textbf{1154}$$

$$-\frac{1}{5}\left(5-4\sqrt{10}\right)\int \frac{1}{4\left(55-17\sqrt{10}\right)-\frac{\left(\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)\right)^2}{2x^2+3x+1}}d\left(-\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{\sqrt{2x^2+3x+1}}\right)-\frac{1}{5}\left(5+4\sqrt{10}\right)\int \frac{1}{4\left(55+17\sqrt{10}\right)-\frac{\left(\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)\right)^2}{2x^2+3x+1}}d\left(-\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{\sqrt{2x^2+3x+1}}\right)$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\left(5-4\sqrt{10}\right)\text{arctanh}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}+\frac{\left(5+4\sqrt{10}\right)\text{arctanh}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55+17\sqrt{10}}}$$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*Sqrt[1 + 3*x + 2*x^2]),x]
```

output
```
((5 - 4*Sqrt[10])*ArcTanh[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt
[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/(10*Sqrt[55 - 17*Sqrt[10]]) +
((5 + 4*Sqrt[10])*ArcTanh[(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt
[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/(10*Sqrt[55 + 17*Sqrt[10]])
```

3.28.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.28.4 Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.23

method	result
default	$\frac{\left(8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{55+17\sqrt{10}+\frac{9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55+17\sqrt{10}}\sqrt{18\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^{2}+9\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+55+17\sqrt{10}}}\right)}{20\sqrt{55+17\sqrt{10}}}+\frac{\left(-8+\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\right.}{}$
trager	$\operatorname{RootOf}\left(\_Z^{2}+4\operatorname{RootOf}\left(2000\_Z^{4}-1000\_Z^{2}+27\right)^{2}-2\right)\ln\left(-\frac{10000x\operatorname{RootOf}\left(2000\_Z^{4}-1000\_Z^{2}+27\right)^{4}\operatorname{RootOf}\left(\_Z^{2}+4\operatorname{RootOf}\right.}{}\right.$

input
```
int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/20*(8+10^(1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*
10^(1/2)+(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/
(18*(x-2/3-1/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+1
7*10^(1/2))^(1/2))+1/20*(-8+10^(1/2))*10^(1/2)/(55-17*10^(1/2))^(1/2)*arct
anh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+1/3*10^(1/2)))/(55
-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4/3*10^(1/2))*(x-2/
3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))
```

3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(103) = 206.

Time = 0.31 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.62

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= \frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x+\left(\sqrt{10}x-4x\right)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)$$
$$-\frac{1}{10}\sqrt{7\sqrt{10}+25}\log\left(-\frac{3\sqrt{10}x-\left(\sqrt{10}x-4x\right)\sqrt{7\sqrt{10}+25}+6x-6\sqrt{2x^2+3x+1}+6}{x}\right)$$
$$+\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x+\left(\sqrt{10}x+4x\right)\sqrt{-7\sqrt{10}+25}-6x+6\sqrt{2x^2+3x+1}-6}{x}\right)$$
$$-\frac{1}{10}\sqrt{-7\sqrt{10}+25}\log\left(\frac{3\sqrt{10}x-\left(\sqrt{10}x+4x\right)\sqrt{-7\sqrt{10}+25}-6x+6\sqrt{2x^2+3x+1}-6}{x}\right)$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="fricas")
```

output
```
1/10*sqrt(7*sqrt(10) + 25)*log(-(3*sqrt(10)*x + (sqrt(10)*x - 4*x)*sqrt(7*
sqrt(10) + 25) + 6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) - 1/10*sqrt(7*sqrt(
10) + 25)*log(-(3*sqrt(10)*x - (sqrt(10)*x - 4*x)*sqrt(7*sqrt(10) + 25) +
6*x - 6*sqrt(2*x^2 + 3*x + 1) + 6)/x) + 1/10*sqrt(-7*sqrt(10) + 25)*log((3
*sqrt(10)*x + (sqrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x
^2 + 3*x + 1) - 6)/x) - 1/10*sqrt(-7*sqrt(10) + 25)*log((3*sqrt(10)*x - (s
qrt(10)*x + 4*x)*sqrt(-7*sqrt(10) + 25) - 6*x + 6*sqrt(2*x^2 + 3*x + 1) -
6)/x)
```

3.28.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= -\int \frac{x}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$
$$-\int \frac{2}{3x^2\sqrt{2x^2+3x+1}-4x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}\,dx$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(1/2),x)
```

output
```
-Integral(x/(3*x**2*sqrt(2*x**2 + 3*x + 1) - 4*x*sqrt(2*x**2 + 3*x + 1) -
2*sqrt(2*x**2 + 3*x + 1)), x) - Integral(2/(3*x**2*sqrt(2*x**2 + 3*x + 1)
- 4*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(103) = 206$.

Time = 0.31 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.40

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx$$
$$= \frac{1}{60}\sqrt{10}\left(\frac{3\sqrt{10}\log\left(\frac{2}{9}\sqrt{10}+\frac{2\sqrt{2x^2+3x+1}\sqrt{17\sqrt{10}+55}}{3|6x-2\sqrt{10}-4|}+\frac{34\sqrt{10}}{9|6x-2\sqrt{10}-4|}+\frac{110}{9|6x-2\sqrt{10}-4|}+\frac{17}{18}\right)}{\sqrt{17\sqrt{10}+55}}+\frac{\sqrt{10}\log\left(-\right.}{}\right.$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="maxima")
```

output
```
1/60*sqrt(10)*(3*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqr
t(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*
sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10)
+ 55) + sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sq
rt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(
10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) +
55/9) + 24*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x + 1)*sqrt(17*sqrt(10) +
 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(6*x - 2*sqrt(10) - 4) +
 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/sqrt(17*sqrt(10) + 55) - 8*log(-
2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9*sqrt(10) + 55/9)/abs(6*x
 + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sqrt(10) - 4) + 110/9/abs(6
*x + 2*sqrt(10) - 4) + 17/18)/sqrt(-17/9*sqrt(10) + 55/9))
```

3.28.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.62

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = 0.169235232112667 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 0.686556214893333 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 0.686556214893333 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.169235232112667 \log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(1/2),x, algorithm="giac")
```

output
```
0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.9097693271200
0) - 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.17652715
6327000) + 0.686556214893333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.91
9278730509000) - 0.169235232112667*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1)
- 1.04272727395000)
```

3.28.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\sqrt{1+3x+2x^2}}\,dx = \int \frac{x+2}{\sqrt{2\,x^2+3\,x+1}\,(-3\,x^2+4\,x+2)}\,dx$$

input
```
int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x + 2*x^2 + 1)^(1/2)*(4*x - 3*x^2 + 2)), x)
```

3.29 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$

3.29.1 Optimal result

Integrand size = 30, antiderivative size = 174

$$\begin{aligned}\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx &= \frac{2(21+22x)}{5\sqrt{1+3x+2x^2}}\\ &-\frac{1}{10}\sqrt{\frac{3}{5}\left(2065+653\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\\ &+\frac{1}{10}\sqrt{\frac{3}{5}\left(2065-653\sqrt{10}\right)}\operatorname{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\end{aligned}$$

output
```
2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)+1/50*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*
10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+17*10^(1/2))^(1/2))*(30975-9795*10^(1/2
))^(1/2)-1/50*arctanh(1/2*(x*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^
(1/2)/(55-17*10^(1/2))^(1/2))*(30975+9795*10^(1/2))^(1/2)
```

3.29.2 Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.75

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = \frac{1}{25}\left(\frac{5(42+44x)}{\sqrt{1+3x+2x^2}} - \sqrt{30975+9795\sqrt{10}}\operatorname{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right) + \frac{45\operatorname{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)}{\sqrt{2065+653\sqrt{10}}}\right)$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

output
```
((5*(42 + 44*x))/Sqrt[1 + 3*x + 2*x^2] - Sqrt[30975 + 9795*Sqrt[10]]*ArcTa
nh[(Sqrt[1 - Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)] + (45*ArcTanh[(S
qrt[1 + Sqrt[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/Sqrt[2065 + 653*Sqrt
[10]])/25
```

3.29.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1349, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)(2x^2+3x+1)^{3/2}}\,dx$$

$$\Big\downarrow \; 1349$$

$$\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}-\frac{2}{15}\int -\frac{9(16-9x)}{2\left(-3x^2+4x+2\right)\sqrt{2x^2+3x+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{5}\int \frac{16-9x}{\left(-3x^2+4x+2\right)\sqrt{2x^2+3x+1}}dx+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

$$\Big\downarrow \text{ 1365}$$

$$\frac{3}{5}\left(-3\left(3+\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx-3\left(3-\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx\right)+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3}{5}\left(-\frac{3}{2}\left(3+\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx-\frac{3}{2}\left(3-\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx\right)+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{3}{5}\left(3\left(3+\sqrt{10}\right)\int \frac{1}{4\left(55-17\sqrt{10}\right)-\frac{\left(\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)\right)^2}{2x^2+3x+1}}d\left(-\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{\sqrt{2x^2+3x+1}}\right)+3\left(3-\sqrt{10}\right) \cdots\right)+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{3}{5}\left(-\frac{3\left(3+\sqrt{10}\right)\operatorname{arctanh}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{2\sqrt{55-17\sqrt{10}}}-\frac{3\left(3-\sqrt{10}\right)\operatorname{arctanh}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{2\sqrt{55+17\sqrt{10}}}\right)+\frac{2(22x+21)}{5\sqrt{2x^2+3x+1}}$$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(3/2)),x]
```

3.29. $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx$

output
```
(2*(21 + 22*x))/(5*Sqrt[1 + 3*x + 2*x^2]) + (3*((-3*(3 + Sqrt[10])*ArcTanh
[(3*(4 - Sqrt[10]) + (17 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1
 + 3*x + 2*x^2])])/(2*Sqrt[55 - 17*Sqrt[10]]) - (3*(3 - Sqrt[10])*ArcTanh[
(3*(4 + Sqrt[10]) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1
+ 3*x + 2*x^2])])/(2*Sqrt[55 + 17*Sqrt[10]])))/5
```

3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.29.4 Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.20

method	result
risch	$\frac{\frac{42}{5}+\frac{44x}{5}}{\sqrt{2x^2+3x+1}}-\frac{9\left(10+3\sqrt{10}\right)\sqrt{10}\,\operatorname{arctanh}\left(\frac{55-17\sqrt{10}+\frac{9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)}{2}}{\sqrt{55-17\sqrt{10}}\sqrt{18\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)^2+9\left(\frac{17}{3}-\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}+\frac{\sqrt{10}}{3}\right)+55-17\sqrt{10}}}\right)}{100\sqrt{55-17\sqrt{10}}}-\frac{9\left(-10+3\right.}{}$
trager	$\frac{\frac{42}{5}+\frac{44x}{5}}{\sqrt{2x^2+3x+1}}-\frac{\operatorname{RootOf}\left(\_Z^2+144\operatorname{RootOf}\left(3840\_Z^4-66080\_Z^2+9\right)^2-2478\right)\ln\left(\frac{928000x\operatorname{RootOf}\left(3840\_Z^4-66080\_Z^2+9\right)}{}\right.}{}$
default	$-\frac{\left(8+\sqrt{10}\right)\sqrt{10}\left(\frac{1}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\sqrt{2\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)^2+\left(\frac{17}{3}+\frac{4\sqrt{10}}{3}\right)\left(x-\frac{2}{3}-\frac{\sqrt{10}}{3}\right)+\frac{55}{9}+\frac{17\sqrt{10}}{9}}}-\frac{}{3\left(\frac{55}{9}+\frac{17\sqrt{10}}{9}\right)\left(\frac{440}{9}+\frac{136\sqrt{10}}{9}-\left(\frac{17}{3}+\frac{4\sqrt{}}{}\right.\right.}\right.}{}$

input
```
int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(21+22*x)/(2*x^2+3*x+1)^(1/2)-9/100*(10+3*10^(1/2))*10^(1/2)/(55-17*10
^(1/2))^(1/2)*arctanh(9/2*(110/9-34/9*10^(1/2)+(17/3-4/3*10^(1/2))*(x-2/3+
1/3*10^(1/2)))/(55-17*10^(1/2))^(1/2)/(18*(x-2/3+1/3*10^(1/2))^2+9*(17/3-4
/3*10^(1/2))*(x-2/3+1/3*10^(1/2))+55-17*10^(1/2))^(1/2))-9/100*(-10+3*10^(
1/2))*10^(1/2)/(55+17*10^(1/2))^(1/2)*arctanh(9/2*(110/9+34/9*10^(1/2)+(17
/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2)))/(55+17*10^(1/2))^(1/2)/(18*(x-2/3-1
/3*10^(1/2))^2+9*(17/3+4/3*10^(1/2))*(x-2/3-1/3*10^(1/2))+55+17*10^(1/2))^
(1/2))
```

3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(122) = 244$.

Time = 0.32 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.10

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = \frac{\sqrt{5}(2x^2+3x+1)\sqrt{1959\sqrt{10}+6195}\log\left(-\frac{45\sqrt{10}x+\left(41\sqrt{10}\sqrt{5}x-13\right.}{}\right.}{}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="fricas")
```

output
```
1/50*(sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(1959*sqrt(10) + 6195)*log(-(45*sqrt(1
0)*x + (41*sqrt(10)*sqrt(5)*x - 130*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195)
+ 90*x - 90*sqrt(2*x^2 + 3*x + 1) + 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1)*sqr
t(1959*sqrt(10) + 6195)*log(-(45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x - 130
*sqrt(5)*x)*sqrt(1959*sqrt(10) + 6195) + 90*x - 90*sqrt(2*x^2 + 3*x + 1) +
 90)/x) + sqrt(5)*(2*x^2 + 3*x + 1)*sqrt(-1959*sqrt(10) + 6195)*log((45*sq
rt(10)*x + (41*sqrt(10)*sqrt(5)*x + 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6
195) - 90*x + 90*sqrt(2*x^2 + 3*x + 1) - 90)/x) - sqrt(5)*(2*x^2 + 3*x + 1
)*sqrt(-1959*sqrt(10) + 6195)*log((45*sqrt(10)*x - (41*sqrt(10)*sqrt(5)*x
+ 130*sqrt(5)*x)*sqrt(-1959*sqrt(10) + 6195) - 90*x + 90*sqrt(2*x^2 + 3*x
+ 1) - 90)/x) + 840*x^2 + 20*sqrt(2*x^2 + 3*x + 1)*(22*x + 21) + 1260*x +
420)/(2*x^2 + 3*x + 1)
```

3.29.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx =$$
$$-\int \frac{x}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}$$
$$-\int \frac{2}{6x^4\sqrt{2x^2+3x+1}+x^3\sqrt{2x^2+3x+1}-13x^2\sqrt{2x^2+3x+1}-10x\sqrt{2x^2+3x+1}-2\sqrt{2x^2+3x+1}}$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(3/2),x)
```

output
```
-Integral(x/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqrt(2*x**2 + 3*x + 1) -
 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x
**2 + 3*x + 1)), x) - Integral(2/(6*x**4*sqrt(2*x**2 + 3*x + 1) + x**3*sqr
t(2*x**2 + 3*x + 1) - 13*x**2*sqrt(2*x**2 + 3*x + 1) - 10*x*sqrt(2*x**2 +
3*x + 1) - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(122) = 244$.

Time = 0.30 (sec) , antiderivative size = 668, normalized size of antiderivative = 3.84

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx =$$

$$-\frac{1}{60}\sqrt{10}\left(\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}+55\sqrt{2x^2+3x+1}}-\frac{588\sqrt{10}x}{17\sqrt{10}\sqrt{2x^2+3x+1}-55\sqrt{2x^2+3x+1}}+\right.$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="maxima")
```

output
```
-1/60*sqrt(10)*(588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqr
t(2*x^2 + 3*x + 1)) - 588*sqrt(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) -
55*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55
*sqrt(2*x^2 + 3*x + 1)) + 2112*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*s
qrt(2*x^2 + 3*x + 1)) - 27*sqrt(10)*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*
x + 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/ab
s(6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqr
t(10) + 55)^(3/2) - sqrt(10)*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*s
qrt(-17/9*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6
*x + 2*sqrt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqr
t(10) + 55/9)^(3/2) + 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55
*sqrt(2*x^2 + 3*x + 1)) - 450*sqrt(10)/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1)
- 55*sqrt(2*x^2 + 3*x + 1)) - 216*log(2/9*sqrt(10) + 2/3*sqrt(2*x^2 + 3*x
+ 1)*sqrt(17*sqrt(10) + 55)/abs(6*x - 2*sqrt(10) - 4) + 34/9*sqrt(10)/abs(
6*x - 2*sqrt(10) - 4) + 110/9/abs(6*x - 2*sqrt(10) - 4) + 17/18)/(17*sqrt(
10) + 55)^(3/2) + 8*log(-2/9*sqrt(10) + 2*sqrt(2*x^2 + 3*x + 1)*sqrt(-17/9
*sqrt(10) + 55/9)/abs(6*x + 2*sqrt(10) - 4) - 34/9*sqrt(10)/abs(6*x + 2*sq
rt(10) - 4) + 110/9/abs(6*x + 2*sqrt(10) - 4) + 17/18)/(-17/9*sqrt(10) + 5
5/9)^(3/2) + 1656/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x
 + 1)) + 1656/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x ...
```

3.29.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.64

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{3/2}}\,dx = \frac{2\,(22\,x+21)}{5\,\sqrt{2\,x^2+3\,x+1}} + 0.0140045514133333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 4.97793168620000\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 4.97793168620000\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.0140045514125333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(3/2),x, algorithm="giac")
```

output
```
2/5*(22*x + 21)/sqrt(2*x^2 + 3*x + 1) + 0.0140045514133333*log(-sqrt(2)*x
+ sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) - 4.97793168620000*log(-sqrt(2
)*x + sqrt(2*x^2 + 3*x + 1) - 0.176527156327000) + 4.97793168620000*log(-s
qrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.919278730509000) - 0.0140045514125333
*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.04272727395000)
```

3.29.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)\left(1+3x+2x^2\right)^{3/2}}\,dx = \int \frac{x+2}{\left(2\,x^2+3\,x+1\right)^{3/2}\,\left(-3\,x^2+4\,x+2\right)}\,dx$$

input
```
int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x + 2*x^2 + 1)^(3/2)*(4*x - 3*x^2 + 2)), x)
```

3.30 $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

3.30.1 Optimal result

Integrand size = 30, antiderivative size = 197

$$\begin{aligned}\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx &= \frac{2(21+22x)}{15(1+3x+2x^2)^{3/2}}+\frac{2(273+230x)}{15\sqrt{1+3x+2x^2}}\\ &\quad-\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115+1544809\sqrt{10}\right)}\,\mathrm{arctanh}\left(\frac{3(4-\sqrt{10})+(17-4\sqrt{10})\,x}{2\sqrt{55-17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\\ &\quad+\frac{1}{50}\sqrt{\frac{1}{3}\left(4885115-1544809\sqrt{10}\right)}\,\mathrm{arctanh}\left(\frac{3(4+\sqrt{10})+(17+4\sqrt{10})\,x}{2\sqrt{55+17\sqrt{10}}\sqrt{1+3x+2x^2}}\right)\end{aligned}$$

output
```
2/15*(21+22*x)/(2*x^2+3*x+1)^(3/2)+2/15*(273+230*x)/(2*x^2+3*x+1)^(1/2)+1/
150*arctanh(1/2*(12+3*10^(1/2)+x*(17+4*10^(1/2)))/(2*x^2+3*x+1)^(1/2)/(55+
17*10^(1/2))^(1/2))*(14655345-4634427*10^(1/2))^(1/2)-1/150*arctanh(1/2*(x
*(17-4*10^(1/2))+12-3*10^(1/2))/(2*x^2+3*x+1)^(1/2)/(55-17*10^(1/2))^(1/2)
)*(14655345+4634427*10^(1/2))^(1/2)
```

3.30.2 Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.78

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2\sqrt{1+3x+2x^2}(294+1071x+1236x^2+460x^3)}{15(1+x)^2(1+2x)^2}$$
$$-\frac{1}{75}\sqrt{14655345+4634427\sqrt{10}}\text{arctanh}\left(\frac{\sqrt{1-\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)$$
$$+\frac{81\text{arctanh}\left(\frac{\sqrt{1+\sqrt{\frac{2}{5}}}\sqrt{1+3x+2x^2}}{1+2x}\right)}{5\sqrt{24425575+7724045\sqrt{10}}}$$

input
```
Integrate[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[1 + 3*x + 2*x^2]*(294 + 1071*x + 1236*x^2 + 460*x^3))/(15*(1 + x)^
2*(1 + 2*x)^2) - (Sqrt[14655345 + 4634427*Sqrt[10]]*ArcTanh[(Sqrt[1 - Sqrt
[2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/75 + (81*ArcTanh[(Sqrt[1 + Sqrt[
2/5]]*Sqrt[1 + 3*x + 2*x^2])/(1 + 2*x)])/(5*Sqrt[24425575 + 7724045*Sqrt[1
0]])
```

3.30.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {1349, 27, 2135, 27, 1365, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+2}{(-3x^2+4x+2)(2x^2+3x+1)^{5/2}}\,dx$$
$$\Big\downarrow \text{ 1349}$$
$$\frac{2(22x+21)}{15(2x^2+3x+1)^{3/2}}-\frac{2}{45}\int -\frac{3(-264x^2+271x+320)}{2(-3x^2+4x+2)(2x^2+3x+1)^{3/2}}dx$$
$$\Big\downarrow \text{ 27}$$

$$\frac{1}{15}\int \frac{-264x^2+271x+320}{(-3x^2+4x+2)\left(2x^2+3x+1\right)^{3/2}}dx+\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 2135

$$\frac{1}{15}\left(\frac{2(230x+273)}{\sqrt{2x^2+3x+1}}-\frac{2}{15}\int -\frac{45(346-201x)}{2\left(-3x^2+4x+2\right)\sqrt{2x^2+3x+1}}dx\right)+\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(3\int \frac{346-201x}{\left(-3x^2+4x+2\right)\sqrt{2x^2+3x+1}}dx+\frac{2(230x+273)}{\sqrt{2x^2+3x+1}}\right)+\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 1365

$$\frac{1}{15}\left(3\left(-\frac{3}{5}\left(335+106\sqrt{10}\right)\int \frac{1}{2\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx-\frac{3}{5}\left(335-106\sqrt{10}\right)\int \frac{1}{2\left(-3x+\sqrt{10}+2\right)\right.\right.\right.$$
$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 27

$$\frac{1}{15}\left(3\left(-\frac{3}{10}\left(335+106\sqrt{10}\right)\int \frac{1}{\left(-3x-\sqrt{10}+2\right)\sqrt{2x^2+3x+1}}dx-\frac{3}{10}\left(335-106\sqrt{10}\right)\int \frac{1}{\left(-3x+\sqrt{10}+2\right)\right.\right.\right.$$
$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 1154

$$\frac{1}{15}\left(3\left(\frac{3}{5}\left(335+106\sqrt{10}\right)\int \frac{1}{4\left(55-17\sqrt{10}\right)-\frac{\left(\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)\right)^2}{2x^2+3x+1}}d\left(-\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{\sqrt{2x^2+3x+1}}\right)+\right.\right.$$
$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

$\big\downarrow$ 219

$$\frac{1}{15}\left(3\left(-\frac{3\left(335+106\sqrt{10}\right)\operatorname{arctanh}\left(\frac{\left(17-4\sqrt{10}\right)x+3\left(4-\sqrt{10}\right)}{2\sqrt{55-17\sqrt{10}}\sqrt{2x^2+3x+1}}\right)}{10\sqrt{55-17\sqrt{10}}}-\frac{3\left(335-106\sqrt{10}\right)\operatorname{arctanh}\left(\frac{\left(17+4\sqrt{10}\right)x+3\left(4+\sqrt{10}\right)}{2\sqrt{55+17\sqrt{10}}\sqrt{2x^2+3x+1}}\right.}{10\sqrt{55+17\sqrt{10}}}\right.\right.$$
$$\frac{2(22x+21)}{15\left(2x^2+3x+1\right)^{3/2}}$$

3.30. $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

input
```
Int[(2 + x)/((2 + 4*x - 3*x^2)*(1 + 3*x + 2*x^2)^(5/2)),x]
```

output
```
(2*(21 + 22*x))/(15*(1 + 3*x + 2*x^2)^(3/2)) + ((2*(273 + 230*x))/Sqrt[1 +
 3*x + 2*x^2] + 3*((-3*(335 + 106*Sqrt[10])*ArcTanh[(3*(4 - Sqrt[10]) + (1
7 - 4*Sqrt[10])*x)/(2*Sqrt[55 - 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])])/(10*
Sqrt[55 - 17*Sqrt[10]]) - (3*(335 - 106*Sqrt[10])*ArcTanh[(3*(4 + Sqrt[10]
) + (17 + 4*Sqrt[10])*x)/(2*Sqrt[55 + 17*Sqrt[10]]*Sqrt[1 + 3*x + 2*x^2])]
)/(10*Sqrt[55 + 17*Sqrt[10]])))/15
```

3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.30. $\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx$

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.30.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 2.52 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.39

method	result
trager	$\frac{\frac{184}{3}x^3+\frac{824}{5}x^2+\frac{714}{5}x+\frac{196}{5}}{(2x^2+3x+1)^{\frac{3}{2}}}+\frac{\operatorname{RootOf}\left(\_Z^2+3600\operatorname{RootOf}\left(96000\_Z^4-781618400\_Z^2+6561\right)^2-29310690\right)\ln\left(\frac{-504352000x\,\mathrm{R}}{}\right.}{}$
default	Expression too large to display

input
```
int((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(460*x^3+1236*x^2+1071*x+294)/(2*x^2+3*x+1)^(3/2)+1/150*RootOf(_Z^2+3
600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)*ln((-504352000*x*Ro
otOf(96000*_Z^4-781618400*_Z^2+6561)^4*RootOf(_Z^2+3600*RootOf(96000*_Z^4-
781618400*_Z^2+6561)^2-29310690)+20232850257120*RootOf(96000*_Z^4-78161840
0*_Z^2+6561)^2*RootOf(_Z^2+3600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-2
9310690)*x+1601657971200*(2*x^2+3*x+1)^(1/2)*RootOf(96000*_Z^4-781618400*_
Z^2+6561)^2+11686912631520*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2*RootOf
(_Z^2+3600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)-108791059443
3*RootOf(_Z^2+3600*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-29310690)*x+30
15957496555836*(2*x^2+3*x+1)^(1/2)-628400494638*RootOf(_Z^2+3600*RootOf(96
000*_Z^4-781618400*_Z^2+6561)^2-29310690))/(1200*x*RootOf(96000*_Z^4-78161
8400*_Z^2+6561)^2-7974733*x-3089618))-2/5*RootOf(96000*_Z^4-781618400*_Z^2
+6561)*ln(-(-22695840000*RootOf(96000*_Z^4-781618400*_Z^2+6561)^5*x-540905
633498400*RootOf(96000*_Z^4-781618400*_Z^2+6561)^3*x+1201243478400*(2*x^2+
3*x+1)^(1/2)*RootOf(96000*_Z^4-781618400*_Z^2+6561)^2-525911068418400*Root
Of(96000*_Z^4-781618400*_Z^2+6561)^3+5908432074489101275*RootOf(96000*_Z^4
-781618400*_Z^2+6561)*x-12042322347390237*(2*x^2+3*x+1)^(1/2)+428186513697
2328150*RootOf(96000*_Z^4-781618400*_Z^2+6561))/(1200*x*RootOf(96000*_Z^4-
781618400*_Z^2+6561)^2-1795497*x+3089618))
```

3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(141) = 282$.

Time = 0.30 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.21

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{23520\,x^4 + 70560\,x^3 + \sqrt{3}(4\,x^4 + 12\,x^3 + 13\,x^2 + 6\,x + 1)\sqrt{15448}}{}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="fricas")
```

output
```
1/150*(23520*x^4 + 70560*x^3 + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1)
*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x + (893*sqrt(10)*sqr
t(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*sq
rt(2*x^2 + 3*x + 1) + 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x + 1
)*sqrt(1544809*sqrt(10) + 4885115)*log(-(243*sqrt(10)*x - (893*sqrt(10)*sq
rt(3)*x - 2824*sqrt(3)*x)*sqrt(1544809*sqrt(10) + 4885115) + 486*x - 486*s
qrt(2*x^2 + 3*x + 1) + 486)/x) + sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x +
1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x + (893*sqrt(10)*s
qrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 486
*sqrt(2*x^2 + 3*x + 1) - 486)/x) - sqrt(3)*(4*x^4 + 12*x^3 + 13*x^2 + 6*x
+ 1)*sqrt(-1544809*sqrt(10) + 4885115)*log((243*sqrt(10)*x - (893*sqrt(10)
*sqrt(3)*x + 2824*sqrt(3)*x)*sqrt(-1544809*sqrt(10) + 4885115) - 486*x + 4
86*sqrt(2*x^2 + 3*x + 1) - 486)/x) + 76440*x^2 + 20*(460*x^3 + 1236*x^2 +
1071*x + 294)*sqrt(2*x^2 + 3*x + 1) + 35280*x + 5880)/(4*x^4 + 12*x^3 + 13
*x^2 + 6*x + 1)
```

3.30.6 Sympy [F]

$$\int \frac{2+x}{(2+4x-3x^2)\left(1+3x+2x^2\right)^{5/2}}\,dx =$$
$$-\int \frac{x}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+$$
$$-\int \frac{2}{12x^6\sqrt{2x^2+3x+1}+20x^5\sqrt{2x^2+3x+1}-17x^4\sqrt{2x^2+3x+1}-58x^3\sqrt{2x^2+3x+1}-47x^2\sqrt{2x^2+$$

input
```
integrate((2+x)/(-3*x**2+4*x+2)/(2*x**2+3*x+1)**(5/2),x)
```

output
```
-Integral(x/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqrt(2*x**2 + 3*x +
1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2 + 3*x + 1) - 47*
x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1) - 2*sqrt(2*x**2
+ 3*x + 1)), x) - Integral(2/(12*x**6*sqrt(2*x**2 + 3*x + 1) + 20*x**5*sqr
t(2*x**2 + 3*x + 1) - 17*x**4*sqrt(2*x**2 + 3*x + 1) - 58*x**3*sqrt(2*x**2
 + 3*x + 1) - 47*x**2*sqrt(2*x**2 + 3*x + 1) - 16*x*sqrt(2*x**2 + 3*x + 1)
 - 2*sqrt(2*x**2 + 3*x + 1)), x)
```

3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(141) = 282.

Time = 0.32 (sec) , antiderivative size = 1276, normalized size of antiderivative = 6.48

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="maxima")
```

output
```
-1/300*sqrt(10)*(980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*
(2*x^2 + 3*x + 1)^(3/2)) - 980*sqrt(10)*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(
3/2) - 55*(2*x^2 + 3*x + 1)^(3/2)) + 5292*sqrt(10)*x/(374*sqrt(10)*sqrt(2*
x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1)) - 5292*sqrt(10)*x/(374*sqrt(1
0)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 15680*sqrt(10)*x/
(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 15680*sqr
t(10)*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 3
520*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x + 1)^(3/2)) +
 3520*x/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2 + 3*x + 1)^(3/2))
 + 19008*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1183*sqrt(2*x^2 + 3*x + 1
)) + 19008*x/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 1183*sqrt(2*x^2 + 3*x +
 1)) - 56320*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x +
1)) - 56320*x/(17*sqrt(10)*sqrt(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1
)) + 750*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) + 55*(2*x^2 + 3*x +
 1)^(3/2)) - 750*sqrt(10)/(17*sqrt(10)*(2*x^2 + 3*x + 1)^(3/2) - 55*(2*x^2
 + 3*x + 1)^(3/2)) + 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x + 1) + 1
183*sqrt(2*x^2 + 3*x + 1)) - 4050*sqrt(10)/(374*sqrt(10)*sqrt(2*x^2 + 3*x
+ 1) - 1183*sqrt(2*x^2 + 3*x + 1)) - 11760*sqrt(10)/(17*sqrt(10)*sqrt(2*x^
2 + 3*x + 1) + 55*sqrt(2*x^2 + 3*x + 1)) + 11760*sqrt(10)/(17*sqrt(10)*sqr
t(2*x^2 + 3*x + 1) - 55*sqrt(2*x^2 + 3*x + 1)) + 2760/(17*sqrt(10)*(2*x...
```

3.30.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \frac{2\left((4\,(115\,x+309)x+1071)x+294\right)}{15\,(2\,x^2+3\,x+1)^{\frac{3}{2}}} + 0.00115890443050800\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}+5.90976932712000\right) - 36.0928986365333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.176527156327000\right) + 36.0928986365333\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-0.919278730509000\right) - 0.00115890442528267\,\log\left(-\sqrt{2}x+\sqrt{2\,x^2+3\,x+1}-1.04272727395000\right)$$

input
```
integrate((2+x)/(-3*x^2+4*x+2)/(2*x^2+3*x+1)^(5/2),x, algorithm="giac")
```

output
```
2/15*((4*(115*x + 309)*x + 1071)*x + 294)/(2*x^2 + 3*x + 1)^(3/2) + 0.0011
5890443050800*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) + 5.90976932712000) -
 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.1765271563270
00) + 36.0928986365333*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 0.91927873
0509000) - 0.00115890442528267*log(-sqrt(2)*x + sqrt(2*x^2 + 3*x + 1) - 1.
04272727395000)
```

3.30.9 Mupad [F(-1)]

Timed out.

$$\int \frac{2+x}{(2+4x-3x^2)(1+3x+2x^2)^{5/2}}\,dx = \int \frac{x+2}{(2\,x^2+3\,x+1)^{5/2}\,(-3\,x^2+4\,x+2)}\,dx$$

input
```
int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)),x)
```

output
```
int((x + 2)/((3*x + 2*x^2 + 1)^(5/2)*(4*x - 3*x^2 + 2)), x)
```

3.31 $\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

3.31.1 Optimal result

Integrand size = 26, antiderivative size = 15

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\text{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

output
```
-arctanh((x^2+2*x+5)^(1/2))
```

3.31.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\text{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

input
```
Integrate[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
-ArcTanh[Sqrt[5 + 2*x + x^2]]
```

3.31.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{1357}$$

$$-2\int \frac{1}{2-2(x^2+2x+5)}d\sqrt{x^2+2x+5}$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

input
```
Int[(1 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
-ArcTanh[Sqrt[5 + 2*x + x^2]]
```

3.31.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

3.31.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
default	$-\operatorname{arctanh}\left(\sqrt{x^{2}+2x+5}\right)$	14
pseudoelliptic	$-\operatorname{arctanh}\left(\sqrt{x^{2}+2x+5}\right)$	14
trager	$-\frac{\ln\left(\frac{x^{2}+2\sqrt{x^{2}+2x+5}+2x+6}{x^{2}+2x+4}\right)}{2}$	35

input
```
int((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh((x^2+2*x+5)^(1/2))
```

3.31.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. $2(13) = 26$.

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.27

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{1}{2}\log\left(x^{2}-\sqrt{x^{2}+2\,x+5}(x+2)+3\,x+6\right) - \frac{1}{2}\log\left(x^{2}-\sqrt{x^{2}+2\,x+5}x+x+4\right)$$

input
```
integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(
x^2 + 2*x + 5)*x + x + 4)
```

3.31.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. $2(14) = 28$.

Time = 1.96 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \frac{\log\left(\sqrt{x^{2}+2x+5}-1\right)}{2} - \frac{\log\left(\sqrt{x^{2}+2x+5}+1\right)}{2}$$

input
```
integrate((1+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

output
```
log(sqrt(x**2 + 2*x + 5) - 1)/2 - log(sqrt(x**2 + 2*x + 5) + 1)/2
```

3.31.7 Maxima [F]

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+1}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

input
```
integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 1)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(13) = 26$.

Time = 0.28 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\frac{1}{2}\log\left(\sqrt{x^2+2\,x+5}+1\right)+\frac{1}{2}\log\left(\sqrt{x^2+2\,x+5}-1\right)$$

input
```
integrate((1+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(sqrt(x^2 + 2*x + 5) + 1) + 1/2*log(sqrt(x^2 + 2*x + 5) - 1)
```

3.31.9 Mupad [B] (verification not implemented)

Time = 12.56 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\mathrm{atanh}\left(\sqrt{x^2+2\,x+5}\right)$$

input
```
int((x + 1)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
-atanh((2*x + x^2 + 5)^(1/2))
```

3.32 $\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx$

3.32.1 Optimal result

Integrand size = 26, antiderivative size = 44

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \sqrt{3}\arctan\left(\frac{1+x}{\sqrt{3}\sqrt{5+2x+x^2}}\right)-\operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

output
```
-arctanh((x^2+2*x+5)^(1/2))+arctan(1/3*(1+x)*3^(1/2)/(x^2+2*x+5)^(1/2))*3^
(1/2)
```

3.32.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = -\sqrt{3}\arctan\left(\frac{4+2x+x^2-(1+x)\sqrt{5+2x+x^2}}{\sqrt{3}}\right) - \operatorname{arctanh}\left(\sqrt{5+2x+x^2}\right)$$

input
```
Integrate[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
-(Sqrt[3]*ArcTan[(4 + 2*x + x^2 - (1 + x)*Sqrt[5 + 2*x + x^2])/Sqrt[3]]) -
 ArcTanh[Sqrt[5 + 2*x + x^2]]
```

3.32.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1358, 27, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 1358}$$

$$3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx+\frac{1}{2}\int \frac{2(x+1)}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 27}$$

$$3\int \frac{1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx+\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx$$

$$\Big\downarrow \text{ 1313}$$

$$\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx-12\int \frac{1}{-\frac{8(x+1)^2}{x^2+2x+5}-24}d\frac{2(x+1)}{\sqrt{x^2+2x+5}}$$

$$\Big\downarrow \text{ 217}$$

$$\int \frac{x+1}{(x^2+2x+4)\sqrt{x^2+2x+5}}dx+\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)$$

$$\Big\downarrow \text{ 1357}$$

$$\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-2\int \frac{1}{2-2(x^2+2x+5)}d\sqrt{x^2+2x+5}$$

$$\Big\downarrow \text{ 219}$$

$$\sqrt{3}\arctan\left(\frac{x+1}{\sqrt{3}\sqrt{x^2+2x+5}}\right)-\text{arctanh}\left(\sqrt{x^2+2x+5}\right)$$

input
```
Int[(4 + x)/((4 + 2*x + x^2)*Sqrt[5 + 2*x + x^2]),x]
```

output
```
Sqrt[3]*ArcTan[(1 + x)/(Sqrt[3]*Sqrt[5 + 2*x + x^2])] - ArcTanh[Sqrt[5 + 2
*x + x^2]]
```

3.32.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.32.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.91

method	result
default	$-\operatorname{arctanh}\left(\sqrt{x^2+2x+5}\right)+\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,(2x+2)}{6\sqrt{x^2+2x+5}}\right)$
trager	$\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)\ln\left(\frac{40\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)^2 x+21\sqrt{x^2+2x+5}\,\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)+51\operatorname{RootOf}\left(\_Z\right.}{\operatorname{RootOf}\left(\_Z^2+\_Z+1\right)x+}\right.$

input
```
int((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh((x^2+2*x+5)^(1/2))+3^(1/2)*arctan(1/6*3^(1/2)/(x^2+2*x+5)^(1/2)*(
2*x+2))
```

3.32.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. 2(37) = 74.

Time = 0.32 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

$$\begin{aligned}\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}(x+2)+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)\\ &\quad+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}x+\frac{1}{3}\sqrt{3}\sqrt{x^2+2x+5}\right)\\ &\quad+\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}(x+2)+3x+6\right)\\ &\quad-\frac{1}{2}\log\left(x^2-\sqrt{x^2+2x+5}x+x+4\right)\end{aligned}$$

input
```
integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(3)*arctan(-1/3*sqrt(3)*(x + 2) + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) +
sqrt(3)*arctan(-1/3*sqrt(3)*x + 1/3*sqrt(3)*sqrt(x^2 + 2*x + 5)) + 1/2*log
(x^2 - sqrt(x^2 + 2*x + 5)*(x + 2) + 3*x + 6) - 1/2*log(x^2 - sqrt(x^2 + 2
*x + 5)*x + x + 4)
```

3.32.6 Sympy [F]

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{(x^2+2x+4)\sqrt{x^2+2x+5}}\,dx$$

input
```
integrate((4+x)/(x**2+2*x+4)/(x**2+2*x+5)**(1/2),x)
```

output
```
Integral((x + 4)/((x**2 + 2*x + 4)*sqrt(x**2 + 2*x + 5)), x)
```

3.32.7 Maxima [F]

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{\sqrt{x^2+2\,x+5}(x^2+2\,x+4)}\,dx$$

input
```
integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((x + 4)/(sqrt(x^2 + 2*x + 5)*(x^2 + 2*x + 4)), x)
```

3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(37) = 74$.

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.45

$$\begin{aligned}\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx &= -\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}+2\right)\right)\\&\quad+\sqrt{3}\arctan\left(-\frac{1}{3}\sqrt{3}\left(x-\sqrt{x^2+2\,x+5}\right)\right)\\&\quad+\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+4\,x-4\,\sqrt{x^2+2\,x+5}\right.\\&\quad\left.+7\right)-\frac{1}{2}\log\left(\left(x-\sqrt{x^2+2\,x+5}\right)^2+3\right)\end{aligned}$$

input
```
integrate((4+x)/(x^2+2*x+4)/(x^2+2*x+5)^(1/2),x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5) + 2)) + sqrt(3)*arct
an(-1/3*sqrt(3)*(x - sqrt(x^2 + 2*x + 5))) + 1/2*log((x - sqrt(x^2 + 2*x +
 5))^2 + 4*x - 4*sqrt(x^2 + 2*x + 5) + 7) - 1/2*log((x - sqrt(x^2 + 2*x +
5))^2 + 3)
```

3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{4+x}{(4+2x+x^2)\sqrt{5+2x+x^2}}\,dx = \int \frac{x+4}{(x^2+2\,x+4)\,\sqrt{x^2+2\,x+5}}\,dx$$

input
```
int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)),x)
```

output
```
int((x + 4)/((2*x + x^2 + 4)*(2*x + x^2 + 5)^(1/2)), x)
```

3.33 $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx$

3.33.1 Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

output
```
-arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

3.33.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}} dx = -\sqrt{2}\text{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)$$

input
```
Integrate[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

3.33.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+1}{(x^2+x+3)\sqrt{x^2+x+5}}dx$$

$$\Big\downarrow \; 1357$$

$$-2\int \frac{1}{-x^2-x-3}d\sqrt{x^2+x+5}$$

$$\Big\downarrow \; 219$$

$$-\sqrt{2}\text{arctanh}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)$$

input
```
Int[(1 + 2*x)/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

output
```
-(Sqrt[2]*ArcTanh[Sqrt[5 + x + x^2]/Sqrt[2]])
```

3.33.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

3.33.4 Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

method	result	size
default	$-\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}$	20
pseudoelliptic	$-\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}$	20
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-2\right)x^2+\operatorname{RootOf}\left(\_Z^2-2\right)x+4\sqrt{x^2+x+5}+7\operatorname{RootOf}\left(\_Z^2-2\right)}{x^2+x+3}\right)}{2}$	56

input
```
int((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)
```

3.33.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.42

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \frac{1}{2}\sqrt{2}\log\left(\frac{x^2-2\sqrt{2}\sqrt{x^2+x+5}+x+7}{x^2+x+3}\right)$$

input
```
integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*log((x^2 - 2*sqrt(2)*sqrt(x^2 + x + 5) + x + 7)/(x^2 + x + 3))
```

3.33.6 Sympy [A] (verification not implemented)

Time = 1.74 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \frac{\sqrt{2}\left(\log\left(\sqrt{x^2+x+5}-\sqrt{2}\right)-\log\left(\sqrt{x^2+x+5}+\sqrt{2}\right)\right)}{2}$$

input
```
integrate((1+2*x)/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

output
```
sqrt(2)*(log(sqrt(x**2 + x + 5) - sqrt(2)) - log(sqrt(x**2 + x + 5) + sqrt
(2)))/2
```

3.33.7 Maxima [F]

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{2\,x+1}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

input
```
integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 1)/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(19) = 38$.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\frac{1}{2}\sqrt{2}\log\left(\sqrt{2}+\sqrt{x^2+x+5}\right) + \frac{1}{2}\sqrt{2}\log\left(-\sqrt{2}+\sqrt{x^2+x+5}\right)$$

input
```
integrate((1+2*x)/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*log(sqrt(2) + sqrt(x^2 + x + 5)) + 1/2*sqrt(2)*log(-sqrt(2) +
 sqrt(x^2 + x + 5))
```

3.33.9 Mupad [B] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = -\sqrt{2}\,\mathrm{atanh}\left(\frac{\sqrt{2}\,\sqrt{x^2+x+5}}{2}\right)$$

input
```
int((2*x + 1)/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

output
```
-2^(1/2)*atanh((2^(1/2)*(x + x^2 + 5)^(1/2))/2)
```

3.33. $\int \frac{1+2x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

3.34 $\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$

3.34.1 Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx=-\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(1+2x)}{\sqrt{5+x+x^2}}\right)}{\sqrt{22}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{5+x+x^2}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)
*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

3.34.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.64

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$$
$$=\text{RootSum}\Big[23-2\#1+3\#1^2-2\#1^3+\#1^4\&,\frac{-5\log\left(-x+\sqrt{5+x+x^2}-\#1\right)+\log\left(-x+\sqrt{5+x+x^2}-\#1\right)\#1^2}{-1+3\#1-3\#1^2+2\#1^3}\&\Big]$$

input
```
Integrate[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

output
```
RootSum[23 - 2*#1 + 3*#1^2 - 2*#1^3 + #1^4 & , (-5*Log[-x + Sqrt[5 + x + x
^2] - #1] + Log[-x + Sqrt[5 + x + x^2] - #1]*#1^2)/(-1 + 3*#1 - 3*#1^2 + 2
*#1^3) & ]
```

3.34.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1358, 1313, 217, 1357, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}dx$$

$$\Big\downarrow \text{1358}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+3)\sqrt{x^2+x+5}}dx-\frac{1}{2}\int \frac{1}{(x^2+x+3)\sqrt{x^2+x+5}}dx$$

$$\Big\downarrow \text{1313}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+3)\sqrt{x^2+x+5}}dx+\int \frac{1}{-\frac{2(2x+1)^2}{x^2+x+5}-11}d\frac{2x+1}{\sqrt{x^2+x+5}}$$

$$\Big\downarrow \text{217}$$

$$\frac{1}{2}\int \frac{2x+1}{(x^2+x+3)\sqrt{x^2+x+5}}dx-\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}$$

$$\Big\downarrow \text{1357}$$

$$-\int \frac{1}{-x^2-x-3}d\sqrt{x^2+x+5}-\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\arctan\left(\frac{\sqrt{\frac{2}{11}}(2x+1)}{\sqrt{x^2+x+5}}\right)}{\sqrt{22}}-\frac{\text{arctanh}\left(\frac{\sqrt{x^2+x+5}}{\sqrt{2}}\right)}{\sqrt{2}}$$

input
```
Int[x/((3 + x + x^2)*Sqrt[5 + x + x^2]),x]
```

output
```
-(ArcTan[(Sqrt[2/11]*(1 + 2*x))/Sqrt[5 + x + x^2]]/Sqrt[22]) - ArcTanh[Sqr
t[5 + x + x^2]/Sqrt[2]]/Sqrt[2]
```

3.34.3.1 Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.34.4 Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

method	result
default	$-\frac{\operatorname{arctanh}\left(\frac{\sqrt{x^2+x+5}\,\sqrt{2}}{2}\right)\sqrt{2}}{2}-\frac{\arctan\left(\frac{(1+2x)\sqrt{22}}{11\sqrt{x^2+x+5}}\right)\sqrt{22}}{22}$
trager	$-\operatorname{RootOf}\left(484\_Z^4-110\_Z^2+9\right)\ln\left(\frac{133342\operatorname{RootOf}\left(484\_Z^4-110\_Z^2+9\right)^5 x-34298\operatorname{RootOf}\left(484\_Z^4-110\_\right.}{}\right.$

input
```
int(x/(x^2+x+3)/(x^2+x+5)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*arctanh(1/2*(x^2+x+5)^(1/2)*2^(1/2))*2^(1/2)-1/22*arctan(1/11*(1+2*x)
*22^(1/2)/(x^2+x+5)^(1/2))*22^(1/2)
```

3.34.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 203, normalized size of antiderivative = 3.62

$$\begin{aligned}\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = &-\frac{1}{22}\sqrt{11}\sqrt{i\sqrt{11}+5}\log\left(\left(\sqrt{11}-i\right)\sqrt{i\sqrt{11}+5}-6x\right.\\&\left.+3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)\\&+\frac{1}{22}\sqrt{11}\sqrt{i\sqrt{11}+5}\log\left(-\left(\sqrt{11}-i\right)\sqrt{i\sqrt{11}+5}\right.\\&\left.-6x+3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)\\&-\frac{1}{22}\sqrt{11}\sqrt{-i\sqrt{11}+5}\log\left(\left(\sqrt{11}+i\right)\sqrt{-i\sqrt{11}+5}\right.\\&\left.-6x-3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)\\&+\frac{1}{22}\sqrt{11}\sqrt{-i\sqrt{11}+5}\log\left(-\left(\sqrt{11}+i\right)\sqrt{-i\sqrt{11}+5}\right.\\&\left.-6x-3i\sqrt{11}+6\sqrt{x^2+x+5}-3\right)\end{aligned}$$

input
```
integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="fricas")
```

output
```
-1/22*sqrt(11)*sqrt(I*sqrt(11) + 5)*log((sqrt(11) - I)*sqrt(I*sqrt(11) + 5
) - 6*x + 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5) - 3) + 1/22*sqrt(11)*sqrt(I*s
qrt(11) + 5)*log(-(sqrt(11) - I)*sqrt(I*sqrt(11) + 5) - 6*x + 3*I*sqrt(11)
 + 6*sqrt(x^2 + x + 5) - 3) - 1/22*sqrt(11)*sqrt(-I*sqrt(11) + 5)*log((sqr
t(11) + I)*sqrt(-I*sqrt(11) + 5) - 6*x - 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5
) - 3) + 1/22*sqrt(11)*sqrt(-I*sqrt(11) + 5)*log(-(sqrt(11) + I)*sqrt(-I*s
qrt(11) + 5) - 6*x - 3*I*sqrt(11) + 6*sqrt(x^2 + x + 5) - 3)
```

3.34.6 Sympy [F]

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

input
```
integrate(x/(x**2+x+3)/(x**2+x+5)**(1/2),x)
```

output
```
Integral(x/((x**2 + x + 3)*sqrt(x**2 + x + 5)), x)
```

3.34.7 Maxima [F]

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{\sqrt{x^2+x+5}(x^2+x+3)}\,dx$$

input
```
integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/(sqrt(x^2 + x + 5)*(x^2 + x + 3)), x)
```

3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(44) = 88.

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.38

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx$$
$$= \frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right)$$
$$-\frac{1}{22}\sqrt{11}\sqrt{2}\arctan\left(-\frac{1}{11}\sqrt{11}\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)\right)$$
$$+\frac{1}{4}\sqrt{2}\log\left(324\left(2x+2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right)$$
$$-\frac{1}{4}\sqrt{2}\log\left(324\left(2x-2\sqrt{2}-2\sqrt{x^2+x+5}+1\right)^2+3564\right)$$

input
```
integrate(x/(x^2+x+3)/(x^2+x+5)^(1/2),x, algorithm="giac")
```

output
```
1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x + 2*sqrt(2) - 2*sqrt(x^2
+ x + 5) + 1)) - 1/22*sqrt(11)*sqrt(2)*arctan(-1/11*sqrt(11)*(2*x - 2*sqrt
(2) - 2*sqrt(x^2 + x + 5) + 1)) + 1/4*sqrt(2)*log(324*(2*x + 2*sqrt(2) - 2
*sqrt(x^2 + x + 5) + 1)^2 + 3564) - 1/4*sqrt(2)*log(324*(2*x - 2*sqrt(2) -
 2*sqrt(x^2 + x + 5) + 1)^2 + 3564)
```

3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(3+x+x^2)\sqrt{5+x+x^2}}\,dx = \int \frac{x}{(x^2+x+3)\sqrt{x^2+x+5}}\,dx$$

input
```
int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)),x)
```

output
```
int(x/((x + x^2 + 3)*(x + x^2 + 5)^(1/2)), x)
```

3.35 $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

3.35.1 Optimal result

Integrand size = 36, antiderivative size = 249

$$\begin{aligned}\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx &= -\frac{((Ab-2aB)e-b(Be-2Af)x)\sqrt{d+ex+fx^2}}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)} \\ &\quad+\frac{(Be-2Af)\left(8aef-b(e^2+4df)\right)\operatorname{arctanh}\left(\frac{\sqrt{bd-ae}(e+2fx)}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{2e^{3/2}(bd-ae)^{3/2}f(be-4af)^{3/2}} \\ &\quad+\frac{B\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{2\sqrt{b}(bd-ae)^{3/2}f}\end{aligned}$$

output
```
1/2*(-2*A*f+B*e)*(8*a*e*f-b*(4*d*f+e^2))*arctanh((2*f*x+e)*(-a*e+b*d)^(1/2
)/e^(1/2)/(-4*a*f+b*e)^(1/2)/(f*x^2+e*x+d)^(1/2))/e^(3/2)/(-a*e+b*d)^(3/2)
/f/(-4*a*f+b*e)^(3/2)+1/2*B*arctanh(b^(1/2)*(f*x^2+e*x+d)^(1/2)/(-a*e+b*d)
^(1/2))/(-a*e+b*d)^(3/2)/f/b^(1/2)-((A*b-2*B*a)*e-b*(-2*A*f+B*e)*x)*(f*x^2
+e*x+d)^(1/2)/e/(-a*e+b*d)/(-4*a*f+b*e)/(b*f*x^2+b*e*x+a*e)
```

3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 3.53 (sec) , antiderivative size = 2374, normalized size of antiderivative = 9.53

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
((2*e*Sqrt[d + x*(e + f*x)]*(B*e*(2*a + b*x) - A*b*(e + 2*f*x)))/((b*d - a
*e)*(b*e - 4*a*f)*(a*e + b*x*(e + f*x))) - (2*RootSum[a*e*f^2 - 2*b*Sqrt[d
]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3 +
 a*e*#1^4 & , (-4*A*b^2*d*e*Log[x] + 4*a*b*B*d*e*Log[x] + a*A*b*e^2*Log[x]
 - a^2*B*e^2*Log[x] + 4*a*A*b*d*f*Log[x] + a^2*A*e*f*Log[x] + 4*A*b^2*d*e*
Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 4*a*b*B*d*e*Log[-Sqrt[d] +
Sqrt[d + e*x + f*x^2] - x*#1] - a*A*b*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*
x^2] - x*#1] + a^2*B*e^2*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 4*
a*A*b*d*f*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - a^2*A*e*f*Log[-Sq
rt[d] + Sqrt[d + e*x + f*x^2] - x*#1] - 2*a*A*b*Sqrt[d]*e*Log[x]*#1 + 2*a^
2*B*Sqrt[d]*e*Log[x]*#1 + 2*a*A*b*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x +
f*x^2] - x*#1]*#1 - 2*a^2*B*Sqrt[d]*e*Log[-Sqrt[d] + Sqrt[d + e*x + f*x^2]
 - x*#1]*#1 - a^2*A*e*Log[x]*#1^2 + a^2*A*e*Log[-Sqrt[d] + Sqrt[d + e*x +
f*x^2] - x*#1]*#1^2)/(-(b*Sqrt[d]*e*f) + b*e^2*#1 + 4*b*d*f*#1 - 2*a*e*f*#
1 - 3*b*Sqrt[d]*e*#1^2 + 2*a*e*#1^3) & ])/a^3 + RootSum[a*e*f^2 - 2*b*Sqrt
[d]*e*f*#1 + b*e^2*#1^2 + 4*b*d*f*#1^2 - 2*a*e*f*#1^2 - 2*b*Sqrt[d]*e*#1^3
 + a*e*#1^4 & , (-8*A*b^4*d^2*e^2*Log[x] + 8*a*b^3*B*d^2*e^2*Log[x] + 10*a
*A*b^3*d*e^3*Log[x] - 10*a^2*b^2*B*d*e^3*Log[x] - 2*a^2*A*b^2*e^4*Log[x] +
 a^3*b*B*e^4*Log[x] + 40*a*A*b^3*d^2*e*f*Log[x] - 32*a^2*b^2*B*d^2*e*f*Log
[x] - 46*a^2*A*b^2*d*e^2*f*Log[x] + 38*a^3*b*B*d*e^2*f*Log[x] + 7*a^3*A...
```

3.35.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {1349, 27, 1358, 1313, 221, 1357, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$$

$$\Big\downarrow \; 1349$$

$$-\frac{\int -\frac{b(bd-ae)f^2\left(2bBde-2a(Be-4Af)e-B(be-4af)xe-Ab\left(e^2+4df\right)\right)}{2\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{bef^2(bd-ae)^2(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{2bBde-2a(Be-4Af)e-B(be-4af)xe-Ab\left(e^2+4df\right)}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 1358$$

$$\frac{-\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2f}-\frac{Be(be-4af)\int \frac{e+2fx}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2f}}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 1313$$

$$\frac{\frac{e(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\int \frac{1}{e^2(be-4af)-\frac{e(bd-ae)(e+2fx)^2}{fx^2+ex+d}}d\frac{e+2fx}{\sqrt{fx^2+ex+d}}}{f}-\frac{Be(be-4af)\int \frac{e+2fx}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2f}}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{e}f\sqrt{bd-ae}\sqrt{be-4af}}-\frac{Be(be-4af)\int \frac{e+2fx}{\sqrt{fx^2+ex+d}\left(bfx^2+bex+ae\right)}dx}{2f}}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

$$\Big\downarrow \; 1357$$

$$\frac{\frac{Be^2(be-4af)\int \frac{1}{e(bd-ae)-be\left(fx^2+ex+d\right)}d\sqrt{fx^2+ex+d}}{f}+\frac{(Be-2Af)\left(8aef-b\left(4df+e^2\right)\right)\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{e}f\sqrt{bd-ae}\sqrt{be-4af}}}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}\left(e(Ab-2aB)-bx(Be-2Af)\right)}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

3.35. $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx$

$$\Big\downarrow \; 221$$

$$\frac{\frac{(Be-2Af)(8aef-b(4df+e^2))\text{arctanh}\left(\frac{(e+2fx)\sqrt{bd-ae}}{\sqrt{e}\sqrt{be-4af}\sqrt{d+ex+fx^2}}\right)}{\sqrt{e}f\sqrt{bd-ae}\sqrt{be-4af}}+\frac{Be(be-4af)\text{arctanh}\left(\frac{\sqrt{b}\sqrt{d+ex+fx^2}}{\sqrt{bd-ae}}\right)}{\sqrt{b}f\sqrt{bd-ae}}}{2e(bd-ae)(be-4af)}-\frac{\sqrt{d+ex+fx^2}(e(Ab-2aB)-bx(Be-2Af))}{e(bd-ae)(be-4af)\left(ae+bex+bfx^2\right)}$$

input
```
Int[(A + B*x)/(Sqrt[d + e*x + f*x^2]*(a*e + b*e*x + b*f*x^2)^2),x]
```

output
```
-((((A*b - 2*a*B)*e - b*(B*e - 2*A*f)*x)*Sqrt[d + e*x + f*x^2])/(e*(b*d -
a*e)*(b*e - 4*a*f)*(a*e + b*e*x + b*f*x^2))) + (((B*e - 2*A*f)*(8*a*e*f -
b*(e^2 + 4*d*f))*ArcTanh[(Sqrt[b*d - a*e]*(e + 2*f*x))/(Sqrt[e]*Sqrt[b*e -
 4*a*f]*Sqrt[d + e*x + f*x^2])])/(Sqrt[e]*Sqrt[b*d - a*e]*f*Sqrt[b*e - 4*a
*f]) + (B*e*(b*e - 4*a*f)*ArcTanh[(Sqrt[b]*Sqrt[d + e*x + f*x^2])/Sqrt[b*d
 - a*e]])/(Sqrt[b]*Sqrt[b*d - a*e]*f))/(2*e*(b*d - a*e)*(b*e - 4*a*f))
```

3.35.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1313
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[1/(e*(b*e - 4*a*f) - (b*d - a*e
)*x^2), x], x, (e + 2*f*x)/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f,
0]
```

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1357
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e
_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-2*g   Subst[Int[1/(b*d - a*e
- b*x^2), x], x, Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c*e - b*f, 0] &
& EqQ[h*e - 2*g*f, 0]
```

rule 1358
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(h*e - 2*g*f)/(2*f)   Int[1/
((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/(2*f)   Int[(e +
 2*f*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c
, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[c
*e - b*f, 0] && NeQ[h*e - 2*g*f, 0]
```

3.35. $\int \frac{A+Bx}{\sqrt{d+ex+fx^2}(ae+bex+bfx^2)^2}\,dx$

3.35.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1429 vs. $2(219) = 438$.

Time = 1.38 (sec) , antiderivative size = 1430, normalized size of antiderivative = 5.74

method	result	size
default	Expression too large to display	1430

input
```
int((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x,method=_RETURNVERB
OSE)
```

output
```
(2*A*f-B*e)/e/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln
((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))
^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))
/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))
/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))-(2*A
*f-B*e)/e/(4*a*f-b*e)/(-b*e*(4*a*f-b*e))^(1/2)/(-(a*e-b*d)/b)^(1/2)*ln((-2
*(a*e-b*d)/b+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1
/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b
/f)^2*f+(-b*e*(4*a*f-b*e))^(1/2)/b*(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/
b/f)-(a*e-b*d)/b)^(1/2))/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f))-1/2/
f*(2*A*b*f-B*b*e-B*(-b*e*(4*a*f-b*e))^(1/2))/e/(4*a*f-b*e)/b^2*(1/(a*e-b*d
)*b/(x+1/2*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)*((x+1/2*(b*e+(-b*e*(4*a*f-b
*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(-b*e*(4*a*f-b
*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2)+1/2*(-b*e*(4*a*f-b*e))^(1/2)/(a*e-b*d)
/(-(a*e-b*d)/b)^(1/2)*ln((-2*(a*e-b*d)/b-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2
*(b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f)+2*(-(a*e-b*d)/b)^(1/2)*((x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/b/f)^2*f-(-b*e*(4*a*f-b*e))^(1/2)/b*(x+1/2*(b*e+(
-b*e*(4*a*f-b*e))^(1/2))/b/f)-(a*e-b*d)/b)^(1/2))/(x+1/2*(b*e+(-b*e*(4*a*f
-b*e))^(1/2))/b/f)))-1/2/f*(2*A*b*f-B*b*e+B*(-b*e*(4*a*f-b*e))^(1/2))/e/(4
*a*f-b*e)/b^2*(1/(a*e-b*d)*b/(x-1/2*(-b*e+(-b*e*(4*a*f-b*e))^(1/2))/b/f...
```

3.35.5 Fricas [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\text{Timed out}$$

input
```
integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="
fricas")
```

output
```
Timed out
```

3.35.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/(b*f*x**2+b*e*x+a*e)**2/(f*x**2+e*x+d)**(1/2),x)
```

output
```
Timed out
```

3.35.7 Maxima [F]

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = \int \frac{Bx+A}{\left(bfx^2+bex+ae\right)^2\sqrt{fx^2+ex+d}}\,dx$$

input
```
integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="
maxima")
```

output
```
integrate((B*x + A)/((b*f*x^2 + b*e*x + a*e)^2*sqrt(f*x^2 + e*x + d)), x)
```

3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5022 vs. 2(218) = 436.

Time = 0.53 (sec) , antiderivative size = 5022, normalized size of antiderivative = 20.17

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\left(ae+bex+bfx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/(b*f*x^2+b*e*x+a*e)^2/(f*x^2+e*x+d)^(1/2),x, algorithm="
giac")
```

output
```
-1/2*((B*b*e^2*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt
(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 -
 4*B*a*e*f*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2
*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^2 + 8*B
*b*d*e*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt
(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) - 8
*B*a*e^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sq
rt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))) -
 4*A*b*e^2*sqrt(f)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*
sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))
 - 16*A*b*d*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4
*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2))
) + 32*A*a*e*f^(3/2)*(e/sqrt(f) - sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 +
4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)
)) - 12*B*b*d*e^2 + 8*B*a*e^3 + 4*A*b*e^3 + 16*B*a*d*e*f + 16*A*b*d*e*f -
32*A*a*e^2*f)*log(-sqrt(f)*x + sqrt(f*x^2 + e*x + d) - 1/2*e/sqrt(f) + 1/2
*sqrt((b*e^2*f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f -
4*a*b*d*e*f^2 + 4*a^2*e^2*f^2)*f)/(b*f^2)))/(b*f*(e/sqrt(f) - sqrt((b*e^2*
f + 4*b*d*f^2 - 8*a*e*f^2 + 4*sqrt(b^2*d*e^2*f - a*b*e^3*f - 4*a*b*d*e*f^2
 + 4*a^2*e^2*f^2)*f)/(b*f^2)))^3 - 3*b*e*sqrt(f)*(e/sqrt(f) - sqrt((b*e...
```

3.35.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{d+ex+fx^2}\,(ae+bex+bfx^2)^2}\,dx=\int \frac{A+B\,x}{{\left(b\,f\,x^2+b\,e\,x+a\,e\right)}^2\,\sqrt{f\,x^2+e\,x+d}}\,dx$$

input
```
int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/((a*e + b*e*x + b*f*x^2)^2*(d + e*x + f*x^2)^(1/2)), x)
```

3.36 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx$

3.36.1 Optimal result

Integrand size = 36, antiderivative size = 48

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = -\frac{2(bg-2ah+(2cg-bh)x)}{(b^2-4ac)\,d^2\sqrt{a+bx+cx^2}}$$

output
```
-2*(b*g-2*a*h+(-b*h+2*c*g)*x)/(-4*a*c+b^2)/d^2/(c*x^2+b*x+a)^(1/2)
```

3.36.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{-2bg+4ah-4cgx+2bhx}{(b^2-4ac)\,d^2\sqrt{a+x(b+cx)}}$$

input
```
Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

output
```
(-2*b*g + 4*a*h - 4*c*g*x + 2*b*h*x)/((b^2 - 4*a*c)*d^2*Sqrt[a + x*(b + c*
x)])
```

3.36.3 Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1329, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}dx$$

$$\Big\downarrow \ 1329$$

$$\frac{\int \frac{g+hx}{(cx^2+bx+a)^{3/2}}dx}{d^2}$$

$$\Big\downarrow \ 1158$$

$$-\frac{2(-2ah+x(2cg-bh)+bg)}{d^2(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^2,x]
```

output
```
(-2*(b*g - 2*a*h + (2*c*g - b*h)*x))/((b^2 - 4*a*c)*d^2*Sqrt[a + b*x + c*x
^2])
```

3.36.3.1 Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1329
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(c/f)^p   Int[(g + h
*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f, g, h, p
, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] && (IntegerQ[p] || GtQ[c
/f, 0]) && ( !IntegerQ[q] || LeafCount[d + e*x + f*x^2] <= LeafCount[a + b*
x + c*x^2])
```

3.36.4 Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

method	result	size
gosper	$-\frac{2(bhx-2cgx+2ah-bg)}{\sqrt{c\,x^{2}+bx+a}\,d^{2}(4ac-b^{2})}$	48
trager	$-\frac{2(bhx-2cgx+2ah-bg)}{\sqrt{c\,x^{2}+bx+a}\,d^{2}(4ac-b^{2})}$	48
default	$\frac{\frac{2g(2cx+b)}{\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}+h\left(-\frac{1}{c\sqrt{c\,x^{2}+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^{2}\right)\sqrt{c\,x^{2}+bx+a}}\right)}{d^{2}}$	95

input
```
int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x,method=_RETURNVERB
OSE)
```

output
```
-2/(c*x^2+b*x+a)^(1/2)*(b*h*x-2*c*g*x+2*a*h-b*g)/d^2/(4*a*c-b^2)
```

3.36.5 Fricas [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.77

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = -\frac{2\,\sqrt{cx^2+bx+a}(bg-2\,ah+(2\,cg-bh)x)}{(b^2c-4\,ac^2)d^2x^2+(b^3-4\,abc)d^2x+(ab^2-4\,a^2c)d^2}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="
fricas")
```

output
```
-2*sqrt(c*x^2 + b*x + a)*(b*g - 2*a*h + (2*c*g - b*h)*x)/((b^2*c - 4*a*c^2
)*d^2*x^2 + (b^3 - 4*a*b*c)*d^2*x + (a*b^2 - 4*a^2*c)*d^2)
```

3.36.6 Sympy [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{\int \frac{g}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx+\int \frac{hx}{a\sqrt{a+bx+cx^2}+bx\sqrt{a+bx+cx^2}+cx^2\sqrt{a+bx+cx^2}}\,dx}{d^2}$$

input
```
integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**2,x)
```

output
```
(Integral(g/(a*sqrt(a + b*x + c*x**2) + b*x*sqrt(a + b*x + c*x**2) + c*x**
2*sqrt(a + b*x + c*x**2)), x) + Integral(h*x/(a*sqrt(a + b*x + c*x**2) + b
*x*sqrt(a + b*x + c*x**2) + c*x**2*sqrt(a + b*x + c*x**2)), x))/d**2
```

3.36.7 Maxima [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx=\int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^2}\,dx$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="
maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^2, x)
```

3.36.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.69

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx=-\frac{2\left(\frac{(2\,cd^2g-bd^2h)x}{b^2d^4-4\,acd^4}+\frac{bd^2g-2\,ad^2h}{b^2d^4-4\,acd^4}\right)}{\sqrt{cx^2+bx+a}}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^2,x, algorithm="
giac")
```

output
```
-2*((2*c*d^2*g - b*d^2*h)*x/(b^2*d^4 - 4*a*c*d^4) + (b*d^2*g - 2*a*d^2*h)/
(b^2*d^4 - 4*a*c*d^4))/sqrt(c*x^2 + b*x + a)
```

3.36.9 Mupad [B] (verification not implemented)

Time = 12.42 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.02

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^2}\,dx = \frac{4\,a\,h-2\,b\,g+2\,b\,h\,x-4\,c\,g\,x}{(b^2\,d^2-4\,a\,c\,d^2)\,\sqrt{c\,x^2+b\,x+a}}$$

input
```
int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^2,x)
```

output
```
(4*a*h - 2*b*g + 2*b*h*x - 4*c*g*x)/((b^2*d^2 - 4*a*c*d^2)*(a + b*x + c*x^
2)^(1/2))
```

3.37 $\int \frac{3+2x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.37.1 Optimal result

Integrand size = 32, antiderivative size = 17

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
arctanh(x/(-x^2-4*x-3)^(1/2))
```

3.37.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

3.37.3 Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.062, Rules used = {1360, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1360}$$

$$3\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

input
```
Int[(3 + 2*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

3.37.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

3.37.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.06

method	result	size
pseudoelliptic	$\operatorname{arctanh}\left(\frac{\sqrt{-x^{2}-4x-3}}{x}\right)$	18
trager	$\frac{\ln\left(\frac{2x\sqrt{-x^{2}-4x-3}-4x-3}{2x^{2}+4x+3}\right)}{2}$	37
default	$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^{2}}{\left(-\frac{3}{2}-x\right)^{2}}-12}\,\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^{2}}{\left(-\frac{3}{2}-x\right)^{2}}-12}}\right)}{6\sqrt{\frac{\frac{x^{2}}{\left(-\frac{3}{2}-x\right)^{2}}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^{2}}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$	94

input
```
int((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arctanh((-x^2-4*x-3)^(1/2)/x)
```

3.37.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(15) = 30.

Time = 0.31 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.29

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

input
```
integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*log((2*sqrt(-x^2
 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.37.6 Sympy [F]

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2x+3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate((3+2*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral((2*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.37.7 Maxima [F]

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2\,x+3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((2*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.37.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(15) = 30$.

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 5.76

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) - \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

input
```
integrate((3+2*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1
)^2/(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(
-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.37.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3+2x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{2\,x+3}{\sqrt{-x^2-4\,x-3}\,(2\,x^2+4\,x+3)}\,dx$$

input
```
int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int((2*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.38 $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} \, dx$

3.38.1 Optimal result

Integrand size = 32, antiderivative size = 86

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} \, dx = \sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
arctanh(x/(-x^2-4*x-3)^(1/2))+arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1/2))*2^(
1/2))*2^(1/2)-arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)
```

3.38.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.59

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} \, dx = -\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + \operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-(Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]) + ArcTanh[x/Sq
rt[-3 - 4*x - x^2]]
```

3.38.3 Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.344, Rules used = {1361, 27, 1317, 27, 1359, 27, 1360, 219, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \ 1361$$

$$-3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \ 27$$

$$2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \ 1317$$

$$2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-3\left(\frac{1}{6}\int -\frac{4x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{6}\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)$$

$$\Big\downarrow \ 27$$

$$2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{2}{3}\int \frac{x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)$$

$$\Big\downarrow \ 1359$$

$$2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{16}{3}\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)$$

$$\Big\downarrow \ 27$$

$$2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-$$
$$3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)$$

$$\Big\downarrow \text{ 1360}$$

$$6\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}-$$
$$3\left(\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$3\left(\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)$$

$$\Big\downarrow \text{ 1475}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{}{\frac{(x+3)^2}{9(-x^2-4x-}}\right.\right.$$

$$\Big\downarrow \text{ 1083}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-}\right.\right.$$

$$\Big\downarrow \text{ 217}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$3\left(\frac{2}{3}\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)$$

input
```
Int[(3 + 4*x)/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

3.38. $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

output
```
-3*((2*Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[-3 - 4*x - x^2])])/3 + ArcTa
nh[x/Sqrt[-3 - 4*x - x^2]]/3) + 2*ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

3.38. $\int \frac{3+4x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

rule 1361
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(2*h*d - g*e)/e   Int[1/((a +
b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/e   Int[(2*d + e*x)/((
a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e,
0] && NeQ[2*h*d - g*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.38.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.43

method	result
default	$\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{6\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$-\ln\left(\frac{4\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)^2x+4\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)x+12\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)-6\sqrt{-x^2-4x-3}-3x-6}{2\operatorname{RootOf}\left(4\_Z^2-4\_Z+3\right)x-3x-3}\right)$

input
```
int((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2
/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^
(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

3.38.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.53

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{2}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

input
```
integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) + 1/2*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3)
)/(2*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/4*lo
g((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.38.6 Sympy [F]

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{4x+3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate((3+4*x)/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral((4*x + 3)/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.38.7 Maxima [F]

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx = \int \frac{4\,x+3}{\left(2\,x^2+4\,x+3\right)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate((4*x + 3)/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.38.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(73) = 146.

Time = 0.30 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.90

$$\begin{aligned}\int \frac{3+4x}{\sqrt{-3-4x-x^2}\left(3+4x+2x^2\right)}\,dx &= \sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right)\\ &\quad+\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)\\ &\quad+\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}\right.\\ &\quad\left.+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right)\\ &\quad-\frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}\right.\\ &\quad\left.+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)\end{aligned}$$

input
```
integrate((3+4*x)/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + s
qrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/2*
log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/
(x + 2)^2 + 1) - 1/2*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2
 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.38.9 Mupad [F(-1)]

Timed out.

$$\int \frac{3+4x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{4\,x+3}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

input
```
int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int((4*x + 3)/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.39 $\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx$

3.39.1 Optimal result

Integrand size = 38, antiderivative size = 136

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx = -\frac{(2cg-bh)\sqrt{a+bx+cx^2}\,\text{arctanh}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right)}{c\sqrt{b^2-4ac}\,d\sqrt{ad+bdx+cdx^2}} + \frac{h\sqrt{a+bx+cx^2}\log\left(a+bx+cx^2\right)}{2cd\sqrt{ad+bdx+cdx^2}}$$

output
```
1/2*h*ln(c*x^2+b*x+a)*(c*x^2+b*x+a)^(1/2)/c/d/(c*d*x^2+b*d*x+a*d)^(1/2)-(-
b*h+2*c*g)*arctanh((2*c*x+b)/(-4*a*c+b^2)^(1/2))*(c*x^2+b*x+a)^(1/2)/c/d/(
-4*a*c+b^2)^(1/2)/(c*d*x^2+b*d*x+a*d)^(1/2)
```

3.39.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.79

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{\left(ad+bdx+cdx^2\right)^{3/2}}\,dx = \frac{(a+x(b+cx))^{3/2}\left((4cg-2bh)\arctan\left(\frac{b+2cx}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,h\log(a+\right.}{2c\sqrt{-b^2+4ac}(d(a+x(b+cx)))^{3/2}}$$

input
```
Integrate[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),
x]
```

output
```
((a + x*(b + c*x))^(3/2)*((4*c*g - 2*b*h)*ArcTan[(b + 2*c*x)/Sqrt[-b^2 + 4
*a*c]] + Sqrt[-b^2 + 4*a*c]*h*Log[a + x*(b + c*x)]))/(2*c*Sqrt[-b^2 + 4*a*
c]*(d*(a + x*(b + c*x)))^(3/2))
```

3.39.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.82, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1330, 1142, 27, 1083, 219, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1330}$$

$$\frac{\sqrt{a+bx+cx^2}\int \frac{g+hx}{cdx^2+bdx+ad}dx}{\sqrt{ad+bdx+cdx^2}}$$

$$\Big\downarrow \text{1142}$$

$$\frac{\sqrt{a+bx+cx^2}\left(\frac{(2cg-bh)\int \frac{1}{cdx^2+bdx+ad}dx}{2c}+\frac{h\int \frac{d(b+2cx)}{cdx^2+bdx+ad}dx}{2cd}\right)}{\sqrt{ad+bdx+cdx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx+cx^2}\left(\frac{(2cg-bh)\int \frac{1}{cdx^2+bdx+ad}dx}{2c}+\frac{h\int \frac{b+2cx}{cdx^2+bdx+ad}dx}{2c}\right)}{\sqrt{ad+bdx+cdx^2}}$$

$$\Big\downarrow \text{1083}$$

$$\frac{\sqrt{a+bx+cx^2}\left(\frac{h\int \frac{b+2cx}{cdx^2+bdx+ad}dx}{2c}-\frac{(2cg-bh)\int \frac{1}{(b^2-4ac)d^2-(bd+2cxd)^2}d(bd+2cxd)}{c}\right)}{\sqrt{ad+bdx+cdx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+bx+cx^2}\left(\frac{h\int \frac{b+2cx}{cdx^2+bdx+ad}dx}{2c}-\frac{(2cg-bh)\text{arctanh}\left(\frac{bd+2cdx}{d\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}}\right)}{\sqrt{ad+bdx+cdx^2}}$$

$$\Big\downarrow \text{1103}$$

$$\frac{\sqrt{a+bx+cx^2}\left(\frac{h\log(a+bx+cx^2)}{2cd}-\frac{(2cg-bh)\text{arctanh}\left(\frac{bd+2cdx}{d\sqrt{b^2-4ac}}\right)}{cd\sqrt{b^2-4ac}}\right)}{\sqrt{ad+bdx+cdx^2}}$$

input
```
Int[((g + h*x)*Sqrt[a + b*x + c*x^2])/(a*d + b*d*x + c*d*x^2)^(3/2),x]
```

output
```
(Sqrt[a + b*x + c*x^2]*(-(((2*c*g - b*h)*ArcTanh[(b*d + 2*c*d*x)/(Sqrt[b^2
 - 4*a*c]*d)])/(c*Sqrt[b^2 - 4*a*c]*d)) + (h*Log[a + b*x + c*x^2])/(2*c*d)
))/Sqrt[a*d + b*d*x + c*d*x^2]
```

3.39.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1330
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[a^IntPart[p]*((a + b
*x + c*x^2)^FracPart[p]/(d^IntPart[p]*(d + e*x + f*x^2)^FracPart[p]))   Int
[(g + h*x)^m*(d + e*x + f*x^2)^(p + q), x], x] /; FreeQ[{a, b, c, d, e, f,
g, h, p, q}, x] && EqQ[c*d - a*f, 0] && EqQ[b*d - a*e, 0] &&  !IntegerQ[p]
&&  !IntegerQ[q] &&  !GtQ[c/f, 0]
```

3.39.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{d(c x^2+bx+a)}\left(h\ln\left(c x^2+bx+a\right)\sqrt{4ac-b^2}-2\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)bh+4\arctan\left(\frac{2cx+b}{\sqrt{4ac-b^2}}\right)cg\right)}{2\sqrt{c x^2+bx+a}\, d^2 c\sqrt{4ac-b^2}}$
risch	$\frac{\sqrt{c x^2+bx+a}\left(4ach-b^2h+\sqrt{-(bh-2cg)^2(4ac-b^2)}\right)\ln\left(-4abch+8a c^2 g+b^3h-2b^2cg-2\sqrt{-(bh-2cg)^2(4ac-b^2)}\, cx-\sqrt{-(bh-2cg)^2}\right.}{2d\sqrt{d(c x^2+bx+a)}\, c(4ac-b^2)}$

input
```
int((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x,method=_RETURN
VERBOSE)
```

output
```
1/2/(c*x^2+b*x+a)^(1/2)*(d*(c*x^2+b*x+a))^(1/2)*(h*ln(c*x^2+b*x+a)*(4*a*c-
b^2)^(1/2)-2*arctan((2*c*x+b)/(4*a*c-b^2)^(1/2))*b*h+4*arctan((2*c*x+b)/(4
*a*c-b^2)^(1/2))*c*g)/d^2/c/(4*a*c-b^2)^(1/2)
```

3.39.5 Fricas [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorit
hm="fricas")
```

output
```
integral(sqrt(c*d*x^2 + b*d*x + a*d)*sqrt(c*x^2 + b*x + a)*(h*x + g)/(c^2*
d^2*x^4 + 2*b*c*d^2*x^3 + 2*a*b*d^2*x + (b^2 + 2*a*c)*d^2*x^2 + a^2*d^2),
x)
```

3.39.6 Sympy [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(d(a+bx+cx^2))^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)*(c*x**2+b*x+a)**(1/2)/(c*d*x**2+b*d*x+a*d)**(3/2),x)
```

output
```
Integral((g + h*x)*sqrt(a + b*x + c*x**2)/(d*(a + b*x + c*x**2))**(3/2), x
)
```

3.39.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorit
hm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.39.8 Giac [F]

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx+a}(hx+g)}{(cdx^2+bdx+ad)^{\frac{3}{2}}}\,dx$$

input
```
integrate((h*x+g)*(c*x^2+b*x+a)^(1/2)/(c*d*x^2+b*d*x+a*d)^(3/2),x, algorit
hm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)*(h*x + g)/(c*d*x^2 + b*d*x + a*d)^(3/2), x
)
```

3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(g+hx)\sqrt{a+bx+cx^2}}{(ad+bdx+cdx^2)^{3/2}}\,dx = \int \frac{(g+h\,x)\,\sqrt{c\,x^2+b\,x+a}}{(c\,d\,x^2+b\,d\,x+a\,d)^{3/2}}\,dx$$

input
```
int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2),x)
```

output
```
int(((g + h*x)*(a + b*x + c*x^2)^(1/2))/(a*d + b*d*x + c*d*x^2)^(3/2), x)
```

3.40 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

3.40.1 Optimal result

Integrand size = 35, antiderivative size = 212

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = -\frac{acx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)} + \frac{bx^2\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{5d(a+bx)} - \frac{(8bc-15adx)\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{60d^2(a+bx)} - \frac{ac^2\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}$$

output
```
1/5*b*x^2*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/60*(-15*a*d*x+8*b*
c)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+a)-1/8*a*c^2*arctanh(x*d^(1/
2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*a*c*x*((b*x+a)^2
)^(1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

3.40.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$=\frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}(15adx(c+2dx^2)+8b(-2c^2+cdx^2+3d^2x^4))+15ac^2\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right.\right.}{120d^2(a+bx)}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(15*a*d*x*(c + 2*d*x^2) + 8*b*(-2*c^2
+ c*d*x^2 + 3*d^2*x^4)) + 15*a*c^2*Sqrt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x
^2]]))/(120*d^2*(a + b*x))
```

3.40.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.71, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1334, 27, 533, 533, 25, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$\Big\downarrow \text{1334}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2bx^2(a+bx)\sqrt{dx^2+c}dx}{2b(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)\sqrt{dx^2+c}dx}{a+bx}$$
$$\Big\downarrow \text{533}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2(c+dx^2)^{3/2}}{5d}-\frac{\int x(2bc-5adx)\sqrt{dx^2+c}dx}{5d}\right)}{a+bx}$$
$$\Big\downarrow \text{533}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{-\frac{\int -cd(5a+8bx)\sqrt{dx^2+c}dx}{4d}-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{\int cd(5a+8bx)\sqrt{dx^2+c}dx}{4d}-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{4}c\int (5a+8bx)\sqrt{dx^2+c}dx-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 455$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{4}c\left(5a\int \sqrt{dx^2+c}dx+\frac{8b\left(c+dx^2\right)^{3/2}}{3d}\right)-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 211$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{4}c\left(5a\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{8b\left(c+dx^2\right)^{3/2}}{3d}\right)-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{4}c\left(5a\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{8b\left(c+dx^2\right)^{3/2}}{3d}\right)-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2\right)^{3/2}}{5d}-\frac{\frac{1}{4}c\left(5a\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{8b\left(c+dx^2\right)^{3/2}}{3d}\right)-\frac{5}{4}ax\left(c+dx^2\right)^{3/2}}{5d}\right)}{a+bx}$$

3.40. $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*x^2*(c + d*x^2)^(3/2))/(5*d) - ((-5*a*x
*(c + d*x^2)^(3/2))/4 + (c*((8*b*(c + d*x^2)^(3/2))/(3*d) + 5*a*((x*Sqrt[c
 + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]))))/4)/
(5*d)))/(a + b*x)
```

3.40.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1334
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*(b + 2*c*x)^(
2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && E
qQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.40.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.45 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.49

method	result	s
default	$\frac{\mathrm{csgn}(bx+a)\left(24(d\,x^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}b\,x^2+30(d\,x^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}ax-16(d\,x^2+c)^{\frac{3}{2}}\sqrt{d}\,bc-15\sqrt{d\,x^2+c}\,d^{\frac{3}{2}}acx-15\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)a\,c^2d\right)}{120d^{\frac{5}{2}}}$	1
risch	$\frac{(24b\,x^4d^2+30a\,x^3d^2+8bc\,x^2d+15acxd-16b\,c^2)\sqrt{d\,x^2+c}\,\sqrt{(bx+a)^2}}{120d^2(bx+a)}-\frac{c^2a\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)\sqrt{(bx+a)^2}}{8d^{\frac{3}{2}}(bx+a)}$	1

input
```
int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/120*csgn(b*x+a)*(24*(d*x^2+c)^(3/2)*d^(3/2)*b*x^2+30*(d*x^2+c)^(3/2)*d^(
3/2)*a*x-16*(d*x^2+c)^(3/2)*d^(1/2)*b*c-15*(d*x^2+c)^(1/2)*d^(3/2)*a*c*x-1
5*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*d)/d^(5/2)
```

3.40.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.83

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$=\left[\frac{15\,ac^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(24\,bd^2x^4+30\,ad^2x^3+8\,bcdx^2+15\,acdx-16\,bc^2)\sqrt{d}}{240\,d^2}\right.$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/240*(15*a*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
 2*(24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)*sqr
t(d*x^2 + c))/d^2, 1/120*(15*a*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 +
 c)) + (24*b*d^2*x^4 + 30*a*d^2*x^3 + 8*b*c*d*x^2 + 15*a*c*d*x - 16*b*c^2)
*sqrt(d*x^2 + c))/d^2]
```

3.40.6 Sympy [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int x^2\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.40.7 Maxima [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int \sqrt{dx^2+c}\sqrt{(bx+a)^2}x^2\,dx$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x^2, x)
```

3.40.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.55

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\frac{ac^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}(bx+a)}{8\,d^{\frac{3}{2}}}$$
$$+\frac{1}{120}\sqrt{dx^2+c}\left(\left(2\left(3\left(4\,bx\mathrm{sgn}(bx+a)+5\,a\mathrm{sgn}(bx+a)\right)x+\frac{4\,bc\mathrm{sgn}(bx+a)}{d}\right)x+\frac{15\,ac\mathrm{sgn}(bx+a)}{d}\right)x$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*a*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/
120*sqrt(d*x^2 + c)*((2*(3*(4*b*x*sgn(b*x + a) + 5*a*sgn(b*x + a))*x + 4*b
*c*sgn(b*x + a)/d)*x + 15*a*c*sgn(b*x + a)/d)*x - 16*b*c^2*sgn(b*x + a)/d^
2)
```

3.40.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int x^2\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

output
```
int(x^2*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.41 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

3.41.1 Optimal result

Integrand size = 33, antiderivative size = 161

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=-\frac{bcx\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{8d(a+bx)}+\frac{(4a+3bx)\sqrt{a^2+2abx+b^2x^2}(c+dx^2)^{3/2}}{12d(a+bx)}-\frac{bc^2\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}(a+bx)}$$

output
```
1/12*(3*b*x+4*a)*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/8*b*c^2*arc
tanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-1/8*b*c*
x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/d/(b*x+a)
```

3.41.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.59

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$=\frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{c+dx^2}(8a(c+dx^2)+3bx(c+2dx^2))+3bc^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{24d^{3/2}(a+bx)}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(Sqrt[d]*Sqrt[c + d*x^2]*(8*a*(c + d*x^2) + 3*b*x*(c +
2*d*x^2)) + 3*b*c^2*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(24*d^(3/2)*(a +
 b*x))
```

3.41.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.76, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {1334, 27, 533, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{1334}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2bx(a+bx)\sqrt{dx^2+c}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x(a+bx)\sqrt{dx^2+c}dx}{a+bx}$$

$$\Big\downarrow \text{533}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx(c+dx^2)^{3/2}}{4d}-\frac{\int(bc-4adx)\sqrt{dx^2+c}dx}{4d}\right)}{a+bx}$$

$$\Big\downarrow \text{455}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx(c+dx^2)^{3/2}}{4d}-\frac{bc\int\sqrt{dx^2+c}dx-\frac{4}{3}a(c+dx^2)^{3/2}}{4d}\right)}{a+bx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx(c+dx^2)^{3/2}}{4d}-\frac{bc\left(\frac{1}{2}c\int\frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{4}{3}a(c+dx^2)^{3/2}}{4d}\right)}{a+bx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx(c+dx^2)^{3/2}}{4d}-\frac{bc\left(\frac{1}{2}c\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{4}{3}a(c+dx^2)^{3/2}}{4d}\right)}{a+bx}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx(c+dx^2)^{3/2}}{4d}-\frac{bc\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)-\frac{4}{3}a(c+dx^2)^{3/2}}{4d}\right)}{a+bx}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*x*(c + d*x^2)^(3/2))/(4*d) - ((-4*a*(c
+ d*x^2)^(3/2))/3 + b*c*((x*Sqrt[c + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sq
rt[c + d*x^2]])/(2*Sqrt[d])))/(4*d)))/(a + b*x)
```

3.41.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 533
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
 Simp[d*x^m*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 2))), x] - Simp[1/(b*(m + 2*
p + 2))   Int[x^(m - 1)*(a + b*x^2)^p*Simp[a*d*m - b*c*(m + 2*p + 2)*x, x],
 x], x] /; FreeQ[{a, b, c, d, p}, x] && IGtQ[m, 0] && GtQ[p, -1] && Integer
Q[2*p]
```

rule 1334
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*(b + 2*c*x)^(
2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && E
qQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.41.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(6\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,bx+8a\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}-3\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bcx-3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)b\,c^{2}\right)}{24d^{\frac{3}{2}}}$	83
risch	$\frac{\left(6bd\,x^{3}+8ad\,x^{2}+3bcx+8ac\right)\sqrt{d\,x^{2}+c}\,\sqrt{\left(bx+a\right)^{2}}}{24d\left(bx+a\right)}-\frac{c^{2}b\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)\sqrt{\left(bx+a\right)^{2}}}{8d^{\frac{3}{2}}\left(bx+a\right)}$	97

input
```
int(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*csgn(b*x+a)*(6*(d*x^2+c)^(3/2)*d^(1/2)*b*x+8*a*(d*x^2+c)^(3/2)*d^(1/2
)-3*(d*x^2+c)^(1/2)*d^(1/2)*b*c*x-3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/d
^(3/2)
```

3.41.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$
$$= \left[\frac{3\,bc^2\sqrt{d}\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(6\,bd^2x^3+8\,ad^2x^2+3\,bcdx+8\,acd)\sqrt{dx^2+c}}{48\,d^2}, \frac{3\,bc^2\sqrt{-\ldots}}{\ldots}\right.$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*b*c^2*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2
*(6*b*d^2*x^3 + 8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2, 1
/24*(3*b*c^2*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (6*b*d^2*x^3 +
8*a*d^2*x^2 + 3*b*c*d*x + 8*a*c*d)*sqrt(d*x^2 + c))/d^2]
```

3.41.6 Sympy [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int x\sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.41.7 Maxima [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int \sqrt{dx^2+c}\sqrt{(bx+a)^2}x\,dx$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)*x, x)
```

3.41.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.61

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{bc^2\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{8\,d^{\frac{3}{2}}} + \frac{1}{24}\sqrt{dx^2+c}\left(\left(2\,(3\,bx\operatorname{sgn}(bx+a)+4\,a\operatorname{sgn}(bx+a))x+\frac{3\,bc\operatorname{sgn}(bx+a)}{d}\right)x+\frac{8\,ac\operatorname{sgn}(bx+a)}{d}\right)$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
1/8*b*c^2*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/d^(3/2) + 1/
24*sqrt(d*x^2 + c)*((2*(3*b*x*sgn(b*x + a) + 4*a*sgn(b*x + a))*x + 3*b*c*s
gn(b*x + a)/d)*x + 8*a*c*sgn(b*x + a)/d)
```

3.41.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \int x\,\sqrt{{(a+b\,x)}^2}\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

output
```
int(x*((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.42 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$

3.42.1 Optimal result

Integrand size = 32, antiderivative size = 148

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{ax\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{ac\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)}$$

output
```
1/3*b*(d*x^2+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/2*a*c*arctanh(x*d^(1/2
)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1/2*a*x*((b*x+a)^2)^(
1/2)*(d*x^2+c)^(1/2)/(b*x+a)
```

3.42.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.57

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \frac{\sqrt{(a+bx)^2}\left(\sqrt{c+dx^2}(3adx+2b(c+dx^2))-3ac\sqrt{d}\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{6d(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(Sqrt[c + d*x^2]*(3*a*d*x + 2*b*(c + d*x^2)) - 3*a*c*Sq
rt[d]*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(6*d*(a + b*x))
```

3.42.3 Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {1298, 27, 455, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx$$

$$\Big\downarrow \text{1298}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2b(a+bx)\sqrt{dx^2+c}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int (a+bx)\sqrt{dx^2+c}dx}{a+bx}$$

$$\Big\downarrow \text{455}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a\int \sqrt{dx^2+c}dx+\frac{b\left(c+dx^2\right)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \text{211}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a\left(\frac{1}{2}c\int \frac{1}{\sqrt{dx^2+c}}dx+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{b\left(c+dx^2\right)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a\left(\frac{1}{2}c\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{b\left(c+dx^2\right)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a\left(\frac{c\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}}+\frac{1}{2}x\sqrt{c+dx^2}\right)+\frac{b(c+dx^2)^{3/2}}{3d}\right)}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*(c + d*x^2)^(3/2))/(3*d) + a*((x*Sqrt[c
 + d*x^2])/2 + (c*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(2*Sqrt[d]))))/(a
+ b*x)
```

3.42.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 455
```
Int[((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[d*((
a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] + Simp[c   Int[(a + b*x^2)^p, x], x]
/; FreeQ[{a, b, c, d, p}, x] &&  !LeQ[p, -1]
```

rule 1298
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_.), x_
Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)
^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a
, b, c, d, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.42.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.44

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(2\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,b+3a\sqrt{d\,x^{2}+c}\,d^{\frac{3}{2}}x+3\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)acd\right)}{6d^{\frac{3}{2}}}$	65
risch	$\frac{\left(2bd\,x^{2}+3adx+2bc\right)\sqrt{d\,x^{2}+c}\,\sqrt{\left(bx+a\right)^{2}}}{6d\left(bx+a\right)}+\frac{ac\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)\sqrt{\left(bx+a\right)^{2}}}{2\sqrt{d}\,\left(bx+a\right)}$	88

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/6*csgn(b*x+a)*(2*(d*x^2+c)^(3/2)*d^(1/2)*b+3*a*(d*x^2+c)^(1/2)*d^(3/2)*x
+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c*d)/d^(3/2)
```

3.42.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.86

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx = \left[\frac{3\,ac\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,(2\,bdx^2+3\,adx+2\,bc)\sqrt{dx^2+c}}{12\,d}, -\frac{3\,ac\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-(2\,bdx^2+3\,adx+2\,bc)\sqrt{dx^2+c}}{6\,d}\right]$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*a*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*(
2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2 + c))/d, -1/6*(3*a*c*sqrt(-d)*arct
an(sqrt(-d)*x/sqrt(d*x^2 + c)) - (2*b*d*x^2 + 3*a*d*x + 2*b*c)*sqrt(d*x^2
+ c))/d]
```

3.42.6 Sympy [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int \sqrt{c+dx^2}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2), x)
```

3.42.7 Maxima [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int \sqrt{dx^2+c}\sqrt{(bx+a)^2}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2), x)
```

3.42.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.53

$$\begin{aligned}
&\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx \\
&=-\frac{ac\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)}{2\sqrt{d}} \\
&\quad+\frac{1}{6}\sqrt{dx^2+c}\left((2\,bx\operatorname{sgn}(bx+a)+3\,a\operatorname{sgn}(bx+a))x+\frac{2\,bc\operatorname{sgn}(bx+a)}{d}\right)
\end{aligned}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*a*c*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a)/sqrt(d) + 1/6
*sqrt(d*x^2 + c)*((2*b*x*sgn(b*x + a) + 3*a*sgn(b*x + a))*x + 2*b*c*sgn(b*
x + a)/d)
```

3.42.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}\,dx=\int \sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}\,dx$$

input
```
int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2), x)
```

3.43 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$

3.43.1 Optimal result

Integrand size = 35, antiderivative size = 160

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{(2a+bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2(a+bx)} + \frac{bc\sqrt{a^2+2abx+b^2x^2}\,\mathrm{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}(a+bx)} - \frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\,\mathrm{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

output
```
-a*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/2*
b*c*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)+1
/2*(b*x+2*a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/(b*x+a)
```

3.43.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \frac{\sqrt{(a+bx)^2}\left(\sqrt{d}(2a+bx)\sqrt{c+dx^2}+4a\sqrt{c}\sqrt{d}\,\mathrm{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-bc\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{2\sqrt{d}(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*(Sqrt[d]*(2*a + b*x)*Sqrt[c + d*x^2] + 4*a*Sqrt[c]*Sqrt
[d]*ArcTanh[(Sqrt[d]*x - Sqrt[c + d*x^2])/Sqrt[c]] - b*c*Log[-(Sqrt[d]*x)
+ Sqrt[c + d*x^2]]))/(2*Sqrt[d]*(a + b*x))
```

3.43.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {1334, 27, 535, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}dx$$

$$\Big\downarrow \text{1334}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+c}}{x}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+c}}{x}dx}{a+bx}$$

$$\Big\downarrow \text{535}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\int \frac{2a+bx}{x\sqrt{dx^2+c}}dx+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(2a\int \frac{1}{x\sqrt{dx^2+c}}dx+b\int \frac{1}{\sqrt{dx^2+c}}dx\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(2a\int \frac{1}{x\sqrt{dx^2+c}}dx+b\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(2a\int\frac{1}{x\sqrt{dx^2+c}}dx+\frac{b\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$\downarrow$ 243

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(a\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+\frac{b\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$\downarrow$ 73

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(\frac{2a\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+\frac{b\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

$\downarrow$ 221

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}c\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{2a\,\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)+\frac{1}{2}(2a+bx)\sqrt{c+dx^2}\right)}{a+bx}$$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((2*a + b*x)*Sqrt[c + d*x^2])/2 + (c*((b*A
rcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d] - (2*a*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]])/Sqrt[c]))/2))/(a + b*x)
```

3.43.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 535
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_), x_Symbol] :> Sim
p[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^p/(2*p*(2*p + 1))), x] + Simp[a/(2*p
 + 1)   Int[(c*(2*p + 1) + 2*d*p*x)*((a + b*x^2)^(p - 1)/x), x], x] /; Free
Q[{a, b, c, d}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1334
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*(b + 2*c*x)^(
2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && E
qQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.43. $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$

3.43.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.49 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{\operatorname{csgn}(bx+a)\left(\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bx-2\sqrt{d}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\sqrt{c}\,a+2\sqrt{d\,x^{2}+c}\,\sqrt{d}\,a+\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)bc\right)}{2\sqrt{d}}$	92

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*csgn(b*x+a)*((d*x^2+c)^(1/2)*d^(1/2)*b*x-2*d^(1/2)*ln(2*(c^(1/2)*(d*x^
2+c)^(1/2)+c)/x)*c^(1/2)*a+2*(d*x^2+c)^(1/2)*d^(1/2)*a+ln(d^(1/2)*x+(d*x^2
+c)^(1/2))*b*c)/d^(1/2)
```

3.43.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx$$

$$= \left[\frac{bc\sqrt{d}\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+2\,a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(bdx+2\,ad)\sqrt{dx^2+c}}{4\,d}\right.$$

$$-\frac{bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}d\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d},\ \frac{4\,a\sqrt{-c}d\arctan\left(\cdots\right.}{}$$

$$\left.-\frac{bc\sqrt{-d}\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-2\,a\sqrt{-c}d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-(bdx+2\,ad)\sqrt{dx^2+c}}{2\,d}\right]$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*a*sq
rt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*(b*d*x + 2
*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2
+ c)) - a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) -
(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, 1/4*(4*a*sqrt(-c)*d*arctan(sqrt(-c)/sq
rt(d*x^2 + c)) + b*c*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d, -1/2*(b*c*sqrt(-d)*arctan(sqrt(
-d)*x/sqrt(d*x^2 + c)) - 2*a*sqrt(-c)*d*arctan(sqrt(-c)/sqrt(d*x^2 + c)) -
 (b*d*x + 2*a*d)*sqrt(d*x^2 + c))/d]
```

3.43.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x, x)
```

3.43.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x, x)
```

3.43.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.43.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x, x)
```

3.44 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx$

3.44.1 Optimal result

Integrand size = 35, antiderivative size = 156

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x(a+bx)} + \frac{a\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{b\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx}$$

output
```
-b*arctanh((d*x^2+c)^(1/2)/c^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+a*ar
ctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-(-b*x+a
)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x/(b*x+a)
```

3.44.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \frac{\sqrt{(a+bx)^2}\left((-a+bx)\sqrt{c+dx^2}+2b\sqrt{c}x\operatorname{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-a\sqrt{d}x\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)}{x(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

output
```
(Sqrt[(a + b*x)^2]*((-a + b*x)*Sqrt[c + d*x^2] + 2*b*Sqrt[c]*x*ArcTanh[(Sq
rt[d]*x - Sqrt[c + d*x^2])/Sqrt[c]] - a*Sqrt[d]*x*Log[-(Sqrt[d]*x) + Sqrt[
c + d*x^2]]))/(x*(a + b*x))
```

3.44.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {1334, 27, 536, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}dx$$

$$\Big\downarrow \text{1334}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+c}}{x^2}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+c}}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{536}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\int \frac{bc+adx}{x\sqrt{dx^2+c}}dx-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(ad\int \frac{1}{\sqrt{dx^2+c}}dx+bc\int \frac{1}{x\sqrt{dx^2+c}}dx-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(ad\int \frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}+bc\int \frac{1}{x\sqrt{dx^2+c}}dx-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(bc\int \frac{1}{x\sqrt{dx^2+c}}dx+a\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}bc\int \frac{1}{x^2\sqrt{dx^2+c}}dx^2+a\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bc\int \frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+a\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{(a-bx)\sqrt{c+dx^2}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(a\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)-\frac{(a-bx)\sqrt{c+dx^2}}{x}-b\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{a+bx}$$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^2,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(((a - b*x)*Sqrt[c + d*x^2])/x) + a*Sqrt[
d]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]] - b*Sqrt[c]*ArcTanh[Sqrt[c + d*x^2
]/Sqrt[c]]))/(a + b*x)
```

3.44.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 536
```
Int[(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_))/(x_)^2, x_Symbol] :> S
imp[(-(2*c*p - d*x))*((a + b*x^2)^p/(2*p*x)), x] + Int[(a*d + 2*b*c*p*x)*((
a + b*x^2)^(p - 1)/x), x] /; FreeQ[{a, b, c, d}, x] && GtQ[p, 0] && Integer
Q[2*p]
```

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1334
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*(b + 2*c*x)^(
2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && E
qQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.44.4 Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{a\sqrt{d\,x^{2}+c}\,\sqrt{\left(bx+a\right)^{2}}}{x\left(bx+a\right)}+\frac{\left(a\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\sqrt{d\,x^{2}+c}\,b-b\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\right)\sqrt{\left(bx+a\right)^{2}}}{bx+a}$
default	$-\frac{\operatorname{csgn}\left(bx+a\right)\left(-a\,d^{\frac{3}{2}}\,x^{2}\sqrt{d\,x^{2}+c}+c^{\frac{3}{2}}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^{2}+c}}{x}\right)\sqrt{d}\,bx+a\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}-\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bcx-\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)acdx\right)}{cx\sqrt{d}}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*(d*x^2+c)^(1/2)/x*((b*x+a)^2)^(1/2)/(b*x+a)+(a*d^(1/2)*ln(d^(1/2)*x+(d*
x^2+c)^(1/2))+(d*x^2+c)^(1/2)*b-b*c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1/2
))/x))*((b*x+a)^2)^(1/2)/(b*x+a)
```

3.44.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.13

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx$$
$$= \left[\frac{a\sqrt{d}x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,\sqrt{dx^2+c}(bx-a)}{2\,x},\right.$$
$$-\frac{2\,a\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-b\sqrt{c}x\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,\sqrt{dx^2+c}(bx-a)}{2\,x},\ \frac{2\,b\sqrt{-c}x\arctan\left(\right.}{}$$
$$\left.-\frac{a\sqrt{-d}x\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-b\sqrt{-c}x\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)-\sqrt{dx^2+c}(bx-a)}{x}\right]$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*(a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + b*sqrt
(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2*sqrt(d*x^2 +
 c)*(b*x - a))/x, -1/2*(2*a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c))
- b*sqrt(c)*x*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*sqrt
(d*x^2 + c)*(b*x - a))/x, 1/2*(2*b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 +
 c)) + a*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) + 2*sqr
t(d*x^2 + c)*(b*x - a))/x, -(a*sqrt(-d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c
)) - b*sqrt(-c)*x*arctan(sqrt(-c)/sqrt(d*x^2 + c)) - sqrt(d*x^2 + c)*(b*x
- a))/x]
```

3.44.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^2}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**2, x)
```

3.44.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^2, x)
```

3.44.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \frac{2\,bc\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\mathrm{sgn}(bx+a)}{\sqrt{-c}} - a\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\mathrm{sgn}(bx+a) + \sqrt{dx^2+c}\,b\,\mathrm{sgn}(bx+a) + \frac{2\,ac\sqrt{d}\,\mathrm{sgn}(bx+a)}{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*b*c*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c
) - a*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + sqrt(d
*x^2 + c)*b*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a)/((sqrt(d)*x - sqrt(d
*x^2 + c))^2 - c)
```

3.44.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^2}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+c}}{x^2}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^2, x)
```

3.45 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3} dx$

3.45.1 Optimal result

Integrand size = 35, antiderivative size = 161

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3} dx = -\frac{(a+2bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{2x^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{a+bx} - \frac{ad\sqrt{a^2+2abx+b^2x^2}\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}(a+bx)}$$

output
```
-1/2*a*d*arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/c^(1/2
)+b*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1
/2*(2*b*x+a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2/(b*x+a)
```

3.45.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3} dx = \frac{\sqrt{(a+bx)^2}\left(2adx^2\text{arctanh}\left(\frac{\sqrt{d}x-\sqrt{c+dx^2}}{\sqrt{c}}\right)-\sqrt{c}\left((a+2bx)\sqrt{c+dx^2}+2b\sqrt{d}x^2\log\left(-\sqrt{d}x+\sqrt{c+dx^2}\right)\right)\right)}{2\sqrt{c}x^2(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

output
```
(Sqrt[(a + b*x)^2]*(2*a*d*x^2*ArcTanh[(Sqrt[d]*x - Sqrt[c + d*x^2])/Sqrt[c
]] - Sqrt[c]*((a + 2*b*x)*Sqrt[c + d*x^2] + 2*b*Sqrt[d]*x^2*Log[-(Sqrt[d]*
x) + Sqrt[c + d*x^2]])))/(2*Sqrt[c]*x^2*(a + b*x))
```

3.45.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1334, 27, 537, 25, 538, 224, 219, 243, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}dx$$

$$\Big\downarrow \text{1334}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+c}}{x^3}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+c}}{x^3}dx}{a+bx}$$

$$\Big\downarrow \text{537}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{1}{2}d\int -\frac{a+2bx}{x\sqrt{dx^2+c}}dx-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\int \frac{a+2bx}{x\sqrt{dx^2+c}}dx-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$$\Big\downarrow \text{538}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(a\int \frac{1}{x\sqrt{dx^2+c}}dx+2b\int \frac{1}{\sqrt{dx^2+c}}dx\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$$\Big\downarrow \text{224}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(a\int\frac{1}{x\sqrt{dx^2+c}}dx+2b\int\frac{1}{1-\frac{dx^2}{dx^2+c}}d\frac{x}{\sqrt{dx^2+c}}\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(a\int\frac{1}{x\sqrt{dx^2+c}}dx+\frac{2b\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$\Big\downarrow$ 243

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(\frac{1}{2}a\int\frac{1}{x^2\sqrt{dx^2+c}}dx^2+\frac{2b\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$\Big\downarrow$ 73

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(\frac{a\int\frac{1}{\frac{x^4}{d}-\frac{c}{d}}d\sqrt{dx^2+c}}{d}+\frac{2b\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

$\Big\downarrow$ 221

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}d\left(\frac{2b\text{arctanh}\left(\frac{\sqrt{d}x}{\sqrt{c+dx^2}}\right)}{\sqrt{d}}-\frac{a\text{arctanh}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{\sqrt{c}}\right)-\frac{(a+2bx)\sqrt{c+dx^2}}{2x^2}\right)}{a+bx}$$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + d*x^2])/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/2*((a + 2*b*x)*Sqrt[c + d*x^2])/x^2 + (
d*((2*b*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/Sqrt[d] - (a*ArcTanh[Sqrt[c
+ d*x^2]/Sqrt[c]])/Sqrt[c]))/2))/(a + b*x)
```

3.45. $\int\frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$

3.45.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 537
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :>
Simp[x^(m + 1)*(c*(m + 2) + d*(m + 1)*x)*((a + b*x^2)^p/((m + 1)*(m + 2))),
 x] - Simp[2*b*(p/((m + 1)*(m + 2)))   Int[x^(m + 2)*(c*(m + 2) + d*(m + 1)
*x)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[m, -2] &&
 GtQ[p, 0] &&  !ILtQ[m + 2*p + 3, 0] && IntegerQ[2*p]
```

3.45. $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$

rule 538
```
Int[((c_) + (d_.)*(x_))/((x_)*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> Simp
[c   Int[1/(x*Sqrt[a + b*x^2]), x], x] + Simp[d   Int[1/Sqrt[a + b*x^2], x]
, x] /; FreeQ[{a, b, c, d}, x]
```

rule 1334
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4
*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*(b + 2*c*x)^(
2*p)*(d + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, f, g, h, m, p, q}, x] && E
qQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.45.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result
risch	$-\frac{(2bx+a)\sqrt{(bx+a)^2}\,\sqrt{d\,x^2+c}}{2x^2(bx+a)}+\frac{\left(\sqrt{d}\,b\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)-\frac{da\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2\sqrt{c}}\right)\sqrt{(bx+a)^2}}{bx+a}$
default	$-\frac{\operatorname{csgn}(bx+a)\left(-2b\,d^{\frac{3}{2}}x^3\sqrt{d\,x^2+c}+\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)d^{\frac{3}{2}}a\,x^2+2\left(d\,x^2+c\right)^{\frac{3}{2}}\sqrt{d}\,bx-a\,d^{\frac{3}{2}}x^2\sqrt{d\,x^2+c}-2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^2+c}\right)b\right.}{2c\,x^2\sqrt{d}}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*x+a)*((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^2/(b*x+a)+(d^(1/2)*b*ln
(d^(1/2)*x+(d*x^2+c)^(1/2))-1/2*d*a/c^(1/2)*ln((2*c+2*c^(1/2)*(d*x^2+c)^(1
/2))/x))*((b*x+a)^2)^(1/2)/(b*x+a)
```

3.45.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.34

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$$

$$= \left[\frac{2\,bc\sqrt{d}x^2\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\sqrt{d}x-c\right)+a\sqrt{c}dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)-2\,(2\,bcx+ac)\sqrt{dx^2}}{4\,cx^2}\right.$$

$$-\frac{4\,bc\sqrt{-d}x^2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{c}dx^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx+ac)\sqrt{dx^2+c}}{4\,cx^2},\ \frac{a\sqrt{-c}dx^2}{}$$

$$\left.-\frac{2\,bc\sqrt{-d}x^2\arctan\left(\frac{\sqrt{-d}x}{\sqrt{dx^2+c}}\right)-a\sqrt{-c}dx^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(2\,bcx+ac)\sqrt{dx^2+c}}{2\,cx^2}\right]$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/4*(2*b*c*sqrt(d)*x^2*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) +
a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) - 2*(2
*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), -1/4*(4*b*c*sqrt(-d)*x^2*arctan(sq
rt(-d)*x/sqrt(d*x^2 + c)) - a*sqrt(c)*d*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c
)*sqrt(c) + 2*c)/x^2) + 2*(2*b*c*x + a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*(a
*sqrt(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + b*c*sqrt(d)*x^2*log(-2*
d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x - c) - (2*b*c*x + a*c)*sqrt(d*x^2 + c)
)/(c*x^2), -1/2*(2*b*c*sqrt(-d)*x^2*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - a
*sqrt(-c)*d*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x + a*c)*sqrt(d*
x^2 + c))/(c*x^2)]
```

3.45.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx = \int \frac{\sqrt{c+dx^2}\sqrt{(a+bx)^2}}{x^3}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c + d*x**2)*sqrt((a + b*x)**2)/x**3, x)
```

3.45.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx=\int \frac{\sqrt{dx^2+c}\sqrt{(bx+a)^2}}{x^3}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + c)*sqrt((b*x + a)^2)/x^3, x)
```

3.45.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.24

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx$$
$$=\frac{ad\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx+a)}{\sqrt{-c}}-b\sqrt{d}\log\left(\left|-\sqrt{d}x+\sqrt{dx^2+c}\right|\right)\operatorname{sgn}(bx+a)$$
$$+\frac{\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^3ad\operatorname{sgn}(bx+a)+2\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2bc\sqrt{d}\operatorname{sgn}(bx+a)+\left(\sqrt{d}x-\sqrt{dx^2+c}\right)ac}{\left(\left(\sqrt{d}x-\sqrt{dx^2+c}\right)^2-c\right)^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
a*d*arctan(-(sqrt(d)*x - sqrt(d*x^2 + c))/sqrt(-c))*sgn(b*x + a)/sqrt(-c)
- b*sqrt(d)*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))*sgn(b*x + a) + ((sqrt(d
)*x - sqrt(d*x^2 + c))^3*a*d*sgn(b*x + a) + 2*(sqrt(d)*x - sqrt(d*x^2 + c)
)^2*b*c*sqrt(d)*sgn(b*x + a) + (sqrt(d)*x - sqrt(d*x^2 + c))*a*c*d*sgn(b*x
 + a) - 2*b*c^2*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 + c))^2 - c
)^2
```

3.45.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2}}{x^3}\,dx=\int \frac{\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+c}}{x^3}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + d*x^2)^(1/2))/x^3, x)
```

3.46 $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

3.46.1 Optimal result

Integrand size = 38, antiderivative size = 317

$$\begin{aligned}
&\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx \\
&= -\frac{(2ad(4cd-5e^2)-b(12cde-7e^3))\,(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{128d^4(a+bx)} \\
&\quad+\frac{bx^2\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{5d(a+bx)} \\
&\quad-\frac{(32bcd+50ade-35be^2-6d(10ad-7be)x)\,\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{240d^3(a+bx)} \\
&\quad-\frac{(4cd-e^2)\,(8acd^2-12bcde-10ade^2+7be^3)\,\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{256d^{9/2}(a+bx)}
\end{aligned}$$

output
```
1/5*b*x^2*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)-1/240*(32*b*c*d+
50*a*d*e-35*b*e^2-6*d*(10*a*d-7*b*e)*x)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1
/2)/d^3/(b*x+a)-1/256*(4*c*d-e^2)*(8*a*c*d^2-10*a*d*e^2-12*b*c*d*e+7*b*e^3
)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(
9/2)/(b*x+a)-1/128*(2*a*d*(4*c*d-5*e^2)-b*(12*c*d*e-7*e^3))*(2*d*x+e)*((b*
x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^4/(b*x+a)
```

3.46.2 Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.74

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \frac{\sqrt{(a+bx)^2}\Big(2\sqrt{d}\sqrt{c+x(e+dx)}(10ad(15e^3-10de^2x+8d^2ex^2+48d^3x^3+4cd(-13e+6dx))+b(-256}{}$$

input
```
Integrate[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(10*a*d*(15*e^3 - 10*d
*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3 + 4*c*d*(-13*e + 6*d*x)) + b*(-256*c^2*d
^2 - 105*e^4 + 70*d*e^3*x - 56*d^2*e^2*x^2 + 48*d^3*e*x^3 + 384*d^4*x^4 +
4*c*d*(115*e^2 - 58*d*e*x + 32*d^2*x^2))) + 15*(4*c*d - e^2)*(2*a*d*(4*c*d
 - 5*e^2) + b*(-12*c*d*e + 7*e^3))*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e
 + d*x)]]))/(3840*d^(9/2)*(a + b*x))
```

3.46.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.71, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {1333, 27, 1236, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\,dx$$

$$\Big\downarrow \text{1333}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2bx^2(a+bx)\sqrt{dx^2+ex+c}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x^2(a+bx)\sqrt{dx^2+ex+c}dx}{a+bx}$$

$$\Big\downarrow \text{1236}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\int -\frac{1}{2}x(4bc-(10ad-7be)x)\sqrt{dx^2+ex+c}dx}{5d}+\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}\right)}{a+bx}$$

$\downarrow$ 27

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\int x(4bc-(10ad-7be)x)\sqrt{dx^2+ex+c}dx}{10d}\right)}{a+bx}$$

$\downarrow$ 1225

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{5\left(8acd^2-10ade^2-12bcde+7be^3\right)\int\sqrt{dx^2+ex+c}dx}{16d^2}+\frac{\left(c+dx^2+ex\right)^{3/2}\left(-6dx(10ad-7be)+50ade+32bcd-\right.}{24d^2}}{10d}\right.}{a+bx}$$

$\downarrow$ 1087

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{5\left(8acd^2-10ade^2-12bcde+7be^3\right)\left(\frac{\left(4cd-e^2\right)\int\frac{1}{\sqrt{dx^2+ex+c}}dx}{8d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{16d^2}+\frac{\left(c+dx^2+ex\right.}{}}{10d}\right.}{a+bx}$$

$\downarrow$ 1092

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{5\left(8acd^2-10ade^2-12bcde+7be^3\right)\left(\frac{\left(4cd-e^2\right)\int\frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{4d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{16d^2}}{10d}\right.}{a+bx}$$

$\downarrow$ 219

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{bx^2\left(c+dx^2+ex\right)^{3/2}}{5d}-\frac{\frac{5\left(8acd^2-10ade^2-12bcde+7be^3\right)\left(\frac{\left(4cd-e^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{16d^2}+}{10d}\right.}{a+bx}$$

3.46. $\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

input
```
Int[x^2*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*x^2*(c + e*x + d*x^2)^(3/2))/(5*d) - ((
(32*b*c*d + 50*a*d*e - 35*b*e^2 - 6*d*(10*a*d - 7*b*e)*x)*(c + e*x + d*x^2
)^(3/2))/(24*d^2) + (5*(8*a*c*d^2 - 12*b*c*d*e - 10*a*d*e^2 + 7*b*e^3)*(((
e + 2*d*x)*Sqrt[c + e*x + d*x^2])/(4*d) + ((4*c*d - e^2)*ArcTanh[(e + 2*d*
x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2))))/(16*d^2))/(10*d)))/(a
 + b*x)
```

3.46.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1333
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*
(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.46.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.84

method	result
risch	$\frac{\left(384b\,x^4d^4+480a\,d^4x^3+48b\,d^3e\,x^3+80a\,d^3e\,x^2+128bc\,d^3x^2-56b\,d^2e^2x^2+240ac\,d^3x-100a\,d^2e^2x-232xbc\,d^2e+70bd\,e^3x-520ac\,d^2e-\right.}{1920d^4(bx+a)}$
default	$\frac{\mathrm{csgn}(bx+a)\left(768\left(d\,x^2+ex+c\right)^{\frac{3}{2}}d^{\frac{9}{2}}b\,x^2+960\left(d\,x^2+ex+c\right)^{\frac{3}{2}}d^{\frac{9}{2}}ax-672\left(d\,x^2+ex+c\right)^{\frac{3}{2}}d^{\frac{7}{2}}bex-800\left(d\,x^2+ex+c\right)^{\frac{3}{2}}d^{\frac{7}{2}}ae-512\left(d\,x^2+e\right.\right.}{}$

input
```
int(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1920*(384*b*d^4*x^4+480*a*d^4*x^3+48*b*d^3*e*x^3+80*a*d^3*e*x^2+128*b*c*
d^3*x^2-56*b*d^2*e^2*x^2+240*a*c*d^3*x-100*a*d^2*e^2*x-232*b*c*d^2*e*x+70*
b*d*e^3*x-520*a*c*d^2*e+150*a*d*e^3-256*b*c^2*d^2+460*b*c*d*e^2-105*b*e^4)
*(d*x^2+e*x+c)^(1/2)/d^4*((b*x+a)^2)^(1/2)/(b*x+a)-1/256*(32*a*c^2*d^3-48*
a*c*d^2*e^2+10*a*d*e^4-48*b*c^2*d^2*e+40*b*c*d*e^3-7*b*e^5)/d^(9/2)*ln((1/
2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)
```

3.46.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 517, normalized size of antiderivative = 1.63

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \left[-\frac{15\,(32\,ac^2d^3-48\,bc^2d^2e-48\,acd^2e^2+40\,bcde^3+10\,ade^4-7\,be^5)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+\right.}{}\right.$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[-1/7680*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^2*e^2 + 40*b*c*d*e^
3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
 e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - 4*(384*b*d^5*x^4 - 256*b*c^
2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 105*b*d*e^4 + 48
*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b*d^3*e^2)*x^2
+ 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e^3)*x)*sqrt(d*
x^2 + e*x + c))/d^5, 1/3840*(15*(32*a*c^2*d^3 - 48*b*c^2*d^2*e - 48*a*c*d^
2*e^2 + 40*b*c*d*e^3 + 10*a*d*e^4 - 7*b*e^5)*sqrt(-d)*arctan(1/2*sqrt(d*x^
2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(384*b*d^5*
x^4 - 256*b*c^2*d^3 - 520*a*c*d^3*e + 460*b*c*d^2*e^2 + 150*a*d^2*e^3 - 10
5*b*d*e^4 + 48*(10*a*d^5 + b*d^4*e)*x^3 + 8*(16*b*c*d^4 + 10*a*d^4*e - 7*b
*d^3*e^2)*x^2 + 2*(120*a*c*d^4 - 116*b*c*d^3*e - 50*a*d^3*e^2 + 35*b*d^2*e
^3)*x)*sqrt(d*x^2 + e*x + c))/d^5]
```

3.46.6 Sympy [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \int x^2\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(x**2*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

output
```
Integral(x**2*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.46.7 Maxima [F]

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int \sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}x^2\,dx$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x^2, x)
```

3.46.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.16

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{dx^2+ex+c}\left(2\left(4\left(6\left(8\,bx\mathrm{sgn}(bx+a)+\frac{10\,ad^4\mathrm{sgn}(bx+a)+bd^3e\mathrm{sgn}(bx+a)}{d^4}\right)x+\frac{16\,bcd^3\mathrm{sgn}(\right.\right.\right.$$
$$+\frac{(32\,ac^2d^3\mathrm{sgn}(bx+a)-48\,bc^2d^2e\mathrm{sgn}(bx+a)-48\,acd^2e^2\mathrm{sgn}(bx+a)+40\,bcde^3\mathrm{sgn}(bx+a)+10\,ade^4\mathrm{sg}}{256\,d^{\frac{9}{2}}}$$

input
```
integrate(x^2*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(d*x^2 + e*x + c)*(2*(4*(6*(8*b*x*sgn(b*x + a) + (10*a*d^4*sgn(
b*x + a) + b*d^3*e*sgn(b*x + a))/d^4)*x + (16*b*c*d^3*sgn(b*x + a) + 10*a*
d^3*e*sgn(b*x + a) - 7*b*d^2*e^2*sgn(b*x + a))/d^4)*x + (120*a*c*d^3*sgn(b
*x + a) - 116*b*c*d^2*e*sgn(b*x + a) - 50*a*d^2*e^2*sgn(b*x + a) + 35*b*d*
e^3*sgn(b*x + a))/d^4)*x - (256*b*c^2*d^2*sgn(b*x + a) + 520*a*c*d^2*e*sgn
(b*x + a) - 460*b*c*d*e^2*sgn(b*x + a) - 150*a*d*e^3*sgn(b*x + a) + 105*b*
e^4*sgn(b*x + a))/d^4) + 1/256*(32*a*c^2*d^3*sgn(b*x + a) - 48*b*c^2*d^2*e
*sgn(b*x + a) - 48*a*c*d^2*e^2*sgn(b*x + a) + 40*b*c*d*e^3*sgn(b*x + a) +
10*a*d*e^4*sgn(b*x + a) - 7*b*e^5*sgn(b*x + a))*log(abs(2*(sqrt(d)*x - sqr
t(d*x^2 + e*x + c))*sqrt(d) + e))/d^(9/2)
```

3.46.9 Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x^2\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

input
```
int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

output
```
int(x^2*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.47 $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

3.47.1 Optimal result

Integrand size = 36, antiderivative size = 227

$$\begin{aligned}\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx &= -\frac{(4bcd+8ade-5be^2)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{64d^3(a+bx)} \\ &\quad+\frac{(8ad-5be+6bdx)\sqrt{a^2+2abx+b^2x^2}(c+ex+dx^2)^{3/2}}{24d^2(a+bx)} \\ &\quad-\frac{(4cd-e^2)(4bcd+8ade-5be^2)\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{128d^{7/2}(a+bx)}\end{aligned}$$

output
```
1/24*(6*b*d*x+8*a*d-5*b*e)*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d^2/(b*x+
a)-1/128*(4*c*d-e^2)*(8*a*d*e+4*b*c*d-5*b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/
2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/d^(7/2)/(b*x+a)-1/64*(8*a*d*e+4*
b*c*d-5*b*e^2)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d^3/(b*x+a)
```

3.47.2 Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.78

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}\sqrt{c+x(e+dx)}(8ad(8cd-3e^2+2dex+8d^2x^2)+b(15e^3-10de^2x+8d^2ex^2+48d^3x^3\right.}{384d^{7/2}(a+}$$

input
```
Integrate[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(2*Sqrt[d]*Sqrt[c + x*(e + d*x)]*(8*a*d*(8*c*d - 3*e^2
+ 2*d*e*x + 8*d^2*x^2) + b*(15*e^3 - 10*d*e^2*x + 8*d^2*e*x^2 + 48*d^3*x^3
 + 4*c*d*(-13*e + 6*d*x))) + 3*(4*c*d - e^2)*(4*b*c*d + 8*a*d*e - 5*b*e^2)
*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*x)]]))/(384*d^(7/2)*(a + b*x)
)
```

3.47.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1333, 27, 1225, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\,dx$$
$$\Big\downarrow \text{1333}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2bx(a+bx)\sqrt{dx^2+ex+c}dx}{2b(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int x(a+bx)\sqrt{dx^2+ex+c}dx}{a+bx}$$
$$\Big\downarrow \text{1225}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(c+dx^2+ex)^{3/2}(8ad+6bdx-5be)}{24d^2}-\frac{(8ade+4bcd-5be^2)\int\sqrt{dx^2+ex+c}dx}{16d^2}\right)}{a+bx}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(c+dx^2+ex)^{3/2}(8ad+6bdx-5be)}{24d^2}-\frac{(8ade+4bcd-5be^2)\left(\frac{(4cd-e^2)\int\frac{1}{\sqrt{dx^2+ex+c}}dx}{8d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{16d^2}\right)}{a+bx}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(c+dx^2+ex)^{3/2}(8ad+6bdx-5be)}{24d^2}-\frac{(8ade+4bcd-5be^2)\left(\frac{(4cd-e^2)\int\frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{4d}+\frac{(2dx+e)\sqrt{c+dx^2+e}}{4d}\right.}{16d^2}\right.}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(c+dx^2+ex)^{3/2}(8ad+6bdx-5be)}{24d^2}-\frac{(8ade+4bcd-5be^2)\left(\frac{(4cd-e^2)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right.}{16d^2}\right.}{a+bx}$$

input
```
Int[x*Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((8*a*d - 5*b*e + 6*b*d*x)*(c + e*x + d*x^
2)^(3/2))/(24*d^2) - ((4*b*c*d + 8*a*d*e - 5*b*e^2)*(((e + 2*d*x)*Sqrt[c +
 e*x + d*x^2])/(4*d) + ((4*c*d - e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[
c + e*x + d*x^2])])/(8*d^(3/2))))/(16*d^2)))/(a + b*x)
```

3.47. $\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

3.47.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

rule 1333
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*
(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.47.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

method	result
risch	$\frac{(48b\,x^3d^3+64a\,d^3x^2+8b\,d^2e\,x^2+16a\,d^2ex+24bc\,d^2x-10bd\,e^2x+64c\,d^2a-24ad\,e^2-52bcde+15b\,e^3)\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{192d^3(bx+a)} - \frac{(32a$
default	$\frac{\operatorname{csgn}(bx+a)\left(96(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}bx+128(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{7}{2}}a-80(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{5}{2}}be-96\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}aex-48\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}b\right.}{}$

input
```
int(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*b*d^3*x^3+64*a*d^3*x^2+8*b*d^2*e*x^2+16*a*d^2*e*x+24*b*c*d^2*x-1
0*b*d*e^2*x+64*a*c*d^2-24*a*d*e^2-52*b*c*d*e+15*b*e^3)*(d*x^2+e*x+c)^(1/2)
/d^3*((b*x+a)^2)^(1/2)/(b*x+a)-1/128*(32*a*c*d^2*e-8*a*d*e^3+16*b*c^2*d^2-
24*b*c*d*e^2+5*b*e^4)/d^(7/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*
((b*x+a)^2)^(1/2)/(b*x+a)
```

3.47.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.72

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \left[\frac{3\,(16\,bc^2d^2+32\,acd^2e-24\,bcde^2-8\,ade^3+5\,be^4)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{\right.}}{}\right.$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/768*(3*(16*b*c^2*d^2 + 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^
4)*sqrt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*s
qrt(d) + 4*c*d + e^2) + 4*(48*b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a
*d^2*e^2 + 15*b*d*e^3 + 8*(8*a*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^
3*e - 5*b*d^2*e^2)*x)*sqrt(d*x^2 + e*x + c))/d^4, 1/384*(3*(16*b*c^2*d^2 +
 32*a*c*d^2*e - 24*b*c*d*e^2 - 8*a*d*e^3 + 5*b*e^4)*sqrt(-d)*arctan(1/2*sq
rt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(48*
b*d^4*x^3 + 64*a*c*d^3 - 52*b*c*d^2*e - 24*a*d^2*e^2 + 15*b*d*e^3 + 8*(8*a
*d^4 + b*d^3*e)*x^2 + 2*(12*b*c*d^3 + 8*a*d^3*e - 5*b*d^2*e^2)*x)*sqrt(d*x
^2 + e*x + c))/d^4]
```

3.47.6 Sympy [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(x*((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

output
```
Integral(x*sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.47.7 Maxima [F]

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int \sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}x\,dx$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)*x, x)
```

3.47.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.17

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\frac{1}{192}\sqrt{dx^2+ex+c}\Bigg(2\left(4\left(6\,bx\mathrm{sgn}(bx+a)+\frac{8\,ad^3\mathrm{sgn}(bx+a)+bd^2e\mathrm{sgn}(bx+a)}{d^3}\right)x+\frac{12\,bcd^2\mathrm{sgn}(bx+}{}\right.$$
$$+\frac{(16\,bc^2d^2\mathrm{sgn}(bx+a)+32\,acd^2e\mathrm{sgn}(bx+a)-24\,bcde^2\mathrm{sgn}(bx+a)-8\,ade^3\mathrm{sgn}(bx+a)+5\,be^4\mathrm{sgn}(bx-}{128\,d^{\frac{7}{2}}}$$

input
```
integrate(x*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(d*x^2 + e*x + c)*(2*(4*(6*b*x*sgn(b*x + a) + (8*a*d^3*sgn(b*x +
 a) + b*d^2*e*sgn(b*x + a))/d^3)*x + (12*b*c*d^2*sgn(b*x + a) + 8*a*d^2*e*
sgn(b*x + a) - 5*b*d*e^2*sgn(b*x + a))/d^3)*x + (64*a*c*d^2*sgn(b*x + a) -
 52*b*c*d*e*sgn(b*x + a) - 24*a*d*e^2*sgn(b*x + a) + 15*b*e^3*sgn(b*x + a)
)/d^3) + 1/128*(16*b*c^2*d^2*sgn(b*x + a) + 32*a*c*d^2*e*sgn(b*x + a) - 24
*b*c*d*e^2*sgn(b*x + a) - 8*a*d*e^3*sgn(b*x + a) + 5*b*e^4*sgn(b*x + a))*l
og(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(7/2)
```

3.47.9 Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int x\,\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

input
```
int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

output
```
int(x*((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.48 $\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$

3.48.1 Optimal result

Integrand size = 35, antiderivative size = 198

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \frac{(2ad-be)(e+2dx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{8d^2(a+bx)} + \frac{b\sqrt{a^2+2abx+b^2x^2}\left(c+ex+dx^2\right)^{3/2}}{3d(a+bx)} + \frac{(2ad-be)\left(4cd-e^2\right)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{16d^{5/2}(a+bx)}$$

output
```
1/3*b*(d*x^2+e*x+c)^(3/2)*((b*x+a)^2)^(1/2)/d/(b*x+a)+1/16*(2*a*d-b*e)*(4*
c*d-e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1
/2)/d^(5/2)/(b*x+a)+1/8*(2*a*d-b*e)*(2*d*x+e)*((b*x+a)^2)^(1/2)*(d*x^2+e*x
+c)^(1/2)/d^2/(b*x+a)
```

3.48.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.87

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \frac{\sqrt{(a+bx)^2}\left(\sqrt{d}\sqrt{c+x(e+dx)}(6ad(e+2dx)+b(8cd-3e^2+2dex+8d^2x^2))+6de(2bc+ae)\text{arctanh}\left(\right.\right.}{24d^{5/2}(a+bx)}$$

input
```
Integrate[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[(a + b*x)^2]*(Sqrt[d]*Sqrt[c + x*(e + d*x)]*(6*a*d*(e + 2*d*x) + b*(
8*c*d - 3*e^2 + 2*d*e*x + 8*d^2*x^2)) + 6*d*e*(2*b*c + a*e)*ArcTanh[(Sqrt[
d]*x)/(Sqrt[c] - Sqrt[c + x*(e + d*x)])] + 3*(8*a*c*d^2 + b*e^3)*ArcTanh[(
Sqrt[d]*x)/(-Sqrt[c] + Sqrt[c + x*(e + d*x)])]))/(24*d^(5/2)*(a + b*x))
```

3.48.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.73, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {1297, 27, 1160, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}\,dx$$
$$\Big\downarrow \text{1297}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int 2b(a+bx)\sqrt{dx^2+ex+c}dx}{2b(a+bx)}$$
$$\Big\downarrow \text{27}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\int (a+bx)\sqrt{dx^2+ex+c}dx}{a+bx}$$
$$\Big\downarrow \text{1160}$$
$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(2ad-be)\int \sqrt{dx^2+ex+c}dx}{2d}+\frac{b(c+dx^2+ex)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \; 1087$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(2ad-be)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{\sqrt{dx^2+ex+c}}dx}{8d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \; 1092$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(2ad-be)\left(\frac{\left(4cd-e^2\right)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{4d}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{(2ad-be)\left(\frac{\left(4cd-e^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8d^{3/2}}+\frac{(2dx+e)\sqrt{c+dx^2+ex}}{4d}\right)}{2d}+\frac{b\left(c+dx^2+ex\right)^{3/2}}{3d}\right)}{a+bx}$$

input
```
Int[Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2],x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*((b*(c + e*x + d*x^2)^(3/2))/(3*d) + ((2*a*
d - b*e)*(((e + 2*d*x)*Sqrt[c + e*x + d*x^2])/(4*d) + ((4*c*d - e^2)*ArcTa
nh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/(8*d^(3/2))))/(2*d)))/(
a + b*x)
```

3.48.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1297
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)
^2)^(q_.), x_Symbol] :> Simp[(a + b*x + c*x^2)^FracPart[p]/((4*c)^IntPart[p
]*(b + 2*c*x)^(2*FracPart[p]))   Int[(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q,
 x], x] /; FreeQ[{a, b, c, d, e, f, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[p]
```

3.48.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(8b\,x^2d^2+12a\,d^2x+2bdex+6ade+8bcd-3b\,e^2)\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{24d^2(bx+a)}+\frac{(8c\,d^2a-2ad\,e^2-4bcde+b\,e^3)\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{16d^{\frac{5}{2}}(bx+a)}$
default	$\frac{\operatorname{csgn}(bx+a)\left(16(d\,x^2+ex+c)^{\frac{3}{2}}d^{\frac{5}{2}}b+24\sqrt{d\,x^2+ex+c}\,d^{\frac{7}{2}}ax-12\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}bex+12\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}ae-6\sqrt{d\,x^2+ex+c}\,d^{\frac{3}{2}}b\,e^2+2\right.}{}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b*d^2*x^2+12*a*d^2*x+2*b*d*e*x+6*a*d*e+8*b*c*d-3*b*e^2)*(d*x^2+e*x
+c)^(1/2)/d^2*((b*x+a)^2)^(1/2)/(b*x+a)+1/16*(8*a*c*d^2-2*a*d*e^2-4*b*c*d*
e+b*e^3)/d^(5/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(
1/2)/(b*x+a)
```

3.48.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.45

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$=\left[\frac{3\,(8\,acd^2-4\,bcde-2\,ade^2+be^3)\sqrt{d}\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+\ldots}{96\,d^3}\right.$$
$$-\frac{3\,(8\,acd^2-4\,bcde-2\,ade^2+be^3)\sqrt{-d}\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-2\,(8\,bd^3x^2+8\,bcd^2+6\,ad^2e-\ldots}{48\,d^3}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="fricas")
```

output
```
[1/96*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^2 + b*e^3)*sqrt(d)*log(8*d^2*x^2
 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) +
4*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*e^2 + 2*(6*a*d^3 + b*d^2*e)
*x)*sqrt(d*x^2 + e*x + c))/d^3, -1/48*(3*(8*a*c*d^2 - 4*b*c*d*e - 2*a*d*e^
2 + b*e^3)*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/
(d^2*x^2 + d*e*x + c*d)) - 2*(8*b*d^3*x^2 + 8*b*c*d^2 + 6*a*d^2*e - 3*b*d*
e^2 + 2*(6*a*d^3 + b*d^2*e)*x)*sqrt(d*x^2 + e*x + c))/d^3]
```

3.48.6 Sympy [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx=\int \sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2),x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2), x)
```

3.48.7 Maxima [F]

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \int \sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2), x)
```

3.48.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.91

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx$$
$$= \frac{1}{24}\sqrt{dx^2+ex+c}\left(2\left(4\,bx\text{sgn}(bx+a)+\frac{6\,ad^2\text{sgn}(bx+a)+bde\text{sgn}(bx+a)}{d^2}\right)x+\frac{8\,bcd\text{sgn}(bx+a)+6\,a\right.$$
$$-\frac{(8\,acd^2\text{sgn}(bx+a)-4\,bcde\text{sgn}(bx+a)-2\,ade^2\text{sgn}(bx+a)+be^3\text{sgn}(bx+a))\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+}\right.\right.\right.}{16\,d^{\frac{5}{2}}}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(d*x^2 + e*x + c)*(2*(4*b*x*sgn(b*x + a) + (6*a*d^2*sgn(b*x + a)
+ b*d*e*sgn(b*x + a))/d^2)*x + (8*b*c*d*sgn(b*x + a) + 6*a*d*e*sgn(b*x + a
) - 3*b*e^2*sgn(b*x + a))/d^2) - 1/16*(8*a*c*d^2*sgn(b*x + a) - 4*b*c*d*e*
sgn(b*x + a) - 2*a*d*e^2*sgn(b*x + a) + b*e^3*sgn(b*x + a))*log(abs(2*(sqr
t(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))/d^(5/2)
```

3.48.9 Mupad [F(-1)]

Timed out.

$$\int \sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}\,dx = \int \sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}\,dx$$

input
```
int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2),x)
```

output
```
int(((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2), x)
```

3.49 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$

3.49.1 Optimal result

Integrand size = 38, antiderivative size = 211

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx &= \frac{(4ad+be+2bdx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4d(a+bx)} \\ &\quad+\frac{(4bcd+4ade-be^2)\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{8d^{3/2}(a+bx)} \\ &\quad-\frac{a\sqrt{c}\sqrt{a^2+2abx+b^2x^2}\operatorname{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{a+bx}\end{aligned}$$

output
```
1/8*(4*a*d*e+4*b*c*d-b*e^2)*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d*x^2+e*x+c)^(1
/2))*((b*x+a)^2)^(1/2)/d^(3/2)/(b*x+a)-a*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*
x^2+e*x+c)^(1/2))*c^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)+1/4*(2*b*d*x+4*a*d+b*e
)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/d/(b*x+a)
```

3.49.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$$
$$= \frac{\sqrt{(a+bx)^2}\left((4bcd+4ade-be^2)\operatorname{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+x(e+dx)}}\right)+2\sqrt{d}\left(\sqrt{c+x(e+dx)}(4ad+b(e+2dx))+8\right.\right.}{8d^{3/2}(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

output
```
(Sqrt[(a + b*x)^2]*((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqr
t[d]*Sqrt[c + x*(e + d*x)])] + 2*Sqrt[d]*(Sqrt[c + x*(e + d*x)]*(4*a*d + b
*(e + 2*d*x)) + 8*a*Sqrt[c]*d*ArcTanh[(Sqrt[d]*x - Sqrt[c + x*(e + d*x)])/
Sqrt[c]])))/(8*d^(3/2)*(a + b*x))
```

3.49.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.78, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.237$, Rules used = {1333, 27, 1231, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{x}\,dx$$

$$\Big\downarrow \text{1333}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+ex+c}}{x}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+ex+c}}{x}dx}{a+bx}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}-\frac{\int -\frac{8acd+\left(-be^2+4ade+4bcd\right)x}{2x\sqrt{dx^2+ex+c}}dx}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\int \frac{8acd+\left(-be^2+4ade+4bcd\right)x}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 1269$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\left(4ade+4bcd-be^2\right)\int \frac{1}{\sqrt{dx^2+ex+c}}dx+8acd\int \frac{1}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{2\left(4ade+4bcd-be^2\right)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}+8acd\int \frac{1}{x\sqrt{dx^2+ex+c}}dx}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{8acd\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+\frac{\left(4ade+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 1154$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\frac{\left(4ade+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-16acd\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\frac{\left(4ade+4bcd-be^2\right)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-8a\sqrt{c}d\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{8d}+\frac{\sqrt{c+dx^2+ex}(4ad+2bdx+be)}{4d}\right)}{a+bx}$$

3.49. $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(((4*a*d + b*e + 2*b*d*x)*Sqrt[c + e*x + d*
x^2])/(4*d) + (((4*b*c*d + 4*a*d*e - b*e^2)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]
*Sqrt[c + e*x + d*x^2])])/Sqrt[d] - 8*a*Sqrt[c]*d*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/(8*d)))/(a + b*x)
```

3.49.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1333
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*
(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.49.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.02

method	result
default	$\frac{\operatorname{csgn}(bx+a)\left(4\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}bx-8\sqrt{c}\,d^{\frac{5}{2}}\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)a+8\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}a+2\sqrt{d\,x^2+ex+c}\,d^{\frac{3}{2}}be+4d^2\ln\left(\frac{2\sqrt{d\,x^2}}{}\right.\right.}{8d^{\frac{5}{2}}}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*csgn(b*x+a)*(4*(d*x^2+e*x+c)^(1/2)*d^(5/2)*b*x-8*c^(1/2)*d^(5/2)*ln((2
*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x)*a+8*(d*x^2+e*x+c)^(1/2)*d^(5/2)*a
+2*(d*x^2+e*x+c)^(1/2)*d^(3/2)*b*e+4*d^2*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(
1/2)+2*d*x+e)/d^(1/2))*a*e+4*ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e
)/d^(1/2))*b*c*d^2-ln(1/2*(2*(d*x^2+e*x+c)^(1/2)*d^(1/2)+2*d*x+e)/d^(1/2))
*b*d*e^2)/d^(5/2)
```

3.49.5 Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 651, normalized size of antiderivative = 3.09

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx$$
$$= \left[\frac{8\,a\sqrt{c}d^2\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{c}+8\,c^2}{x^2}\right)-(4\,bcd+4\,ade-be^2)\sqrt{d}\log\left(8\,d^2x^2+8\,dex-\right.}{16\,d^2}\right.$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/16*(8*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e
*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)*sq
rt(d)*log(8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d
) + 4*c*d + e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/
d^2, 1/8*(4*a*sqrt(c)*d^2*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2
+ e*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) - (4*b*c*d + 4*a*d*e - b*e^2)
*sqrt(-d)*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 +
 d*e*x + c*d)) + 2*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^
2, 1/16*(16*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sq
rt(-c)/(c*d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(d)*log(
8*d^2*x^2 + 8*d*e*x - 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d
+ e^2) + 4*(2*b*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2, 1/8*(
8*a*sqrt(-c)*d^2*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(c*
d*x^2 + c*e*x + c^2)) - (4*b*c*d + 4*a*d*e - b*e^2)*sqrt(-d)*arctan(1/2*sq
rt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(2*b
*d^2*x + 4*a*d^2 + b*d*e)*sqrt(d*x^2 + e*x + c))/d^2]
```

3.49.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x,x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x, x)
```

3.49.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x, x)
```

3.49.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.49.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x}\,dx = \int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x, x)
```

3.50 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$

3.50.1 Optimal result

Integrand size = 38, antiderivative size = 202

$$\begin{aligned}\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx &= -\frac{(a-bx)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x(a+bx)} \\ &+\frac{(2ad+be)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{d}(a+bx)} \\ &-\frac{(2bc+ae)\sqrt{a^2+2abx+b^2x^2}\,\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{2\sqrt{c}(a+bx)}\end{aligned}$$

output
```
-1/2*(a*e+2*b*c)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(1/2))*((b*x+
a)^2)^(1/2)/(b*x+a)/c^(1/2)+1/2*(2*a*d+b*e)*arctanh(1/2*(2*d*x+e)/d^(1/2)/
(d*x^2+e*x+c)^(1/2))*((b*x+a)^2)^(1/2)/(b*x+a)/d^(1/2)-(-b*x+a)*((b*x+a)^2
)^(1/2)*(d*x^2+e*x+c)^(1/2)/x/(b*x+a)
```

3.50.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = -\frac{\sqrt{(a+bx)^2}\left(2\sqrt{d}(2bc+ae)x\operatorname{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)+\sqrt{c}\left(2\sqrt{d}(a-bx)\sqrt{c+x(e+dx)}+(2ad+\right.\right.}{2\sqrt{c}\sqrt{d}x(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

output
```
-1/2*(Sqrt[(a + b*x)^2]*(2*Sqrt[d]*(2*b*c + a*e)*x*ArcTanh[(-(Sqrt[d]*x) +
 Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*(2*Sqrt[d]*(a - b*x)*Sqrt[c + x
*(e + d*x)] + (2*a*d + b*e)*x*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt[c + x*(e + d*
x)]])))/(Sqrt[c]*Sqrt[d]*x*(a + b*x))
```

3.50.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.74, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1333, 27, 1230, 25, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{x^2}dx$$

$$\Big\downarrow \text{1333}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+ex+c}}{x^2}dx}{2b(a+bx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+ex+c}}{x^2}dx}{a+bx}$$

$$\Big\downarrow \text{1230}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{1}{2}\int -\frac{2bc+ae+(2ad+be)x}{x\sqrt{dx^2+ex+c}}dx-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\int \frac{2bc+ae+(2ad+be)x}{x\sqrt{dx^2+ex+c}}dx-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\left((2ad+be)\int \frac{1}{\sqrt{dx^2+ex+c}}dx+(ae+2bc)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx\right)-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\left((ae+2bc)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+2(2ad+be)\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}\right)-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\left((ae+2bc)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+\frac{(2ad+be)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}\right)-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\left(\frac{(2ad+be)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-2(ae+2bc)\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}\right)-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{1}{2}\left(\frac{(2ad+be)\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{\sqrt{d}}-\frac{(ae+2bc)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{\sqrt{c}}\right)-\frac{(a-bx)\sqrt{c+dx^2+ex}}{x}\right)}{a+bx}$$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^2,x]
```

3.50. $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-(((a - b*x)*Sqrt[c + e*x + d*x^2])/x) + (
((2*a*d + b*e)*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[c + e*x + d*x^2])])/Sqr
t[d] - ((2*b*c + a*e)*ArcTanh[(2*c + e*x)/(2*Sqrt[c]*Sqrt[c + e*x + d*x^2]
)])/Sqrt[c])/2))/(a + b*x)
```

3.50.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1333
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*
(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.50.4 Maple [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{a\sqrt{d\,x^2+ex+c}\,\sqrt{(bx+a)^2}}{x(bx+a)}+\frac{\left(a\sqrt{d}\,\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)+\frac{be\ln\left(\frac{\frac{e}{2}+dx}{\sqrt{d}}+\sqrt{d\,x^2+ex+c}\right)}{2\sqrt{d}}+b\sqrt{d\,x^2+ex+c}-\frac{\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,\cdots}}{x}\right.}{2\sqrt{c}}\cdots}{bx+a}$
default	$-\frac{\operatorname{csgn}(bx+a)\left(-2\sqrt{d\,x^2+ex+c}\,d^{\frac{5}{2}}\,a\,x^2+2d^{\frac{3}{2}}\,c^{\frac{3}{2}}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)bx+d^{\frac{3}{2}}\,\sqrt{c}\,\ln\left(\frac{2c+ex+2\sqrt{c}\,\sqrt{d\,x^2+ex+c}}{x}\right)aex+2(d\,x^2+\cdots\right.}{\cdots}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-a*(d*x^2+e*x+c)^(1/2)/x*((b*x+a)^2)^(1/2)/(b*x+a)+(a*d^(1/2)*ln((1/2*e+d*
x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))+1/2*b*e*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+
c)^(1/2))/d^(1/2)+b*(d*x^2+e*x+c)^(1/2)-1/2/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*
(d*x^2+e*x+c)^(1/2))/x)*a*e-c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1
/2))/x)*b)*((b*x+a)^2)^(1/2)/(b*x+a)
```

3.50.5 Fricas [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.20

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx$$
$$= \left[\frac{(2\,acd+bce)\sqrt{d}x\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)+(2\,bcd+ade)\sqrt{c}x}{4\,cdx}\right.$$
$$-\frac{2\,(2\,acd+bce)\sqrt{-d}x\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-(2\,bcd+ade)\sqrt{c}x\log\left(\frac{8\,cex+(4\,cd+e^2)x^2-4\,\sqrt{dx^2+ex+c}}{x^2}\right.}{4\,cdx}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/4*((2*a*c*d + b*c*e)*sqrt(d)*x*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 +
 e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) + (2*b*c*d + a*d*e)*sqrt(c)*x
*log((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sq
rt(c) + 8*c^2)/x^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x),
-1/4*(2*(2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(2*d
*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) - (2*b*c*d + a*d*e)*sqrt(c)*x*lo
g((8*c*e*x + (4*c*d + e^2)*x^2 - 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(
c) + 8*c^2)/x^2) - 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/4
*(2*(2*b*c*d + a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2
*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + (2*a*c*d + b*c*e)*sqrt(d)*x*log(8*
d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d +
e^2) + 4*(b*c*d*x - a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x), 1/2*((2*b*c*d +
 a*d*e)*sqrt(-c)*x*arctan(1/2*sqrt(d*x^2 + e*x + c)*(e*x + 2*c)*sqrt(-c)/(
c*d*x^2 + c*e*x + c^2)) - (2*a*c*d + b*c*e)*sqrt(-d)*x*arctan(1/2*sqrt(d*x
^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + 2*(b*c*d*x -
 a*c*d)*sqrt(d*x^2 + e*x + c))/(c*d*x)]
```

3.50.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x^2}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**2, x)
```

3.50.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx = \int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x^2}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^2, x)
```

3.50.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.00

$$\begin{aligned}&\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx\\&=\sqrt{dx^2+ex+c}\,b\mathrm{sgn}(bx+a)\\&+\frac{(2\,bc\mathrm{sgn}(bx+a)+ae\mathrm{sgn}(bx+a))\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}\\&-\frac{(2\,ad\mathrm{sgn}(bx+a)+be\mathrm{sgn}(bx+a))\log\left(\left|-2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}-e\right|\right)}{2\sqrt{d}}\\&+\frac{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)ae\mathrm{sgn}(bx+a)+2\,ac\sqrt{d}\mathrm{sgn}(bx+a)}{\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^2-c}\end{aligned}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^2,x, algorithm="giac")
```

output
```
sqrt(d*x^2 + e*x + c)*b*sgn(b*x + a) + (2*b*c*sgn(b*x + a) + a*e*sgn(b*x +
 a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/sqrt(-c) - 1/2*
(2*a*d*sgn(b*x + a) + b*e*sgn(b*x + a))*log(abs(-2*(sqrt(d)*x - sqrt(d*x^2
 + e*x + c))*sqrt(d) - e))/sqrt(d) + ((sqrt(d)*x - sqrt(d*x^2 + e*x + c))*
a*e*sgn(b*x + a) + 2*a*c*sqrt(d)*sgn(b*x + a))/((sqrt(d)*x - sqrt(d*x^2 +
e*x + c))^2 - c)
```

3.50.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^2}\,dx=\int \frac{\sqrt{{\left(a+b\,x\right)}^2}\,\sqrt{d\,x^2+e\,x+c}}{x^2}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^2, x)
```

3.51 $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$

3.51.1 Optimal result

Integrand size = 38, antiderivative size = 215

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = -\frac{(2ac+(4bc+ae)x)\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{4cx^2(a+bx)} + \frac{b\sqrt{d}\sqrt{a^2+2abx+b^2x^2}\,\mathrm{arctanh}\left(\frac{e+2dx}{2\sqrt{d}\sqrt{c+ex+dx^2}}\right)}{a+bx} - \frac{(4acd+4bce-ae^2)\sqrt{a^2+2abx+b^2x^2}\,\mathrm{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+ex+dx^2}}\right)}{8c^{3/2}(a+bx)}$$

output
```
-1/8*(4*a*c*d-a*e^2+4*b*c*e)*arctanh(1/2*(e*x+2*c)/c^(1/2)/(d*x^2+e*x+c)^(
1/2))*((b*x+a)^2)^(1/2)/c^(3/2)/(b*x+a)+b*arctanh(1/2*(2*d*x+e)/d^(1/2)/(d
*x^2+e*x+c)^(1/2))*d^(1/2)*((b*x+a)^2)^(1/2)/(b*x+a)-1/4*(2*a*c+(a*e+4*b*c
)*x)*((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/c/x^2/(b*x+a)
```

3.51.2 Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx = -\frac{\sqrt{(a+bx)^2}\left((4acd+4bce-ae^2)\,x^2\text{arctanh}\left(\frac{-\sqrt{d}x+\sqrt{c+x(e+dx)}}{\sqrt{c}}\right)+\sqrt{c}\left((2ac+4bcx+aex)\sqrt{c+x(e+\right.\right.}{4c^{3/2}x^2(a+bx)}$$

input
```
Integrate[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

output
```
-1/4*(Sqrt[(a + b*x)^2]*((4*a*c*d + 4*b*c*e - a*e^2)*x^2*ArcTanh[(-(Sqrt[d
]*x) + Sqrt[c + x*(e + d*x)])/Sqrt[c]] + Sqrt[c]*((2*a*c + 4*b*c*x + a*e*x
)*Sqrt[c + x*(e + d*x)] + 4*b*c*Sqrt[d]*x^2*Log[e + 2*d*x - 2*Sqrt[d]*Sqrt
[c + x*(e + d*x)]])))/(c^(3/2)*x^2*(a + b*x))
```

3.51.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {1333, 27, 1229, 27, 1269, 1092, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+dx^2+ex}}{x^3}\,dx$$

$$\Big\downarrow \ 1333$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{2b(a+bx)\sqrt{dx^2+ex+c}}{x^3}dx}{2b(a+bx)}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\int \frac{(a+bx)\sqrt{dx^2+ex+c}}{x^3}dx}{a+bx}$$

$$\Big\downarrow \ 1229$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(-\frac{\int -\frac{4bce+a\left(4cd-e^2\right)+8bcdx}{2x\sqrt{dx^2+ex+c}}dx}{4c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\int \frac{-ae^2+4bce+4acd+8bcdx}{x\sqrt{dx^2+ex+c}}dx}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 1269$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\left(4acd-ae^2+4bce\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+8bcd\int \frac{1}{\sqrt{dx^2+ex+c}}dx}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 1092$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\left(4acd-ae^2+4bce\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+16bcd\int \frac{1}{4d-\frac{(e+2dx)^2}{dx^2+ex+c}}d\frac{e+2dx}{\sqrt{dx^2+ex+c}}}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{\left(4acd-ae^2+4bce\right)\int \frac{1}{x\sqrt{dx^2+ex+c}}dx+8bc\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 1154$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{8bc\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)-2\left(4acd-ae^2+4bce\right)\int \frac{1}{4c-\frac{(2c+ex)^2}{dx^2+ex+c}}d\frac{2c+ex}{\sqrt{dx^2+ex+c}}}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

$$\Big\downarrow \ 219$$

$$\frac{\sqrt{a^2+2abx+b^2x^2}\left(\frac{8bc\sqrt{d}\text{arctanh}\left(\frac{2dx+e}{2\sqrt{d}\sqrt{c+dx^2+ex}}\right)-\frac{\left(4acd-ae^2+4bce\right)\text{arctanh}\left(\frac{2c+ex}{2\sqrt{c}\sqrt{c+dx^2+ex}}\right)}{\sqrt{c}}}{8c}-\frac{\sqrt{c+dx^2+ex}(x(ae+4bc)+2ac)}{4cx^2}\right)}{a+bx}$$

3.51. $\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$

input
```
Int[(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*Sqrt[c + e*x + d*x^2])/x^3,x]
```

output
```
(Sqrt[a^2 + 2*a*b*x + b^2*x^2]*(-1/4*((2*a*c + (4*b*c + a*e)*x)*Sqrt[c + e
*x + d*x^2])/(c*x^2) + (8*b*c*Sqrt[d]*ArcTanh[(e + 2*d*x)/(2*Sqrt[d]*Sqrt[
c + e*x + d*x^2])] - ((4*a*c*d + 4*b*c*e - a*e^2)*ArcTanh[(2*c + e*x)/(2*S
qrt[c]*Sqrt[c + e*x + d*x^2])])/Sqrt[c])/(8*c)))/(a + b*x)
```

3.51.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1333
```
Int[((g_.) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_
) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^Fr
acPart[p]/((4*c)^IntPart[p]*(b + 2*c*x)^(2*FracPart[p]))   Int[(g + h*x)^m*
(b + 2*c*x)^(2*p)*(d + e*x + f*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e, f, g
, h, m, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

3.51.4 Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.73

method	result
risch	$-\frac{\sqrt{d\,x^{2}+e x +c}\,\left(a e x +4 b c x +2 a c \right) \sqrt{\left(b x +a \right)^{2}}}{4 x^{2} c \left(b x +a \right)}+\frac{\left(8 b c \sqrt{d}\,\ln \left(\frac{\frac{e}{2}+d x}{\sqrt{d}}+\sqrt{d\,x^{2}+e x +c}\right)-\frac{\left(4 a c d -e^{2} a +4 b c e \right) \ln \left(\frac{2 c +e x +2 \sqrt{c}\,\sqrt{d\,x^{2}+e x +c}}{x}\right)}{\sqrt{c}} \cdots\right.}{8 c \left(b x +a \right)}$
default	$-\frac{\operatorname{csgn}\left(b x +a \right) \left(4 d^{\frac{5}{2}} c^{\frac{3}{2}} \ln \left(\frac{2 c +e x +2 \sqrt{c}\,\sqrt{d\,x^{2}+e x +c}}{x}\right) a\,x^{2}+2 \sqrt{d\,x^{2}+e x +c}\,d^{\frac{5}{2}} a e\,x^{3}-8 \sqrt{d\,x^{2}+e x +c}\,d^{\frac{5}{2}} b c\,x^{3}+4 d^{\frac{3}{2}} c^{\frac{3}{2}} \ln \left(\frac{2 c +e x +2 \sqrt{c}\,\sqrt{\cdots}}{x}\right.\right.}{\cdots}$

input
```
int(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(d*x^2+e*x+c)^(1/2)*(a*e*x+4*b*c*x+2*a*c)/x^2/c*((b*x+a)^2)^(1/2)/(b*
x+a)+1/8/c*(8*b*c*d^(1/2)*ln((1/2*e+d*x)/d^(1/2)+(d*x^2+e*x+c)^(1/2))-(4*a
*c*d-a*e^2+4*b*c*e)/c^(1/2)*ln((2*c+e*x+2*c^(1/2)*(d*x^2+e*x+c)^(1/2))/x))
*((b*x+a)^2)^(1/2)/(b*x+a)
```

3.51.5 Fricas [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 693, normalized size of antiderivative = 3.22

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$$
$$= \Bigg[\frac{8\,bc^2\sqrt{d}x^2\log\left(8\,d^2x^2+8\,dex+4\,\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{d}+4\,cd+e^2\right)-(4\,acd+4\,bce-ae^2)\sqrt{c}x^2}{16\,c^2x^2}$$
$$-\frac{16\,bc^2\sqrt{-d}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)+(4\,acd+4\,bce-ae^2)\sqrt{c}x^2\log\left(\frac{8\,cex+(4\,cd+e^2)x^2+4\,\sqrt{dx^2+ex+}}{x^2}\right.}{16\,c^2x^2}$$
$$-\frac{8\,bc^2\sqrt{-d}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(2\,dx+e)\sqrt{-d}}{2\,(d^2x^2+dex+cd)}\right)-(4\,acd+4\,bce-ae^2)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{dx^2+ex+c}(ex+2\,c)\sqrt{-c}}{2\,(cdx^2+cex+c^2)}\right)}{8\,c^2x^2}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/16*(8*b*c^2*sqrt(d)*x^2*log(8*d^2*x^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x +
c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(c
)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e*x + c)*(e*x + 2*
c)*sqrt(c) + 8*c^2)/x^2) - 4*(2*a*c^2 + (4*b*c^2 + a*c*e)*x)*sqrt(d*x^2 +
e*x + c))/(c^2*x^2), -1/16*(16*b*c^2*sqrt(-d)*x^2*arctan(1/2*sqrt(d*x^2 +
e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)) + (4*a*c*d + 4*b*c*
e - a*e^2)*sqrt(c)*x^2*log((8*c*e*x + (4*c*d + e^2)*x^2 + 4*sqrt(d*x^2 + e
*x + c)*(e*x + 2*c)*sqrt(c) + 8*c^2)/x^2) + 4*(2*a*c^2 + (4*b*c^2 + a*c*e)
*x)*sqrt(d*x^2 + e*x + c))/(c^2*x^2), 1/8*(4*b*c^2*sqrt(d)*x^2*log(8*d^2*x
^2 + 8*d*e*x + 4*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(d) + 4*c*d + e^2)
+ (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x + c
)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) - 2*(2*a*c^2 + (4*b*c^2 +
a*c*e)*x)*sqrt(d*x^2 + e*x + c))/(c^2*x^2), -1/8*(8*b*c^2*sqrt(-d)*x^2*arc
tan(1/2*sqrt(d*x^2 + e*x + c)*(2*d*x + e)*sqrt(-d)/(d^2*x^2 + d*e*x + c*d)
) - (4*a*c*d + 4*b*c*e - a*e^2)*sqrt(-c)*x^2*arctan(1/2*sqrt(d*x^2 + e*x +
 c)*(e*x + 2*c)*sqrt(-c)/(c*d*x^2 + c*e*x + c^2)) + 2*(2*a*c^2 + (4*b*c^2
+ a*c*e)*x)*sqrt(d*x^2 + e*x + c))/(c^2*x^2)]
```

3.51.6 Sympy [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx=\int \frac{\sqrt{c+dx^2+ex}\sqrt{(a+bx)^2}}{x^3}\,dx$$

input
```
integrate(((b*x+a)**2)**(1/2)*(d*x**2+e*x+c)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(c + d*x**2 + e*x)*sqrt((a + b*x)**2)/x**3, x)
```

3.51.7 Maxima [F]

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx=\int \frac{\sqrt{dx^2+ex+c}\sqrt{(bx+a)^2}}{x^3}\,dx$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x^2 + e*x + c)*sqrt((b*x + a)^2)/x^3, x)
```

3.51.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(160) = 320$.

Time = 0.34 (sec) , antiderivative size = 434, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx$$
$$=-b\sqrt{d}\log\left(\left|2\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)\sqrt{d}+e\right|\right)\operatorname{sgn}(bx+a)$$
$$+\frac{(4\,acd\operatorname{sgn}(bx+a)+4\,bce\operatorname{sgn}(bx+a)-ae^2\operatorname{sgn}(bx+a))\arctan\left(-\frac{\sqrt{d}x-\sqrt{dx^2+ex+c}}{\sqrt{-c}}\right)}{4\sqrt{-c}c}$$
$$+\frac{4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3acd\operatorname{sgn}(bx+a)+4\left(\sqrt{d}x-\sqrt{dx^2+ex+c}\right)^3bce\operatorname{sgn}(bx+a)+\left(\sqrt{d}x-\sqrt{\cdots}\right.}{\cdots}$$

input
```
integrate(((b*x+a)^2)^(1/2)*(d*x^2+e*x+c)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-b*sqrt(d)*log(abs(2*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*sqrt(d) + e))*sgn
(b*x + a) + 1/4*(4*a*c*d*sgn(b*x + a) + 4*b*c*e*sgn(b*x + a) - a*e^2*sgn(b
*x + a))*arctan(-(sqrt(d)*x - sqrt(d*x^2 + e*x + c))/sqrt(-c))/(sqrt(-c)*c
) + 1/4*(4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))^3*a*c*d*sgn(b*x + a) + 4*(s
qrt(d)*x - sqrt(d*x^2 + e*x + c))^3*b*c*e*sgn(b*x + a) + (sqrt(d)*x - sqrt
(d*x^2 + e*x + c))^3*a*e^2*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x
+ c))^2*b*c^2*sqrt(d)*sgn(b*x + a) + 8*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))
^2*a*c*sqrt(d)*e*sgn(b*x + a) + 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c^
2*d*sgn(b*x + a) - 4*(sqrt(d)*x - sqrt(d*x^2 + e*x + c))*b*c^2*e*sgn(b*x +
 a) + (sqrt(d)*x - sqrt(d*x^2 + e*x + c))*a*c*e^2*sgn(b*x + a) - 8*b*c^3*s
qrt(d)*sgn(b*x + a))/(((sqrt(d)*x - sqrt(d*x^2 + e*x + c))^2 - c)^2*c)
```

3.51.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a^2+2abx+b^2x^2}\sqrt{c+ex+dx^2}}{x^3}\,dx=\int \frac{\sqrt{(a+b\,x)^2}\,\sqrt{d\,x^2+e\,x+c}}{x^3}\,dx$$

input
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3,x)
```

output
```
int((((a + b*x)^2)^(1/2)*(c + e*x + d*x^2)^(1/2))/x^3, x)
```

3.52 $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

3.52.1 Optimal result

Integrand size = 27, antiderivative size = 452

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = -\frac{(2e-fx)\sqrt{a+cx^2}}{2f^2}+\frac{(af^2+2c(e^2-df))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2\sqrt{c}f^3}$$

$$-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2df(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{\cdots}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2df(af^2+c(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{\cdots}}\right)}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
1/2*(a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^3/c^(1/2)-
1/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e
^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(
1/2)))^(1/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4
*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(
-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)
))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1
/2))*(-2*d*f*(a*f^2+c*(-d*f+e^2))+e*(a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)
^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2)))^(1/2)
```

3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.36

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{f(-2e+fx)\sqrt{a+cx^2}+\frac{2(af^2+2c(e^2-df))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}}-2\text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(f*(-2*e + f*x)*Sqrt[a + c*x^2] + (2*(a*f^2 + 2*c*(e^2 - d*f))*ArcTanh[(Sq
rt[c]*x)/(-Sqrt[a] + Sqrt[a + c*x^2])])/Sqrt[c] - 2*RootSum[c^2*d + 2*Sqrt
[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c^2*
d*e^2*Log[x] - c^2*d^2*f*Log[x] + a*c*d*f^2*Log[x] - c^2*d*e^2*Log[-Sqrt[a
] + Sqrt[a + c*x^2] - x*#1] + c^2*d^2*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x
*#1] - a*c*d*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*Sqrt[a]*c*e^3*
Log[x]*#1 - 4*Sqrt[a]*c*d*e*f*Log[x]*#1 + 2*a^(3/2)*e*f^2*Log[x]*#1 - 2*Sq
rt[a]*c*e^3*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + 4*Sqrt[a]*c*d*e*f*
Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - 2*a^(3/2)*e*f^2*Log[-Sqrt[a] +
 Sqrt[a + c*x^2] - x*#1]*#1 - c*d*e^2*Log[x]*#1^2 + c*d^2*f*Log[x]*#1^2 -
a*d*f^2*Log[x]*#1^2 + c*d*e^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2
- c*d^2*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 + a*d*f^2*Log[-Sqrt[
a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 - 4*a*f*#1 +
 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/(2*f^3)
```

3.52.3 Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {2139, 2145, 27, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{2139}$$

$$-\frac{\int \frac{-c\left(af^2+2c\left(e^2-df\right)\right)x^2-ce(2cd-af)x+acdf}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{2145}$$

$$-\frac{\frac{\int \frac{2c\left(d\left(af^2+c\left(e^2-df\right)\right)+e\left(af^2+c\left(e^2-2df\right)\right)x\right)}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(af^2+2c\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2c\int \frac{d\left(af^2+c\left(e^2-df\right)\right)+e\left(af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(af^2+2c\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{224}$$

$$-\frac{\frac{2c\int \frac{d\left(af^2+c\left(e^2-df\right)\right)+e\left(af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(af^2+2c\left(e^2-df\right)\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{2c\int \frac{d\left(af^2+c\left(e^2-df\right)\right)+e\left(af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(af^2+2c\left(e^2-df\right)\right)}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{1367}$$

$$-\frac{\frac{2c\left(\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-\right.\right.\right.}{\sqrt{e^2-4df}}}{f}}{2cf^2}$$

$$\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \text{488}$$

3.52. $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

$$-\frac{\frac{2c\left(\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}-\cdots\right)}{f}\cdots}{\cdots}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{2c\left(\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-2df\right)\right)-2df\left(af^2+c\left(e^2-df\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(\cdots\right.\right.\right.}{\cdots}\right)}{f}\cdots}{2cf^2}-\frac{\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

input
```
Int[(x^2*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
-1/2*((2*e - f*x)*Sqrt[a + c*x^2])/f^2 - (-((Sqrt[c]*(a*f^2 + 2*c*(e^2 - d
*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f) + (2*c*(((e*(e - Sqrt[e^2 -
4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcTanh
[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e*(e + Sqrt[e^2
 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*d*f*(a*f^2 + c*(e^2 - d*f)))*ArcT
anh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*
f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/f)/(2*c*f^2)
```

3.52. $\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

3.52.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2139
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*((-c)*e*(2*p + q + 2)) + 2*c*C*f
*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q +
1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   I
nt[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1)
 - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p +
 q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q
 + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x
+ (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*
(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2
*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, c, d, e, f, q}, x] &
& PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0
] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.52.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 841 vs. $2(403) = 806$.

Time = 0.82 (sec) , antiderivative size = 842, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{(-fx+2e)\sqrt{c\,x^2+a}}{2f^2}+\frac{\frac{\left(a\,f^2-2cdf+2c\,e^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f\sqrt{c}}-\frac{\left(-2ae\,f^2\sqrt{-4df+e^2}+4cdef\sqrt{-4df+e^2}-2c\,e^3\sqrt{-4df+e^2}-4ad\,f^3+2a\,e^2f\right.}{}}{}$
default	Expression too large to display

input
```
int(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2+1/2/f^2*(1/f*(a*f^2-2*c*d*f+2*c*e^2)*l
n(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-2*a*e*f^2*(-4*d*f+e^2)^(1/2)+4*
c*d*e*f*(-4*d*f+e^2)^(1/2)-2*c*e^3*(-4*d*f+e^2)^(1/2)-4*a*d*f^3+2*a*e^2*f^
2+4*c*d^2*f^2-8*c*d*e^2*f+2*c*e^4)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f
+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/
2*(-2*a*e*f^2*(-4*d*f+e^2)^(1/2)+4*c*d*e*f*(-4*d*f+e^2)^(1/2)-2*c*e^3*(-4*
d*f+e^2)^(1/2)+4*a*d*f^3-2*a*e^2*f^2-4*c*d^2*f^2+8*c*d*e^2*f-2*c*e^4)/f^2/
(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-
4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)))
```

3.52.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.52.6 Sympy [F]

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate(x**2*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**2*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.52.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.52.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.52.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

output
```
int((x^2*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.53 $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

3.53.1 Optimal result

Integrand size = 25, antiderivative size = 395

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{\sqrt{a+cx^2}}{f} - \frac{\sqrt{c}e\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2} - \frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\arctan\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\left(2cdef-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\arctan\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
-e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^2+(c*x^2+a)^(1/2)/f-1/2*ar
ctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*
f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e
^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f
+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2)))^(1/2))*(2*c*d*e*f-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^
2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^
(1/2)
```

3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.34 (sec) , antiderivative size = 379, normalized size of antiderivative = 0.96

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{f\sqrt{a+cx^2}+\sqrt{c}e\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)-\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1\right.}{}$$

input
```
Integrate[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(f*Sqrt[a + c*x^2] + Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] - RootS
um[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 +
 f*#1^4 & , (a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*c*d*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a
 + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#
1 - c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2
*f*#1^3) & ])/f^2
```

3.53.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.01, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1353, 25, 2027, 2145, 25, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$
$$\Big\downarrow \; 1353$$
$$\frac{\int -\frac{cex^2+(cd-af)x}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}+\frac{\sqrt{a+cx^2}}{f}$$
$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\int \frac{cex^2+(cd-af)x}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 2027$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\int \frac{x(cd-af+cex)}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 2145$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{\int -\frac{cde+\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{ce\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{ce\int \frac{1}{\sqrt{cx^2+a}}dx}{f}-\frac{\int \frac{cde+\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{ce\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}-\frac{\int \frac{cde+\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\int \frac{cde+\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}}{f}$$

$$\Big\downarrow \; 1367$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\frac{\left(2cdef-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\int}{\sqrt{e^2-4df}}}{f}}{f}$$

$$\Big\downarrow \; 488$$

3.53. $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}\cdots}{f}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{a+cx^2}}{f}-\frac{\frac{\sqrt{c}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\frac{\left(2cdef-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(2cdef-\left(e-\sqrt{e^2-4\cdots}\right.\right.}{\cdots}}{f}}{f}$$

input
```
Int[(x*Sqrt[a + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
Sqrt[a + c*x^2]/f - ((Sqrt[c]*e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f -
(-(((2*c*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[
(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d
*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + ((2*c*d*e*f - (e
+ Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))/f)/f
```

3.53.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

3.53. $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1353
```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Simp[1/(2*f*(p + q + 1))   Int[(a + c*x^2)^(p
- 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p
*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
 FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] &&
NeQ[p + q + 1, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.53. $\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

3.53.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(350) = 700$.

Time = 0.79 (sec) , antiderivative size = 769, normalized size of antiderivative = 1.95

method	result
risch	$\frac{\sqrt{c\,x^2+a}}{f}-\frac{\frac{\sqrt{c}\,e\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f}-\frac{\left(-a\,f^2\sqrt{-4df+e^2}+cdf\sqrt{-4df+e^2}-c\,e^2\sqrt{-4df+e^2}+ae\,f^2-3cdef+c\,e^3\right)\sqrt{2}\,\ln\left(\frac{\frac{-\sqrt{-4df+e^2}\,ce+2a\,f^2}{f^2}}{}\right.}{}}{}$
default	Expression too large to display

input
```
int(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/f-1/f*(c^(1/2)*e/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-1/2*(-a*f
^2*(-4*d*f+e^2)^(1/2)+c*d*f*(-4*d*f+e^2)^(1/2)-c*e^2*(-4*d*f+e^2)^(1/2)+a*
e*f^2-3*c*d*e*f+c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(-a*f^2*(-
4*d*f+e^2)^(1/2)+c*d*f*(-4*d*f+e^2)^(1/2)-c*e^2*(-4*d*f+e^2)^(1/2)-a*e*f^2
+3*c*d*e*f-c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

3.53.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.53.6 Sympy [F]

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate(x*(c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x*sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.53.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.53.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.53.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\,\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

output
```
int((x*(a + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.54 $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} \, dx$

3.54.1 Optimal result

Integrand size = 24, antiderivative size = 298

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2} \, dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f} - \frac{\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\,\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}} + \frac{\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\,\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1/2*(2*a*f-c*x*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*
d*f+e^2)^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)
/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(
1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))
)^(1/2))*(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*
d*f+e^2)^(1/2)
```

3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \frac{-\sqrt{c}\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)+\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(-(Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]) + RootSum[a^2*f + 2*a*Sqrt
[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e*
Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*d*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]*#1 - c*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]
*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/f
```

3.54.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1322, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \ 1322$$

$$\frac{c\int \frac{1}{\sqrt{cx^2+a}}dx}{f}-\frac{\int \frac{cd-af+cex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \ 224$$

$$\frac{c\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}-\frac{\int \frac{cd-af+cex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\int \frac{cd-af+cex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$\Big\downarrow$ 1367

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{-\frac{\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2f(cd-af)-ce\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}}{f}$$

$\Big\downarrow$ 488

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\frac{\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}+\frac{\left(2f(cd-af)-ce\left(\sqrt{e^2-4df}+e\right.\right.}{}}{f}$$

$\Big\downarrow$ 219

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f}-\frac{\frac{\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}}+\frac{\left(2f(cd-af)-ce\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e\right.}}}}{f}$$

input
```
Int[Sqrt[a + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f - ((Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f]
)*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a
 + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]) + ((2*f*(c*d - a*f) - c*e*(e + Sq
rt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(S
qrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]))/f
```

3.54. $\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$

3.54.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1322
```
Int[Sqrt[(a_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^2), x_Symbol]
 :> Simp[c/f   Int[1/Sqrt[a + c*x^2], x], x] - Simp[1/f   Int[(c*d - a*f +
c*e*x)/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)), x], x] /; FreeQ[{a, c, d, e, f}
, x] && NeQ[e^2 - 4*d*f, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.54.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. 2(259) = 518.

Time = 0.74 (sec) , antiderivative size = 1212, normalized size of antiderivative = 4.07

method	result	size
default	Expression too large to display	1212

input
```
int((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e+(-4*d*f+e^2)^(1/2)
)/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
)/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2))-1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^
2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e-(-4*d
*f+e^2)^(1/2))/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(x-1/2/f*(-e+(-4*d*
f+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*...
```

3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(257) = 514$.

Time = 138.40 (sec) , antiderivative size = 2384, normalized size of antiderivative = 8.00

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt
(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a
*c*e^2 + sqrt(2)*(c*e^3 - 4*c*d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(
e^2*f^4 - 4*d*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^
2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) +
 2*(a*e^2*f^2 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) - sqrt(2)
*f*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2
*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 - sqrt
(2)*(c*e^3 - 4*c*d*e*f - (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d
*f^5)))*sqrt(c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 + (e^2*f^2 - 4*d*f
^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) + 2*(a*e^2*f^2
 - 4*a*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) + sqrt(2)*f*sqrt((c*e^
2 - 2*c*d*f + 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^
5)))/(e^2*f^2 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 + sqrt(2)*(c*e^3 -
4*c*d*e*f + (e^3*f^2 - 4*d*e*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))*sqrt(
c*x^2 + a)*sqrt((c*e^2 - 2*c*d*f + 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*
e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2 - 4*d*f^3)) - 2*(a*e^2*f^2 - 4*a*d*f^3)
*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/x) - sqrt(2)*f*sqrt((c*e^2 - 2*c*d*f +
 2*a*f^2 - (e^2*f^2 - 4*d*f^3)*sqrt(c^2*e^2/(e^2*f^4 - 4*d*f^5)))/(e^2*f^2
 - 4*d*f^3))*log((4*c^2*d*e*x - 2*a*c*e^2 - sqrt(2)*(c*e^3 - 4*c*d*e*f ...
```

3.54.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(d + e*x + f*x**2), x)
```

3.54.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.54.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.54.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{c\,x^2+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.55 $\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx$

3.55.1 Optimal result

Integrand size = 27, antiderivative size = 358

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \frac{\left(2aef+(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(2aef+(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} - \frac{\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

output
```
-arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*arctanh(1/2*(2*a*f-c*x*(e-
(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d
*f+e^2)^(1/2)))^(1/2))*(2*a*e*f+(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/d*2^(1/
2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1
/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*e*f+(-a*f+c*d)*(e
+(-4*d*f+e^2)^(1/2)))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e
*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.55.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 310, normalized size of antiderivative = 0.87

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$
$$= \frac{\sqrt{a}\left(-\log(x)+\log\left(-\sqrt{a}+\sqrt{a+cx^2}\right)\right)-\text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+4af\#1^2-2\sqrt{a}e\#1^3 \right.}{}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

output
```
(Sqrt[a]*(-Log[x] + Log[-Sqrt[a] + Sqrt[a + c*x^2]]) - RootSum[c^2*d + 2*S
qrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-
(a*c*e*Log[x]) + a*c*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*Sqrt[a]*
c*d*Log[x]*#1 - 2*a^(3/2)*f*Log[x]*#1 - 2*Sqrt[a]*c*d*Log[-Sqrt[a] + Sqrt[
a + c*x^2] - x*#1]*#1 + 2*a^(3/2)*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]
*#1 + a*e*Log[x]*#1^2 - a*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(
Sqrt[a]*c*e - 2*c*d*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/d
```

3.55.3 Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{\sqrt{a+cx^2}(-e-fx)}{d\left(d+ex+fx^2\right)}+\frac{\sqrt{a+cx^2}}{dx}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)(cd-af)+2aef\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\left(\sqrt{e^2-4df}+e\right)(cd-af)+2aef\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

input
```
Int[Sqrt[a + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

output
```
((2*a*e*f + (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - S
qrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((2*a*e*f + (c*d - a*f)*(e + Sqrt
[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqr
t[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*
f])]) - (Sqrt[a]*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d
```

3.55.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.55.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1315 vs. $2(315) = 630$.

Time = 0.68 (sec) , antiderivative size = 1316, normalized size of antiderivative = 3.68

method	result	size
default	Expression too large to display	1316

input
```
int((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*((c*x^2+a)^(1/2)-a^(1/
2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d
*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2*c^(1/2)*(e+(-4*d*f+e^2)^(1/2))/f*ln((
-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2
)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2))-1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*2^(1
/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/
2*c^(1/2)*(e-(-4*d*f+e^2)^(1/2))/f*ln((-1/2*c*(e-(-4*d*f+e^2)^(1/2))/f+c*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))/c^(1/2)+((x-1/2/f*(-e+(-4*d*f+e^2)^(1...
```

3.55.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. $2(313) = 626$.

Time = 24.10 (sec) , antiderivative size = 2266, normalized size of antiderivative = 6.33

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
[-1/4*(sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqr
t(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^
2*e^2 + sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt
(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2
*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*
sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 -
2*a*d*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2
- 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(
a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d
*f + (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d
^3*f)) - (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/x) - s
qrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^
2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f))*log((2*a*c*d*e*x - a^2*e^2 +
sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 +
 a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^
4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*f)) + (a*d^2*e^2 - 4*a*d^3*f)*sqrt(a^2
*e^2/(d^4*e^2 - 4*d^5*f)))/x) + sqrt(2)*d*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f
- (d^2*e^2 - 4*d^3*f)*sqrt(a^2*e^2/(d^4*e^2 - 4*d^5*f)))/(d^2*e^2 - 4*d^3*
f))*log((2*a*c*d*e*x - a^2*e^2 - sqrt(2)*(d^3*e^2 - 4*d^4*f)*sqrt(a^2*e^2/
(d^4*e^2 - 4*d^5*f))*sqrt(c*x^2 + a)*sqrt((2*c*d^2 + a*e^2 - 2*a*d*f - ...
```

3.55.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{a+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

3.55.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x), x)
```

3.55.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.55.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

3.56 $\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx$

3.56.1 Optimal result

Integrand size = 27, antiderivative size = 382

$$\begin{aligned}
&\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx \\
&= -\frac{\sqrt{a+cx^2}}{dx} \\
&\quad - \frac{f\left(2cd^2+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&\quad + \frac{f\left(2cd^2+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} \\
&\quad + \frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}
\end{aligned}$$

output
```
e*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^(1/2)/d^2-(c*x^2+a)^(1/2)/d/x-1/2*f*a
rctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a
*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d^2+a*(e^2-2*d*f+e*(-
4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*
(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1
/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))
^(1/2))*(2*c*d^2+a*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e
^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.56.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.22

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx =$$

$$-\frac{d\sqrt{a+cx^2}-\sqrt{a}ex\log(x)+\sqrt{a}ex\log\left(-\sqrt{a}+\sqrt{a+cx^2}\right)+x\text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

output
```
-((d*Sqrt[a + c*x^2] - Sqrt[a]*e*x*Log[x] + Sqrt[a]*e*x*Log[-Sqrt[a] + Sqr
t[a + c*x^2]] + x*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1
^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d^2*Log[x]) - a*c*e^2*Log[x] + a
*c*d*f*Log[x] + c^2*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + a*c*e^2*L
og[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - a*c*d*f*Log[-Sqrt[a] + Sqrt[a + c*
x^2] - x*#1] - 2*a^(3/2)*e*f*Log[x]*#1 + 2*a^(3/2)*e*f*Log[-Sqrt[a] + Sqrt
[a + c*x^2] - x*#1]*#1 + c*d^2*Log[x]*#1^2 + a*e^2*Log[x]*#1^2 - a*d*f*Log
[x]*#1^2 - c*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 - a*e^2*Log[-
Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 + a*d*f*Log[-Sqrt[a] + Sqrt[a + c*x
^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2
 - 2*d*#1^3) & ])/(d^2*x))
```

3.56.3 Rubi [A] (verified)

Time = 1.45 (sec) , antiderivative size = 382, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^2 (d+ex+fx^2)} \, dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{\sqrt{a+cx^2} (-df+e^2+efx)}{d^2 (d+ex+fx^2)}-\frac{e\sqrt{a+cx^2}}{d^2x}+\frac{\sqrt{a+cx^2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)+2cd^2\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\sqrt{a}e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-\frac{\sqrt{a+cx^2}}{dx}$$

input
```
Int[Sqrt[a + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

output
```
-(Sqrt[a + c*x^2]/(d*x)) - (f*(2*c*d^2 + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*S
qrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) +
(f*(2*c*d^2 + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e
 + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f
^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) + (Sqrt[a]*e*ArcTanh[Sqrt[a +
 c*x^2]/Sqrt[a]])/d^2
```

3.56.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.56.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 775 vs. $2(337) = 674$.

Time = 0.76 (sec) , antiderivative size = 776, normalized size of antiderivative = 2.03

method	result
risch	$-\frac{\sqrt{c x^2+a}}{d x} - \frac{\frac{4 f \sqrt{a}\, e \ln\left(\frac{2a+2\sqrt{a}\sqrt{c x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)} - \frac{\left(f a\sqrt{-4df+e^2} - c d\sqrt{-4df+e^2}+a e f+c d e\right)\sqrt{2}\,\ln\left(\frac{\frac{-\sqrt{-4df+e^2}\,c e+2a f^2-2c d f+c e^2}{f^2} - \cdots}{\cdots}\right)}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+a)^(1/2)/d/x-1/d*(4*f*a^(1/2)*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f
+e^2)^(1/2))*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-(f*a*(-4*d*f+e^2)^(1/2)
-c*d*(-4*d*f+e^2)^(1/2)+a*e*f+c*d*e)/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(
1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^
(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-
4*d*f+e^2)^(1/2))))+(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2)^(1/2)-a*e*f-c
*d*e)/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(
1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

3.56.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2656 vs. $2(335) = 670$.

Time = 176.36 (sec) , antiderivative size = 5324, normalized size of antiderivative = 13.94

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.56.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**2*(d + e*x + f*x**2)), x)
```

3.56.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^2), x)
```

3.56.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.56.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^2\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.57 $\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} \, dx$

3.57.1 Optimal result

Integrand size = 27, antiderivative size = 507

$$\int \frac{\sqrt{a+cx^2}}{x^3(d+ex+fx^2)} \, dx = -\frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x}$$
$$+ \frac{f\left(cd^2\left(e+\sqrt{e^2-4df}\right)+a\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$- \frac{f\left(cd^2\left(e-\sqrt{e^2-4df}\right)+a\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$- \frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d} - \frac{\sqrt{a}(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

output
```
-1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)-(-d*f+e^2)*arctanh((c*x^
2+a)^(1/2)/a^(1/2))*a^(1/2)/d^3-1/2*(c*x^2+a)^(1/2)/d/x^2+e*(c*x^2+a)^(1/2
)/d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^
2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c*d^2*(e+(
-4*d*f+e^2)^(1/2))+a*(e^3-3*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^
(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2
)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/
2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c*
d^2*(e-(-4*d*f+e^2)^(1/2))+a*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d
*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4
*d*f+e^2)^(1/2)))^(1/2)
```

3.57.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.79 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{a+cx^2}}{x^3 (d+ex+fx^2)}\,dx$$
$$= \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{x^2} + \frac{2cd^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - 4\sqrt{a}(e^2-df)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+cx^2}}{\sqrt{a}}\right) - 2\text{RootSum}\left[a^2f+2\right.}{}$$

input
```
Integrate[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

output
```
((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 + (2*c*d^2*ArcTanh[(Sqrt[c]*x - Sqrt
[a + c*x^2])/Sqrt[a]])/Sqrt[a] - 4*Sqrt[a]*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + c*x^2])/Sqrt[a]] - 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d^2*f*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]) - a^2*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1] + a^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(3/
2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*e^3*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*Sqrt[c]*d*e*f*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1]*#1 + c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]*#1^2 + a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*d
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#
1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(2*d^3)
```

3.57.3 Rubi [A] (verified)

Time = 1.78 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+cx^2}}{x^3\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{\sqrt{a+cx^2}\left(e^2-df\right)}{d^3x}+\frac{\sqrt{a+cx^2}\left(-fx\left(e^2-df\right)-e\left(e^2-2df\right)\right)}{d^3\left(d+ex+fx^2\right)}-\frac{e\sqrt{a+cx^2}}{d^2x^2}+\frac{\sqrt{a+cx^2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{a}\left(e^2-df\right)\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+$$

$$\frac{f\left(a\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{f\left(a\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+cd^2\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-\right.}}\right.}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{a}d}+\frac{e\sqrt{a+cx^2}}{d^2x}-\frac{\sqrt{a+cx^2}}{2dx^2}$$

input
```
Int[Sqrt[a + c*x^2]/(x^3*(d + e*x + f*x^2)),x]
```

output
```
-1/2*Sqrt[a + c*x^2]/(d*x^2) + (e*Sqrt[a + c*x^2])/(d^2*x) + (f*(c*d^2*(e
+ Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt
[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqr
t[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*
d*f])]) - (f*(c*d^2*(e - Sqrt[e^2 - 4*d*f]) + a*(e^3 - 3*d*e*f - e^2*Sqrt[
e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 -
4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*
Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 -
2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*S
qrt[a]*d) - (Sqrt[a]*(e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

3.57.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.57.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 789, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2d^2x^2}-\frac{\frac{4f\left(2adf-2e^2a-c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{2f\left(\sqrt{-4df+e^2}\,ae-2adf+e^2a+2c\,d^2\right)\sqrt{2}\,\ln\left(\frac{-\sqrt{-4df+e^2}\,ce+2a\,f}{f^2}\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(4*f*(2*a*d*f-2*a*e^2-c*d^
2)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1/2
)*(c*x^2+a)^(1/2))/x)+2*f*((-4*d*f+e^2)^(1/2)*a*e-2*a*d*f+e^2*a+2*c*d^2)/(
-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/
2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-2*f*((-4*d*f+e^2)^
(1/2)*a*e+2*a*d*f-e^2*a-2*c*d^2)/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)))
```

3.57.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.57.6 Sympy [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx$$

input
```
integrate((c*x**2+a)**(1/2)/x**3/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + c*x**2)/(x**3*(d + e*x + f*x**2)), x)
```

3.57.7 Maxima [F]

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+a}}{(fx^2+ex+d)x^3}\,dx$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + a)/((f*x^2 + e*x + d)*x^3), x)
```

3.57.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+a)^(1/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.57.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+cx^2}}{x^3\,(d+ex+fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+a}}{x^3\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(1/2)/(x^3*(d + e*x + f*x^2)), x)
```

3.58 $\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.58.1 Optimal result

Integrand size = 27, antiderivative size = 795

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{(8e(af^2+c(e^2-2df))-f(3af^2+4c(e^2-df))x)\sqrt{a+cx^2}}{8f^4}$$

$$-\frac{(4e-3fx)(a+cx^2)^{3/2}}{12f^2}$$

$$+\frac{(3a^2f^4+12acf^2(e^2-df)+8c^2(e^4-3de^2f+d^2f^2))\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{8\sqrt{c}f^5}$$

$$-\frac{\left(a^2f^4\left(e^2-2df-e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)+c^2\left(e^6-\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4a}}$$

$$+\frac{\left(a^2f^4\left(e^2-2df+e\sqrt{e^2-4df}\right)+2acf^2\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+c^2\left(e^6-\right.\right.}{\sqrt{2}f^5\sqrt{e^2-4dj}}$$

output
```
-1/12*(-3*f*x+4*e)*(c*x^2+a)^(3/2)/f^2+1/8*(3*a^2*f^4+12*a*c*f^2*(-d*f+e^2
)+8*c^2*(d^2*f^2-3*d*e^2*f+e^4))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^5/c^
(1/2)-1/8*(8*e*(a*f^2+c*(-2*d*f+e^2))-f*(3*a*f^2+4*c*(-d*f+e^2))*x)*(c*x^2
+a)^(1/2)/f^4-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(
c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(a^2*f^
4*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))+2*a*c*f^2*(e^4-4*d*e^2*f+2*d^2*f^2-e^3*
(-4*d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e^6-6*d*e^4*f+9*d^2*e^
2*f^2-2*d^3*f^3-e^5*(-4*d*f+e^2)^(1/2)+4*d*e^3*f*(-4*d*f+e^2)^(1/2)-3*d^2*
e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-
2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e
^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(
1/2)))^(1/2))*(a^2*f^4*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+2*a*c*f^2*(e^4-4*d
*e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/2))+c^2*(e
^6-6*d*e^4*f+9*d^2*e^2*f^2-2*d^3*f^3+e^5*(-4*d*f+e^2)^(1/2)-4*d*e^3*f*(-4*
d*f+e^2)^(1/2)+3*d^2*e*f^2*(-4*d*f+e^2)^(1/2)))/f^5*2^(1/2)/(-4*d*f+e^2)^(
1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.58.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.82 (sec) , antiderivative size = 1239, normalized size of antiderivative = 1.56

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{f\sqrt{a+cx^2}(af^2(-32e+15fx)-2c(12e^3-6e^2fx+4ef(-6d+fx^2)-3f^2x(-2d+}{}$$

input
```
Integrate[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
(f*Sqrt[a + c*x^2]*(a*f^2*(-32*e + 15*f*x) - 2*c*(12*e^3 - 6*e^2*f*x + 4*e
*f*(-6*d + f*x^2) - 3*f^2*x*(-2*d + f*x^2))) + (6*(3*a^2*f^4 + 12*a*c*f^2*
(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*f^2))*ArcTanh[(Sqrt[c]*x)/(-Sqr
t[a] + Sqrt[a + c*x^2])])/Sqrt[c] + 24*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1 -
2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^3*d*e^4*Log[x
]) + 3*c^3*d^2*e^2*f*Log[x] - c^3*d^3*f^2*Log[x] - 2*a*c^2*d*e^2*f^2*Log[x
] + 2*a*c^2*d^2*f^3*Log[x] - a^2*c*d*f^4*Log[x] + c^3*d*e^4*Log[-Sqrt[a] +
 Sqrt[a + c*x^2] - x*#1] - 3*c^3*d^2*e^2*f*Log[-Sqrt[a] + Sqrt[a + c*x^2]
- x*#1] + c^3*d^3*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*a*c^2*d*e
^2*f^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - 2*a*c^2*d^2*f^3*Log[-Sqrt[
a] + Sqrt[a + c*x^2] - x*#1] + a^2*c*d*f^4*Log[-Sqrt[a] + Sqrt[a + c*x^2]
- x*#1] - 2*Sqrt[a]*c^2*e^5*Log[x]*#1 + 8*Sqrt[a]*c^2*d*e^3*f*Log[x]*#1 -
6*Sqrt[a]*c^2*d^2*e*f^2*Log[x]*#1 - 4*a^(3/2)*c*e^3*f^2*Log[x]*#1 + 8*a^(3
/2)*c*d*e*f^3*Log[x]*#1 - 2*a^(5/2)*e*f^4*Log[x]*#1 + 2*Sqrt[a]*c^2*e^5*Lo
g[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - 8*Sqrt[a]*c^2*d*e^3*f*Log[-Sqrt[
a] + Sqrt[a + c*x^2] - x*#1]*#1 + 6*Sqrt[a]*c^2*d^2*e*f^2*Log[-Sqrt[a] + S
qrt[a + c*x^2] - x*#1]*#1 + 4*a^(3/2)*c*e^3*f^2*Log[-Sqrt[a] + Sqrt[a + c*
x^2] - x*#1]*#1 - 8*a^(3/2)*c*d*e*f^3*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#
1]*#1 + 2*a^(5/2)*e*f^4*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + c^2*d*
e^4*Log[x]*#1^2 - 3*c^2*d^2*e^2*f*Log[x]*#1^2 + c^2*d^3*f^2*Log[x]*#1^2...
```

3.58.3 Rubi [A] (verified)

Time = 1.56 (sec) , antiderivative size = 656, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.407, Rules used = {2139, 27, 2139, 25, 2145, 27, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 (a+cx^2)^{3/2}}{d+ex+fx^2} \, dx$$

$$\Big\downarrow \text{2139}$$

$$-\frac{\int \frac{3\sqrt{cx^2+a}(-c(3af^2+4c(e^2-df))x^2-ce(4cd-af)x+acdf)}{fx^2+ex+d}dx}{12cf^2}-\frac{(a+cx^2)^{3/2}(4e-3fx)}{12f^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{cx^2+a}(-c(3af^2+4c(e^2-df))x^2-ce(4cd-af)x+acdf)}{fx^2+ex+d}dx}{4cf^2}-\frac{(a+cx^2)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 2139

$$-\frac{\frac{c\sqrt{a+cx^2}\left(8e\left(af^2+c\left(e^2-2df\right)\right)-fx\left(3af^2+4c\left(e^2-df\right)\right)\right)}{2f^2}-\frac{\int -\frac{-\left(\left(3a^2f^4+12ac\left(e^2-df\right)f^2+8c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x^2c^2\right)+adf\left(5af^2+4c\left(e^2-df\right)\right)c^2 \cdots}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}}{2cf^2}}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 25

$$-\frac{\frac{\int \frac{-\left(\left(3a^2f^4+12ac\left(e^2-df\right)f^2+8c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x^2c^2\right)+adf\left(5af^2+4c\left(e^2-df\right)\right)c^2+e\left(5a^2f^3+4ac\left(e^2-5df\right)f-8c^2d\left(e^2-2df\right)\right)xc^2}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{2cf^2}+\frac{c\sqrt{a+cx^2}(8 \cdots}{\cdots}}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 2145

$$-\frac{\frac{\frac{\int \frac{8c^2\left(d\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)+e\left(ce^2+af^2-3cdf\right)\left(ce^2+af^2-cdf\right)x\right)}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2\left(3a^2f^4+12acf^2\left(e^2-df\right)+8c^2\left(d^2f^2-3de^2f+e^4\right)\right)\int \frac{\cdots}{\sqrt{\cdots}}}{f}}{2cf^2} \cdots}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{8c^2\int \frac{d\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)+e\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2\left(3a^2f^4+12acf^2\left(e^2-df\right)+8c^2\left(d^2f^2-3de^2f+e^4\right)\right)\int \frac{\cdots}{\sqrt{\cdots}}}{f}}{2cf^2} \cdots}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 224

$$-\frac{\frac{\frac{8c^2\int \frac{d\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)+e\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2\left(3a^2f^4+12acf^2\left(e^2-df\right)+8c^2\left(d^2f^2-3de^2f+e^4\right)\right)\int \frac{\cdots}{1 \cdots}}{f}}{2cf^2} \cdots}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\Big\downarrow$ 219

3.58. $\displaystyle\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$-\frac{\frac{\frac{8c^2\int \frac{d\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)+e\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3a^2f^4+12acf^2\left(e^2-df\right)+8c\right.}{f}}{2cf^2}}{4cf^2}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\big\downarrow$ 1367

$$-\frac{\frac{8c^2\left(\frac{\left(e\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)-2df\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(e\left(e-\right.\right.}{}\right.}{f}}{}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\big\downarrow$ 488

$$-\frac{8c^2\left(\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)-2df\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{\right.\right.}{cx^2}}}{\sqrt{e^2-4df}}\right.}{}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

$\big\downarrow$ 219

$$-\frac{8c^2\left(\frac{\left(e\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-3df\right)\right)\left(af^2+c\left(e^2-df\right)\right)-2df\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2\right.}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right.}{}$$

$$\frac{\left(a+cx^2\right)^{3/2}(4e-3fx)}{12f^2}$$

3.58. $\int \frac{x^2\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

input
```
Int[(x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
-1/12*((4*e - 3*f*x)*(a + c*x^2)^(3/2))/f^2 - ((c*(8*e*(a*f^2 + c*(e^2 - 2
*d*f)) - f*(3*a*f^2 + 4*c*(e^2 - d*f))*x)*Sqrt[a + c*x^2])/(2*f^2) + (-((c
^(3/2)*(3*a^2*f^4 + 12*a*c*f^2*(e^2 - d*f) + 8*c^2*(e^4 - 3*d*e^2*f + d^2*
f^2))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f) + (8*c^2*(((e*(e - Sqrt[e^2
 - 4*d*f])*(a*f^2 + c*(e^2 - 3*d*f))*(a*f^2 + c*(e^2 - d*f)) - 2*d*f*(a^2*
f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2
*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f
 - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqr
t[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - ((e*(e + Sqrt[e^2 -
4*d*f])*(a*f^2 + c*(e^2 - 3*d*f))*(a*f^2 + c*(e^2 - d*f)) - 2*d*f*(a^2*f^4
 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*
f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]))))/f)/(2*c*f^2))/(4*c*f^2)
```

3.58.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2139
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*((-c)*e*(2*p + q + 2)) + 2*c*C*f
*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q +
1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   I
nt[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1)
 - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p +
 q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q
 + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x
+ (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*
(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2
*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, c, d, e, f, q}, x] &
& PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0
] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.58.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 1183, normalized size of antiderivative = 1.49

method	result	size
risch	Expression too large to display	1183
default	Expression too large to display	2367

input
```
int(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-6*c*f^3*x^3+8*c*e*f^2*x^2-15*a*f^3*x+12*c*d*f^2*x-12*c*e^2*f*x+32*
a*e*f^2-48*c*d*e*f+24*c*e^3)*(c*x^2+a)^(1/2)/f^4+1/8/f^4*(1/f*(3*a^2*f^4-1
2*a*c*d*f^3+12*a*c*e^2*f^2+8*c^2*d^2*f^2-24*c^2*d*e^2*f+8*c^2*e^4)*ln(x*c^
(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-8*e*f^4*a^2*(-4*d*f+e^2)^(1/2)+32*a*c
*d*e*f^3*(-4*d*f+e^2)^(1/2)-16*a*c*e^3*f^2*(-4*d*f+e^2)^(1/2)-24*c^2*d^2*e
*f^2*(-4*d*f+e^2)^(1/2)+32*c^2*d*e^3*f*(-4*d*f+e^2)^(1/2)-8*c^2*e^5*(-4*d*
f+e^2)^(1/2)+16*a^2*d*f^5-8*a^2*e^2*f^4-32*a*c*d^2*f^4+64*a*c*d*e^2*f^3-16
*a*c*e^4*f^2+16*c^2*d^3*f^3-72*c^2*d^2*e^2*f^2+48*c^2*d*e^4*f-8*c^2*e^6)/f
^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e
+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2
*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-8*e*f^4*a^2*(-4*d*f+e^2)^(1/2)+32*a*c*d*
e*f^3*(-4*d*f+e^2)^(1/2)-16*a*c*e^3*f^2*(-4*d*f+e^2)^(1/2)-24*c^2*d^2*e*f^
2*(-4*d*f+e^2)^(1/2)+32*c^2*d*e^3*f*(-4*d*f+e^2)^(1/2)-8*c^2*e^5*(-4*d*f+e
^2)^(1/2)-16*a^2*d*f^5+8*a^2*e^2*f^4+32*a*c*d^2*f^4-64*a*c*d*e^2*f^3+16*a*
c*e^4*f^2-16*c^2*d^3*f^3+72*c^2*d^2*e^2*f^2-48*c^2*d*e^4*f+8*c^2*e^6)/f^2/
(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*...
```

3.58.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.58.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x**2*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.58.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.58.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.58.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,(c\,x^2+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input

```
int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

output

```
int((x^2*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.59 $\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.59.1 Optimal result

Integrand size = 25, antiderivative size = 553

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{(2(af^2+c(e^2-df))-cefx)\sqrt{a+cx^2}}{2f^3}$$

$$+\frac{(a+cx^2)^{3/2}}{3f}-\frac{\sqrt{c}e(3af^2+2c(e^2-2df))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^4}$$

$$-\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2))\right)\text{arctan}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(2cdef(2af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)(a^2f^4+2acf^2(e^2-df)+c^2(e^4-3de^2f+d^2f^2))\right)\text{arctanh}}{\sqrt{2}f^4\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
1/3*(c*x^2+a)^(3/2)/f-1/2*e*(3*a*f^2+2*c*(-2*d*f+e^2))*arctanh(x*c^(1/2)/(
c*x^2+a)^(1/2))*c^(1/2)/f^4+1/2*(2*a*f^2+2*c*(-d*f+e^2)-c*e*f*x)*(c*x^2+a)
^(1/2)/f^3-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x
^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e*f
*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e
^2*f+e^4))*(e-(-4*d*f+e^2)^(1/2)))/f^4*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2
+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(
-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*
f+e^2)^(1/2)))^(1/2))*(2*c*d*e*f*(2*a*f^2+c*(-2*d*f+e^2))-(a^2*f^4+2*a*c*f
^2*(-d*f+e^2)+c^2*(d^2*f^2-3*d*e^2*f+e^4))*(e+(-4*d*f+e^2)^(1/2)))/f^4*2^(
1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.59.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 755, normalized size of antiderivative = 1.37

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{f\sqrt{a+cx^2}(8af^2+c(6e^2-6df-3efx+2f^2x^2))+3\sqrt{c}e(3af^2+2c(e^2-2df))\log(\cdots}{}$$

input
```
Integrate[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
(f*Sqrt[a + c*x^2]*(8*a*f^2 + c*(6*e^2 - 6*d*f - 3*e*f*x + 2*f^2*x^2)) + 3
*Sqrt[c]*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
]] - 6*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt
[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
] - 3*a*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a*c^2*d^2*f
^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*a^2*c*e^2*f^2*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c
*x^2] - #1] + a^3*f^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(5/2)
*d*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4*c^(5/2)*d^2*e*f*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*e*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^4*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1]*#1^2 + 3*c^2*d*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1
^2 - c^2*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - 2*a*c*e^2
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d*f^3*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*f^4*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2
+ 2*f*#1^3) & ])/(6*f^4)
```

3.59.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 569, normalized size of antiderivative = 1.03, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1353, 27, 2027, 2139, 2145, 27, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}dx$$

$$\Big\downarrow \ 1353$$

$$\frac{\int -\frac{3\sqrt{cx^2+a}\left(cex^2+(cd-af)x\right)}{fx^2+ex+d}dx}{3f}+\frac{\left(a+cx^2\right)^{3/2}}{3f}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-\frac{\int \frac{\sqrt{cx^2+a}\left(cex^2+(cd-af)x\right)}{fx^2+ex+d}dx}{f}$$

$$\Big\downarrow \ 2027$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-\frac{\int \frac{x(cd-af+cex)\sqrt{cx^2+a}}{fx^2+ex+d}dx}{f}$$

$$\Big\downarrow \ 2139$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-\frac{-\frac{\int \frac{-e\left(3af^2+2c\left(e^2-2df\right)\right)x^2c^2+adefc^2-\left(acfe^2+2(cd-af)\left(af^2+c\left(e^2-df\right)\right)\right)xc}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{2cf^2}-\frac{\sqrt{a+cx^2}\left(2\left(af^2+c\left(e^2-df\right)\right)-cefx\right)}{2f^2}}{f}$$

$$\Big\downarrow \ 2145$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-\frac{-\frac{\frac{\int \frac{2c\left(cde\left(2af^2+c\left(e^2-2df\right)\right)+\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x\right)}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2e\left(3af^2+2c\left(e^2-2df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2(af^2+c(e^2-df}{2f^2}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$
$$-\frac{\frac{\frac{2c\int \frac{cde\left(2af^2+c\left(e^2-2df\right)\right)+\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2e\left(3af^2+2c\left(e^2-2df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2(af^2+c(e^2-df))}{2f^2}}{f}$$

$\Big\downarrow$ 224

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$
$$-\frac{\frac{\frac{2c\int \frac{cde\left(2af^2+c\left(e^2-2df\right)\right)+\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^2e\left(3af^2+2c\left(e^2-2df\right)\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2(af^2+c}{2f}}{f}$$

$\Big\downarrow$ 219

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$
$$-\frac{\frac{\frac{2c\int \frac{cde\left(2af^2+c\left(e^2-2df\right)\right)+\left(a^2f^4+2ac\left(e^2-df\right)f^2+c^2\left(e^4-3dfe^2+d^2f^2\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-2df\right)\right)}{f}}{2cf^2}-\frac{\sqrt{a+cx^2}(2(af^2+}{2}}{f}$$

$\Big\downarrow$ 1367

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$
$$-\frac{\frac{2c\left(\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2cdef\left(2af^2+c\left(e^2-2\right.\right.\right.}{}\right.}{f}}{2}$$

$\Big\downarrow$ 488

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$
$$-\frac{2c\left(\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2}{-}}{\sqrt{e^2-4df}}\right.}{}$$

3.59. $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$\Big\downarrow \; 219$$

$$\frac{\left(a+cx^2\right)^{3/2}}{3f}-$$

$$2c\left(\frac{\left(2cdef\left(2af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(a^2f^4+2acf^2\left(e^2-df\right)+c^2\left(d^2f^2-3de^2f+e^4\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right.$$

$$-\frac{\quad}{\quad}$$

input
```
Int[(x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x]
```

output
```
(a + c*x^2)^(3/2)/(3*f) - (-1/2*((2*(a*f^2 + c*(e^2 - d*f)) - c*e*f*x)*Sqr
t[a + c*x^2])/f^2 - (-((c^(3/2)*e*(3*a*f^2 + 2*c*(e^2 - 2*d*f))*ArcTanh[(S
qrt[c]*x)/Sqrt[a + c*x^2]])/f) + (2*c*(-(((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2
*d*f)) - (e - Sqrt[e^2 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e
^4 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/
(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*
x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f])])) + ((2*c*d*e*f*(2*a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2
 - 4*d*f])*(a^2*f^4 + 2*a*c*f^2*(e^2 - d*f) + c^2*(e^4 - 3*d*e^2*f + d^2*f
^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[
e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/f)/(
2*c*f^2))/f
```

3.59.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1353
```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + c*x^2)^p*((d + e*x + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] + Simp[1/(2*f*(p + q + 1))   Int[(a + c*x^2)^(p
- 1)*(d + e*x + f*x^2)^q*Simp[a*h*e*p - a*(h*e - 2*g*f)*(p + q + 1) - 2*h*p
*(c*d - a*f)*x - (h*c*e*p + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /;
 FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 0] &&
NeQ[p + q + 1, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

3.59. $\int \frac{x\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

rule 2139
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*((-c)*e*(2*p + q + 2)) + 2*c*C*f
*(p + q + 1)*x)*(a + c*x^2)^p*((d + e*x + f*x^2)^(q + 1)/(2*c*f^2*(p + q +
1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   I
nt[(a + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[p*((-a)*e)*(C*(c*e)*(q + 1)
 - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(a*c*(C*(2*d*f - e^2*(2*p +
 q + 2)) + f*(B*e - 2*A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e)*(q
 + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(-4*a*c)))*x
+ (p*(c*e)*(C*(c*e)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q + 1)*
(C*f^2*p*(-4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2
*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, c, d, e, f, q}, x] &
& PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*p + 2*q + 3, 0
] && !IGtQ[p, 0] && !IGtQ[q, 0]
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.59.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. 2(499) = 998.

Time = 0.76 (sec) , antiderivative size = 1039, normalized size of antiderivative = 1.88

method	result	size
risch	Expression too large to display	1039
default	Expression too large to display	2294

input
```
int(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*c*f^2*x^2-3*c*e*f*x+8*a*f^2-6*c*d*f+6*c*e^2)*(c*x^2+a)^(1/2)/f^3-1/
2/f^3*(1/f*c^(1/2)*e*(3*a*f^2-4*c*d*f+2*c*e^2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2
))-1/2*(-2*a^2*f^4*(-4*d*f+e^2)^(1/2)+4*a*c*d*f^3*(-4*d*f+e^2)^(1/2)-4*a*c
*e^2*f^2*(-4*d*f+e^2)^(1/2)-2*c^2*d^2*f^2*(-4*d*f+e^2)^(1/2)+6*c^2*d*e^2*f
*(-4*d*f+e^2)^(1/2)-2*c^2*e^4*(-4*d*f+e^2)^(1/2)+2*e*f^4*a^2-12*f^3*a*c*d*
e+4*a*c*e^3*f^2+10*c^2*d^2*e*f^2-10*c^2*d*e^3*f+2*c^2*e^5)/f^2/(-4*d*f+e^2
)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2
)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1
/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2
*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2))))-1/2*(-2*a^2*f^4*(-4*d*f+e^2)^(1/2)+4*a*c*d*f^3*(-4*d
*f+e^2)^(1/2)-4*a*c*e^2*f^2*(-4*d*f+e^2)^(1/2)-2*c^2*d^2*f^2*(-4*d*f+e^2)^
(1/2)+6*c^2*d*e^2*f*(-4*d*f+e^2)^(1/2)-2*c^2*e^4*(-4*d*f+e^2)^(1/2)-2*e*f^
4*a^2+12*f^3*a*c*d*e-4*a*c*e^3*f^2-10*c^2*d^2*e*f^2+10*c^2*d*e^3*f-2*c^2*e
^5)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+...
```

3.59.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.59.6 Sympy [F]

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x(a+cx^2)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

input
```
integrate(x*(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x*(a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

3.59.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.59.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{x\,(c\,x^2+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2),x)
```

output
```
int((x*(a + c*x^2)^(3/2))/(d + e*x + f*x^2), x)
```

3.60 $\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.60.1 Optimal result

Integrand size = 24, antiderivative size = 484

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = -\frac{c(2e-fx)\sqrt{a+cx^2}}{2f^2} + \frac{\sqrt{c}(3af^2+2c(e^2-df))\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2f^3}$$
$$-\frac{\left(ce\left(e-\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\text{arctanh}\left(\frac{2af-c\left(e-\cdots\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2d\cdots\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+\frac{\left(ce\left(e+\sqrt{e^2-4df}\right)(2af^2+c(e^2-2df))-2f(2acdf^2-a^2f^3+c^2d(e^2-df))\right)\text{arctanh}\left(\frac{2af-c\left(e+\cdots\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df\cdots\right.}}\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
1/2*(3*a*f^2+2*c*(-d*f+e^2))*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/f^
3-1/2*c*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d
*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^
2)^(1/2)))^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+c*e*(2*a*f^
2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(
2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c
*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e
*(-4*d*f+e^2)^(1/2)))^(1/2))*(-2*f*(2*a*c*d*f^2-a^2*f^3+c^2*d*(-d*f+e^2))+
c*e*(2*a*f^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^3*2^(1/2)/(-4*d*f+e
^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.60.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.12

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \frac{cf(-2e+fx)\sqrt{a+cx^2}+\sqrt{c}(-2ce^2+2cdf-3af^2)\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)+2\mathrm{Roo}}{}$$

input
```
Integrate[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

output
```
(c*f*(-2*e + f*x)*Sqrt[a + c*x^2] + Sqrt[c]*(-2*c*e^2 + 2*c*d*f - 3*a*f^2)
*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] + 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1
+ 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*e^3*Log[-
(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1] + 2*a^2*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1
] + 2*c^(5/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*c^(5/2
)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d*f^2*Lo
g[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^3*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 - c^2*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1^2 + 2*c^2*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2
- 2*a*c*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e
+ 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(2*f^3)
```

3.60.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1310, 2145, 27, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}dx$$

$$\Big\downarrow \; 1310$$

$$-\frac{\int \frac{-c(3af^2+2c(e^2-df))x^2-ce(2cd-af)x+af(cd-2af)}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{2f^2}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 2145$$

$$-\frac{\frac{\int \frac{2\left(-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)+ce\left(2af^2+c\left(e^2-2df\right)\right)x\right)}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(3af^2+2c\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2f^2}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{2\int \frac{-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)+ce\left(2af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(3af^2+2c\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+a}}dx}{f}}{2f^2}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 224$$

$$-\frac{\frac{2\int \frac{-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)+ce\left(2af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{c\left(3af^2+2c\left(e^2-df\right)\right)\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}}{2f^2}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{2\int \frac{-a^2f^3+2acdf^2+c^2d\left(e^2-df\right)+ce\left(2af^2+c\left(e^2-2df\right)\right)x}{\sqrt{cx^2+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)\left(3af^2+2c\left(e^2-df\right)\right)}{f}}{2f^2}-\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 1367$$

$$-\frac{\frac{2\left(\frac{\left(-2a^2f^4-ce\left(e-\sqrt{e^2-4df}\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(-2a^2f^4-ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c\left(e^2\right.\right.\right.}{}}{f}}{2f^2}$$

$$\frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$$\Big\downarrow \; 488$$

3.60. $\int \frac{\left(a+cx^2\right)^{3/2}}{d+ex+fx^2}\,dx$

$$-\frac{2\left(\frac{\left(-2a^2f^4-ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}\right.}{} - \frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

$\downarrow$ 219

$$-\frac{2\left(\frac{\left(-2a^2f^4-ce\left(\sqrt{e^2-4df}+e\right)\left(2af^2+c\left(e^2-2df\right)\right)+4acdf^3+2c^2df\left(e^2-df\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(-2a^2f^4-ce\left(e-\right.\right.}{}\right.}{f} - \frac{c\sqrt{a+cx^2}(2e-fx)}{2f^2}$$

input

```
Int[(a + c*x^2)^(3/2)/(d + e*x + f*x^2),x]
```

output

```
-1/2*(c*(2*e - f*x)*Sqrt[a + c*x^2])/f^2 - (-((Sqrt[c]*(3*a*f^2 + 2*c*(e^2
 - d*f))*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f) + (2*(-(((4*a*c*d*f^3 -
2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e - Sqrt[e^2 - 4*d*f])*(2*a*f^2 +
 c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f])])) + ((4*a*c*d*f^3 - 2*a^2*f^4 + 2*c^2*d*f*(e^2 - d*f) - c*e*(e + Sqr
t[e^2 - 4*d*f])*(2*a*f^2 + c*(e^2 - 2*d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[
e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2
- 2*d*f + e*Sqrt[e^2 - 4*d*f])])))/f)/(2*f^2)
```

3.60. $\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx$

3.60.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1310
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(-c)*(e*(2*p + q) - 2*f*(p + q)*x)*(a + c*x^2)^(p - 1)*((d
 + e*x + f*x^2)^(q + 1)/(2*f^2*(p + q)*(2*p + 2*q + 1))), x] - Simp[1/(2*f^
2*(p + q)*(2*p + 2*q + 1))   Int[(a + c*x^2)^(p - 2)*(d + e*x + f*x^2)^q*Si
mp[(-a)*c*e^2*(1 - p)*(2*p + q) + a*(p + q)*(-2*a*f^2*(2*p + 2*q + 1) + c*(
2*d*f - e^2*(2*p + q))) + (2*(c*d - a*f)*(c*e)*(1 - p)*(2*p + q) + 4*a*c*e*
f*(1 - p)*(p + q))*x + (p*c^2*e^2*(1 - p) - c*(p + q)*(2*a*f^2*(4*p + 2*q -
 1) + c*(2*d*f*(1 - 2*p) + e^2*(3*p + q - 1))))*x^2, x], x], x] /; FreeQ[{a
, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && GtQ[p, 1] && NeQ[p + q, 0] &
& NeQ[2*p + 2*q + 1, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.60.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(435) = 870$.

Time = 0.78 (sec) , antiderivative size = 886, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{c(-fx+2e)\sqrt{c\,x^2+a}}{2f^2}+\frac{\frac{\sqrt{c}\left(3a\,f^2-2cdf+2c\,e^2\right)\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f}-\frac{\left(-4ace\,f^2\sqrt{-4df+e^2}+4c^2def\sqrt{-4df+e^2}-2c^2e^3\sqrt{-4df+e^2}+4a^2f^4\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*c*(-f*x+2*e)*(c*x^2+a)^(1/2)/f^2+1/2/f^2*(1/f*c^(1/2)*(3*a*f^2-2*c*d*
f+2*c*e^2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-1/2*(-4*a*c*e*f^2*(-4*d*f+e^2)^(1
/2)+4*c^2*d*e*f*(-4*d*f+e^2)^(1/2)-2*c^2*e^3*(-4*d*f+e^2)^(1/2)+4*a^2*f^4-
8*a*c*d*f^3+4*a*c*e^2*f^2+4*c^2*d^2*f^2-8*c^2*d*e^2*f+2*c^2*e^4)/f^2/(-4*d
*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d
*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/
2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/
f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(-4*a*c*e*f^2*(-4*d*f+e^2)^(1/2)+4*c^2*d*e
*f*(-4*d*f+e^2)^(1/2)-2*c^2*e^3*(-4*d*f+e^2)^(1/2)-4*a^2*f^4+8*a*c*d*f^3-4
*a*c*e^2*f^2-4*c^2*d^2*f^2+8*c^2*d*e^2*f-2*c^2*e^4)/f^2/(-4*d*f+e^2)^(1/2)
*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*
c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)))
```

3.60.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.60.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(a+cx^2)^{\frac{3}{2}}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral((a + c*x**2)**(3/2)/(d + e*x + f*x**2), x)
```

3.60.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.60.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{d+ex+fx^2}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((a + c*x^2)^(3/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + c*x^2)^(3/2)/(d + e*x + f*x^2), x)
```

3.61 $\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)} dx$

3.61.1 Optimal result

Integrand size = 27, antiderivative size = 496

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \frac{a\sqrt{a+cx^2}}{d}+\frac{(cd-af)\sqrt{a+cx^2}}{df}-\frac{c^{3/2}e\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}$$
$$-\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+\frac{\left(2ef(c^2d^2-a^2f^2)-(c^2de^2-f(cd-af)^2)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$-\frac{a^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}$$

output
```
-c^(3/2)*e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2-a^(3/2)*arctanh((c*x^2+a
)^(1/2)/a^(1/2))/d+a*(c*x^2+a)^(1/2)/d+(-a*f+c*d)*(c*x^2+a)^(1/2)/d/f-1/2*
arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*
a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)
-(c^2*d*e^2-f*(-a*f+c*d)^2)*(e-(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+
e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(
1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*
(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*f*(-a^2*f^2+c^2*d^2)-(c^2*d*
e^2-f*(-a*f+c*d)^2)*(e+(-4*d*f+e^2)^(1/2)))/d/f^2*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 552, normalized size of antiderivative = 1.11

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \frac{cdf\sqrt{a+cx^2}+2a^{3/2}f^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+c^{3/2}de\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)+}{}$$

input
```
Integrate[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

output
```
(c*d*f*Sqrt[a + c*x^2] + 2*a^(3/2)*f^2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2
])/Sqrt[a]] + c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] + RootSum[a^
2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1
^4 & , (-(a*c^2*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a*c^2*d^
2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1] + a^3*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
 #1] - 2*c^(5/2)*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 2*a^2
*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + c^2*d*e^2*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - c^2*d^2*f*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1^2 + 2*a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]*#1^2 - a^2*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*S
qrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(d*f^2)
```

3.61.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+cx^2\right)^{3/2}}{x\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{\left(a+cx^2\right)^{3/2}(-e-fx)}{d\left(d+ex+fx^2\right)}+\frac{\left(a+cx^2\right)^{3/2}}{dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d}-\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(e-\sqrt{e^2-4df}\right)\left(c^2de^2-f(cd-af)^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2ef\left(c^2d^2-a^2f^2\right)-\left(\sqrt{e^2-4df}+e\right)\left(c^2de^2-f(cd-af)^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}df^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{c^{3/2}e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{f^2}+\frac{\sqrt{a+cx^2}(cd-af)}{df}+\frac{a\sqrt{a+cx^2}}{d}$$

input
```
Int[(a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x]
```

output
```
(a*Sqrt[a + c*x^2])/d + ((c*d - a*f)*Sqrt[a + c*x^2])/(d*f) - (c^(3/2)*e*A
rcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/f^2 - ((2*e*f*(c^2*d^2 - a^2*f^2) - (
c^2*d*e^2 - f*(c*d - a*f)^2)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(
e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[
e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*e*f*(c^2*d^2 - a^2*f
^2) - (c^2*d*e^2 - f*(c*d - a*f)^2)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*
f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*f^2*Sqrt[e^2 - 4*d*f]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (a^(3/2)*ArcTanh[
Sqrt[a + c*x^2]/Sqrt[a]])/d
```

3.61.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2378 vs. $2(443) = 886$.

Time = 0.65 (sec) , antiderivative size = 2379, normalized size of antiderivative = 4.80

method	result	size
default	Expression too large to display	2379

input
```
int((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/3*(c*x^2+a)^(3/2)+a
*((c*x^2+a)^(1/2)-a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)))+2*f/(e+(
-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(1/3*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(3/2)-1/2*c*(e+(-4*d*f+e^
2)^(1/2))/f*(1/4*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(
1/2))/f)/c*((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/
f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2)^(1/2)+1/8*(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/c^(3/2)*ln((-1/2*c*(e+(-4*d*f+e
^2)^(1/2))/f+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)))+1/2
*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2*(1/2*(4*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)
^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-1/2
*c^(1/2)*(e+(-4*d*f+e^2)^(1/2))/f*ln((-1/2*c*(e+(-4*d*f+e^2)^(1/2))/f+c*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^
2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))-1/2*((-4*d*f+e^2)^...
```

3.61.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.61.6 Sympy [F]

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx=\int \frac{(a+cx^2)^{\frac{3}{2}}}{x(d+ex+fx^2)}\,dx$$

input
```
integrate((c*x**2+a)**(3/2)/x/(f*x**2+e*x+d),x)
```

output
```
Integral((a + c*x**2)**(3/2)/(x*(d + e*x + f*x**2)), x)
```

3.61.7 Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx=\int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x}\,dx$$

input
```
integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x), x)
```

3.61.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x(d+ex+fx^2)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(3/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.61.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x\,(d+ex+fx^2)}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{x\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(3/2)/(x*(d + e*x + f*x^2)), x)
```

3.62 $\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx$

3.62.1 Optimal result

Integrand size = 27, antiderivative size = 604

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = -\frac{ae\sqrt{a+cx^2}}{d^2} + \frac{3cx\sqrt{a+cx^2}}{2d} + \frac{(2ae-cdx)\sqrt{a+cx^2}}{2d^2}$$

$$-\frac{(a+cx^2)^{3/2}}{dx} + \frac{3a\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d} + \frac{\sqrt{c}(2cd-3af)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2df}$$

$$-\frac{\left(4acd^2f^2+c^2d^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+a^2f^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$+\frac{\left(4acd^2f^2+a^2f^2\left(e^2-2df-e\sqrt{e^2-4df}\right)+c^2d^2\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-\right.}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{\right.}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$+\frac{a^{3/2}e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}$$

output
```
-(c*x^2+a)^(3/2)/d/x+a^(3/2)*e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2+3/2*a*
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d+1/2*(-3*a*f+2*c*d)*arctanh(x*
c^(1/2)/(c*x^2+a)^(1/2))*c^(1/2)/d/f-a*e*(c*x^2+a)^(1/2)/d^2+3/2*c*x*(c*x^
2+a)^(1/2)/d+1/2*(-c*d*x+2*a*e)*(c*x^2+a)^(1/2)/d^2-1/2*arctanh(1/2*(2*a*f
-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f
-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(4*a*c*d^2*f^2+c^2*d^2*(e^2-2*d*f-e*(-4*d*f
+e^2)^(1/2))+a^2*f^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))/d^2/f*2^(1/2)/(-4*d
*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arcta
nh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2
+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(4*a*c*d^2*f^2+a^2*f^2*(e^2-2*
d*f-e*(-4*d*f+e^2)^(1/2))+c^2*d^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))/d^2/f*
2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1
/2)
```

3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 497, normalized size of antiderivative = 0.82

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx =$$

$$-\frac{2a^{3/2}efx\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+d\left(af\sqrt{a+cx^2}+c^{3/2}dx\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)\right)+x\text{RootSum}\left[a^2f+2\right.}{}$$

input
```
Integrate[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

output
```
-((2*a^(3/2)*e*f*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]] + d*(a*f
*Sqrt[a + c*x^2] + c^(3/2)*d*x*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]]) + x*Ro
otSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^
3 + f*#1^4 & , (-(a*c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) +
a^3*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 2*c^(5/2)*d^3*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 + 4*a*c^(3/2)*d^2*f*Log[-(Sqrt[c]*x)
+ Sqrt[a + c*x^2] - #1]*#1 + 2*a^2*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2
] - #1]*#1 + c^2*d^2*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2
*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d
*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(d^2*f*x))
```

3.62.3 Rubi [A] (verified)

Time = 2.63 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{(a+cx^2)^{3/2}(-df+e^2+efx)}{d^2(d+ex+fx^2)}-\frac{e(a+cx^2)^{3/2}}{d^2x}+\frac{(a+cx^2)^{3/2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^{3/2}e\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2}-$$
$$\frac{\left(a^2f^2\left(e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-\right.}}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2\right.}}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$\frac{\left(a^2f^2\left(-e\sqrt{e^2-4df}-2df+e^2\right)+4acd^2f^2+c^2d^2\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-\right.}}}\right.}{\sqrt{2}d^2f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)(2cd-3af)}{2df}+\frac{3a\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{2d}-\frac{ae\sqrt{a+cx^2}}{d^2}+$$
$$\frac{\sqrt{a+cx^2}(2ae-cdx)}{2d^2}-\frac{(a+cx^2)^{3/2}}{dx}+\frac{3cx\sqrt{a+cx^2}}{2d}$$

input
```
Int[(a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x]
```

output
```
-((a*e*Sqrt[a + c*x^2])/d^2) + (3*c*x*Sqrt[a + c*x^2])/(2*d) + ((2*a*e - c
*d*x)*Sqrt[a + c*x^2])/(2*d^2) - (a + c*x^2)^(3/2)/(d*x) + (3*a*Sqrt[c]*Ar
cTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d) + (Sqrt[c]*(2*c*d - 3*a*f)*ArcTa
nh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(2*d*f) - ((4*a*c*d^2*f^2 + c^2*d^2*(e^2
- 2*d*f - e*Sqrt[e^2 - 4*d*f]) + a^2*f^2*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 +
c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*f*S
qrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) +
((4*a*c*d^2*f^2 + a^2*f^2*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + c^2*d^2*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*
f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt
[a + c*x^2])])/(Sqrt[2]*d^2*f*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]) + (a^(3/2)*e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]
)/d^2
```

3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.62.4 Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 957, normalized size of antiderivative = 1.58

method	result
risch	$-\frac{a\sqrt{c\,x^2+a}}{dx}+\frac{\frac{c^{\frac{3}{2}}d\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f}-\frac{4f\,a^{\frac{3}{2}}e\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}-\frac{\left(2a^2f^3\sqrt{-4df+e^2}-4acd\,f^2\sqrt{-4df+e^2}+2c^2d^2f\sqrt{-4df+e^2}-\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(c*x^2+a)^(1/2)/x+1/d*(c^(3/2)*d/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-4*f*
a^(3/2)*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*ln((2*a+2*a^(1/2)
*(c*x^2+a)^(1/2))/x)-1/2*(2*a^2*f^3*(-4*d*f+e^2)^(1/2)-4*a*c*d*f^2*(-4*d*f
+e^2)^(1/2)+2*c^2*d^2*f*(-4*d*f+e^2)^(1/2)-2*(-4*d*f+e^2)^(1/2)*c^2*d*e^2-
2*a^2*e*f^3-4*a*c*d*e*f^2+6*c^2*d^2*e*f-2*c^2*d*e^3)/f^2/(-4*d*f+e^2)^(1/2
)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c
*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+
1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-2*a^2*f^3*(-4*d*f+e^2)^(1/2)+4*a*c*d*
f^2*(-4*d*f+e^2)^(1/2)-2*c^2*d^2*f*(-4*d*f+e^2)^(1/2)+2*(-4*d*f+e^2)^(1/2)
*c^2*d*e^2-2*a^2*e*f^3-4*a*c*d*e*f^2+6*c^2*d^2*e*f-2*c^2*d*e^3)/f^2/(-4*d*
f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+
1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+...
```

3.62.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.62.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**(3/2)/x**2/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.62.7 Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^2), x)
```

3.62.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+a)^(3/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.62.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{x^2\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(3/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.63 $\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx$

3.63.1 Optimal result

Integrand size = 27, antiderivative size = 668

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \frac{3c\sqrt{a+cx^2}}{2d} + \frac{a(e^2-df)\sqrt{a+cx^2}}{d^3} - \frac{3cex\sqrt{a+cx^2}}{2d^2}$$

$$- \frac{(2(cd^2+a(e^2-df))-cdex)\sqrt{a+cx^2}}{2d^3} - \frac{(a+cx^2)^{3/2}}{2dx^2} + \frac{e(a+cx^2)^{3/2}}{d^2x}$$

$$+ \frac{\left(c^2d^3\left(e-\sqrt{e^2-4df}\right)+2acd^2f\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def+e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}\right)\right)\text{arctan}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$- \frac{\left(2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(e+\sqrt{e^2-4df}\right)+a^2f\left(e^3-3def-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}\right)\right)\text{arcta}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$- \frac{3\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d} - \frac{a^{3/2}(e^2-df)\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}$$

output
```
-1/2*(c*x^2+a)^(3/2)/d/x^2+e*(c*x^2+a)^(3/2)/d^2/x-a^(3/2)*(-d*f+e^2)*arct
anh((c*x^2+a)^(1/2)/a^(1/2))/d^3-3/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))*a^
(1/2)/d+3/2*c*(c*x^2+a)^(1/2)/d+a*(-d*f+e^2)*(c*x^2+a)^(1/2)/d^3-3/2*c*e*x
*(c*x^2+a)^(1/2)/d^2-1/2*(2*c*d^2+2*a*(-d*f+e^2)-c*d*e*x)*(c*x^2+a)^(1/2)/
d^3+1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(
1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(c^2*d^3*(e-(-4*d
*f+e^2)^(1/2))+2*a*c*d^2*f*(e+(-4*d*f+e^2)^(1/2))+a^2*f*(e^3-3*d*e*f+e^2*(
-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh(1/2*(2*a*f-
c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+
e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*c*d^2*f*(e-(-4*d*f+e^2)^(1/2))+c^2*d^3*
(e+(-4*d*f+e^2)^(1/2))+a^2*f*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d
*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4
*d*f+e^2)^(1/2)))^(1/2)
```

3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.78 (sec) , antiderivative size = 617, normalized size of antiderivative = 0.92

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \frac{\frac{ad(-d+2ex)\sqrt{a+cx^2}}{x^2} + 6\sqrt{a}cd^2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right) - 4a^{3/2}(e^2-df)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\cdots}{\cdots}\right)\cdots}{\cdots}$$

input
```
Integrate[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

output
```
((a*d*(-d + 2*e*x)*Sqrt[a + c*x^2])/x^2 + 6*Sqrt[a]*c*d^2*ArcTanh[(Sqrt[c]
*x - Sqrt[a + c*x^2])/Sqrt[a]] - 4*a^(3/2)*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*
x) + Sqrt[a + c*x^2])/Sqrt[a]] - 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c^2*d^3*Log[-(Sqrt[
c]*x) + Sqrt[a + c*x^2] - #1] - 2*a^2*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1] - a^3*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^3*d*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - 4*a*c^(3/2)*d^2*e*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1 + 4*a^2*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1 - c^2*d^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 +
 2*a*c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + a^2*e^2*f*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a^2*d*f^2*Log[-(Sqrt[c]*x) +
Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]
*e*#1^2 + 2*f*#1^3) & ])/(2*d^3)
```

3.63.3 Rubi [A] (verified)

Time = 2.69 (sec) , antiderivative size = 668, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{(a+cx^2)^{3/2}(e^2-df)}{d^3x}+\frac{(a+cx^2)^{3/2}(-fx(e^2-df)-e(e^2-2df))}{d^3(d+ex+fx^2)}-\frac{e(a+cx^2)^{3/2}}{d^2x^2}+\frac{(a+cx^2)^{3/2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3}+$$

$$\frac{\left(a^2f\left(e^2\sqrt{e^2-4df}-df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(\sqrt{e^2-4df}+e\right)+c^2d^3\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\Bigg(}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{\left(a^2f\left(-e^2\sqrt{e^2-4df}+df\sqrt{e^2-4df}-3def+e^3\right)+2acd^2f\left(e-\sqrt{e^2-4df}\right)+c^2d^3\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{3\sqrt{a}c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2d}+\frac{a\sqrt{a+cx^2}(e^2-df)}{d^3}+\frac{e(a+cx^2)^{3/2}}{d^2x}-\frac{3cex\sqrt{a+cx^2}}{2d^2}-\frac{\sqrt{a+cx^2}(2(a(e^2-df)+cd^2)-cdex)}{2d^3}-\frac{(a+cx^2)^{3/2}}{2dx^2}+\frac{3c\sqrt{a+cx^2}}{2d}$$

input
```
Int[(a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x]
```

output
```
(3*c*Sqrt[a + c*x^2])/(2*d) + (a*(e^2 - d*f)*Sqrt[a + c*x^2])/d^3 - (3*c*e
*x*Sqrt[a + c*x^2])/(2*d^2) - ((2*(c*d^2 + a*(e^2 - d*f)) - c*d*e*x)*Sqrt[
a + c*x^2])/(2*d^3) - (a + c*x^2)^(3/2)/(2*d*x^2) + (e*(a + c*x^2)^(3/2))/
(d^2*x) + ((c^2*d^3*(e - Sqrt[e^2 - 4*d*f]) + 2*a*c*d^2*f*(e + Sqrt[e^2 -
4*d*f]) + a^2*f*(e^3 - 3*d*e*f + e^2*Sqrt[e^2 - 4*d*f] - d*f*Sqrt[e^2 - 4*
d*f]))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*
Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) -
 ((2*a*c*d^2*f*(e - Sqrt[e^2 - 4*d*f]) + c^2*d^3*(e + Sqrt[e^2 - 4*d*f]) +
 a^2*f*(e^3 - 3*d*e*f - e^2*Sqrt[e^2 - 4*d*f] + d*f*Sqrt[e^2 - 4*d*f]))*Ar
cTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2
 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*d^3*Sqrt[e^2
- 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - (3*Sqrt[
a]*c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*d) - (a^(3/2)*(e^2 - d*f)*ArcTan
h[Sqrt[a + c*x^2]/Sqrt[a]])/d^3
```

3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.63.4 Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 911, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{a\sqrt{c\,x^{2}+a}\,(-2ex+d)}{2d^{2}x^{2}}-\frac{\frac{4f\sqrt{a}\left(2adf-2e^{2}a-3c\,d^{2}\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)}-\frac{\left(4a^{2}e\,f^{2}\sqrt{-4df+e^{2}}-4\sqrt{-4df+e^{2}}\,c^{2}d^{2}e+8a^{2}d\,f^{3}-4a^{2}e^{2}f^{2}\cdots\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*(c*x^2+a)^(1/2)*(-2*e*x+d)/d^2/x^2-1/2/d^2*(4*f*a^(1/2)*(2*a*d*f-2*
a*e^2-3*c*d^2)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*ln((2*a+2*a^
(1/2)*(c*x^2+a)^(1/2))/x)-1/2*(4*a^2*e*f^2*(-4*d*f+e^2)^(1/2)-4*(-4*d*f+e^
2)^(1/2)*c^2*d^2*e+8*a^2*d*f^3-4*a^2*e^2*f^2-16*a*c*d^2*f^2+8*d^3*f*c^2-4*
c^2*d^2*e^2)/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^
(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-4*a^2
*e*f^2*(-4*d*f+e^2)^(1/2)+4*(-4*d*f+e^2)^(1/2)*c^2*d^2*e+8*a^2*d*f^3-4*a^2
*e^2*f^2-16*a*c*d^2*f^2+8*d^3*f*c^2-4*c^2*d^2*e^2)/(-4*d*f+e^2)^(1/2)/(-e+
(-4*d*f+e^2)^(1/2))/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*
(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))
/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

3.63.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+a)**(3/2)/x**3/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.63.7 Maxima [F]

$$\int \frac{(a+cx^2)^{3/2}}{x^3\,(d+ex+fx^2)}\,dx = \int \frac{(cx^2+a)^{\frac{3}{2}}}{(fx^2+ex+d)x^3}\,dx$$

input
```
integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + a)^(3/2)/((f*x^2 + e*x + d)*x^3), x)
```

3.63.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+cx^2)^{3/2}}{x^3\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+a)^(3/2)/x^3/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.63.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+cx^2)^{3/2}}{x^3(d+ex+fx^2)}\,dx = \int \frac{(c\,x^2+a)^{3/2}}{x^3\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)),x)
```

output
```
int((a + c*x^2)^(3/2)/(x^3*(d + e*x + f*x^2)), x)
```

3.64 $\int \frac{x^3}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.64.1 Optimal result

Integrand size = 27, antiderivative size = 380

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\sqrt{a+cx^2}}{cf}-\frac{e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2} - \frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
-e*arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f^2/c^(1/2)+(c*x^2+a)^(1/2)/c/f-1/2*
arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*
a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-
4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*
(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2
)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(
1/2))*(2*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)
^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.64.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{f\sqrt{a+cx^2}+\sqrt{c}e\log\left(-\sqrt{c}x+\sqrt{a+cx^2}\right)-c\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#\right.}{}$$

input
```
Integrate[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
(f*Sqrt[a + c*x^2] + Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]] - c*Roo
tSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3
 + f*#1^4 & , (a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sq
rt[a + c*x^2] - #1]*#1 - e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2
 + d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d
*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(c*f^2)
```

3.64.3 Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{ 7279}$$
$$\int \left(\frac{x(e^2-df)+de}{f^2\sqrt{a+cx^2}\,(d+ex+fx^2)}-\frac{e}{f^2\sqrt{a+cx^2}}+\frac{x}{f\sqrt{a+cx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{e\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+cx^2}}{cf}$$

input
```
Int[x^3/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
Sqrt[a + c*x^2]/(c*f) - (e*ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]])/(Sqrt[c]*
f^2) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c
*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*
a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*d*e*f - (e^2 - d*f)*
(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sq
rt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2
])]])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt
[e^2 - 4*d*f])])
```

3.64.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.64.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(335) = 670$.

Time = 0.78 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.88

method	result
default	$\frac{\sqrt{c\,x^2+a}}{cf}-\frac{e\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f^2\sqrt{c}}-\frac{\left(e^3-3def+e^2\sqrt{-4df+e^2}-df\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$
risch	$\frac{\sqrt{c\,x^2+a}}{cf}-\frac{\frac{e\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f\sqrt{c}}-\frac{\left(df\sqrt{-4df+e^2}-e^2\sqrt{-4df+e^2}+3def-e^3\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x\cdots\right.}{f}\cdots\right.}{\cdots}}{\cdots}$

input
```
int(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+a)^(1/2)/c/f-e/f^2*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(e^3-3
*d*e*f+e^2*(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/
2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-
4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f))-1/2*(-d*f*(-4*d*f+e^2)^(1/2)+e^2*(-4*d*f+e^2)^(1/2)+3*d*e*f-e^3)/
f^3/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c
*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c
*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f
+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
)/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.64.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.64.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.64.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.64.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.64.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^3/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.65 $\int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.65.1 Optimal result

Integrand size = 27, antiderivative size = 344

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx &= \frac{\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f} \\ &- \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\ &- \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}\end{aligned}$$

output
```
arctanh(x*c^(1/2)/(c*x^2+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e
-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*
d*f+e^2)^(1/2)))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(-4*d*f
+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh
(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c
*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))
/f*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))
^(1/2)
```

3.65.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.76

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+cx^2}}\right)}{\sqrt{c}} + \text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+4af\#1^2-2\sqrt{a}e\#1^3+d\#1^4\&,\frac{-cd\log(x)\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
((2*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + c*x^2])])/Sqrt[c] + RootSum[c
^2*d + 2*Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#
1^4 & , (-(c*d*Log[x]) + c*d*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - 2*Sq
rt[a]*e*Log[x]*#1 + 2*Sqrt[a]*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1
+ d*Log[x]*#1^2 - d*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a
]*c*e) + 2*c*d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/f
```

3.65.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 342, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {2145, 25, 224, 219, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{2145}$$

$$\frac{\int -\frac{d+ex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}+\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{1}{\sqrt{cx^2+a}}dx}{f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 224$$

$$\frac{\int \frac{1}{1-\frac{cx^2}{cx^2+a}}d\frac{x}{\sqrt{cx^2+a}}}{f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \; 1367$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}-\frac{-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}}{f}$$

$$\Big\downarrow \; 488$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}-\frac{\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}+\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{}{4af^2+c\left(e+\sqrt{e^2 \cdots\right.}}}{\cdots}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}f}-\frac{\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-\cdots}\right.}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{\cdots}\right.}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e\cdots\right.}}}{f}$$

input
```
Int[x^2/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

3.65. $\int \frac{x^2}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

output
```
ArcTanh[(Sqrt[c]*x)/Sqrt[a + c*x^2]]/(Sqrt[c]*f) - (((e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt
[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt
[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])
]) + ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2
 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f + e*Sqrt[e^2 - 4*d*f])]))/f
```

3.65.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2145
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + f*x^2], x], x] + Simp[1/c   Int[(A*c - a*
C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{a
, b, c, d, f}, x] && PolyQ[Px, x, 2]
```

3.65.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(300) = 600$.

Time = 0.74 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.93

method	result
default	$\frac{\ln\left(x\sqrt{c}+\sqrt{c\,x^2+a}\right)}{f\sqrt{c}} - \frac{\left(-e\sqrt{-4df+e^2}+2df-e^2\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\cdots}{\cdots}\right.}{2f^2\sqrt{-4df\cdots}}$

input
```
int(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/f*ln(x*c^(1/2)+(c*x^2+a)^(1/2))/c^(1/2)-1/2*(-e*(-4*d*f+e^2)^(1/2)+2*d*f
-e^2)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f
^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)
*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(
-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))
/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^
2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e
^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(
e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-
4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.65.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.65.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.65.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.65.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m i_lex_is_greater E
rror: Bad Argument Value
```

3.65.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^2/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.66 $\int \frac{x}{\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.66.1 Optimal result

Integrand size = 25, antiderivative size = 294

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)
/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e-(-4*d*f+e^2)^(1/2)
)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^
(1/2)-1/2*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)
^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e+(-4*d*f+e^2)
^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1
/2)))^(1/2)
```

3.66.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.53

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3 + f\#1^4\&, \frac{-a\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)+\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)\#1^2}{a\sqrt{c}e+4cd\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\right]$$

input
```
Integrate[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#
1^3 + f*#1^4 & , (-(a*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + Log[-(Sq
rt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1
- 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

3.66.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1367}$$

$$\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx+\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx$$

$$\Big\downarrow \text{488}$$

$$-\left(\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}\right)-$$

$$\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

input
```
Int[x/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-(((1 - e/Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)
/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]))
- ((1 + e/Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)
/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

3.66.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.66.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 621 vs. $2(257) = 514$.

Time = 0.74 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.12

method	result
default	$-\frac{\left(e+\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}}\cdots}{x+\frac{e+\sqrt{-4\cdots}}{2f}}\cdots\right)}{2\sqrt{-4df+e^2}\,f\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-\cdots}{f^2}}\cdots}$

input
```
int(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((((-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((((-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)+1/2*2^(1/2)*((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-e+(-4*d*f+e^2
)^(1/2))/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*
c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2
)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(
1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+
(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.66.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5085 vs. $2(255) = 510$.

Time = 1.03 (sec) , antiderivative size = 5085, normalized size of antiderivative = 17.30

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.66.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(x/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.66.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.66.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.66.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{c\,x^2+a}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.67 $\int \frac{1}{\sqrt{a+cx^2}(d+ex+fx^2)} dx$

3.67.1 Optimal result

Integrand size = 24, antiderivative size = 266

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
-f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(
1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+f*arctanh(1/2*(2*a
*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d
*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.67.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -2\sqrt{c}\text{RootSum}\Big[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3 + f\#1^4\&, \frac{\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)\#1}{a\sqrt{c}e+4cd\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\Big]$$

input
```
Integrate[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-2*Sqrt[c]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*
Sqrt[c]*e*#1^3 + f*#1^4 & , (Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1)/
(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

3.67.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1315, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1315}$$

$$\frac{2f\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{488}$$

$$\frac{2f\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{4af^2+c\left(e-\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e-\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}f\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{\sqrt{2}f\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

input
```
Int[1/(Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-((Sqrt[2]*f*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2
*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e
^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (Sqr
t[2]*f*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2
 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[e^2 - 4
*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

3.67.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1315
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f_.)*(x_)^2]), x_Sym
bol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[1/((b - q + 2*c*x
)*Sqrt[d + f*x^2]), x], x] - Simp[2*(c/q)   Int[1/((b + q + 2*c*x)*Sqrt[d +
 f*x^2]), x], x]] /; FreeQ[{a, b, c, d, f}, x] && NeQ[b^2 - 4*a*c, 0] && Po
sQ[b^2 - 4*a*c]
```

3.67.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 588 vs. 2(232) = 464.

Time = 0.63 (sec) , antiderivative size = 589, normalized size of antiderivative = 2.21

method	result
default	$\dfrac{\sqrt{2}\,\ln\left(\dfrac{\frac{\sqrt{-4df+e^2}\,ce+2a f^2-2cdf+c e^2}{f^2}-\frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a f^2-2cdf+c e^2}{f^2}}\sqrt{4\left(x+\frac{e+\sqrt{-4df+e}}{2f}\right.}}{}}{x+\frac{e+\sqrt{-4df+e^2}}{2f}}\right.}{\sqrt{-4df+e^2}\sqrt{\frac{\sqrt{-4df+e^2}\,ce+2a f^2-2cdf+c e^2}{f^2}}}$

input
```
int(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*f+e^2)^(1/2)*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+
(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.67.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5073 vs. $2(230) = 460$.

Time = 1.03 (sec) , antiderivative size = 5073, normalized size of antiderivative = 19.07

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

3.67.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.67.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.67.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.67.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.68 $\int \frac{1}{x\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.68.1 Optimal result

Integrand size = 27, antiderivative size = 330

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{f\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
-arctanh((c*x^2+a)^(1/2)/a^(1/2))/d/a^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(
e-(-4*d*f+e^2)^(1/2))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4
*d*f+e^2)^(1/2)))^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2
*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2
*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*
f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.68.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.31 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} - \operatorname{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,\frac{af\log\left(-\sqrt{\ldots}\right.}{\ldots}\right.}{d}$$

input
```
Integrate[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
((2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a] - RootSum[a^2*
f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4
 & , (a*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*Sqrt[c]*e*Log[-(Sqr
t[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
- #1]*#1^2)/(-(a*Sqrt[c]*e) - 4*c*d*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f
*#1^3) & ])/d
```

3.68.3 Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\sqrt{a+cx^2}\,(d+ex+fx^2)}+\frac{1}{dx\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(x*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
(f*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(
Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt
[e^2 - 4*d*f])]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt
[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*
d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e
^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(Sq
rt[a]*d)
```

3.68.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.68.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 680 vs. $2(287) = 574$.

Time = 0.67 (sec) , antiderivative size = 681, normalized size of antiderivative = 2.06

method	result
default	$\frac{4f\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^{2}+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)\sqrt{a}} - \frac{2f\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^{2}}\,ce+2a\,f^{2}-2cdf+c\,e^{2}}{f^{2}} - \frac{c\left(e+\sqrt{-4df+e^{2}}\right)\left(x+\frac{e+\sqrt{-4df+e^{2}}}{2f}\right)}{f} + \frac{\sqrt{2}\sqrt{\sqrt{}}}{}\right.}{\left(e+\sqrt{-4d}\right.}$

input
```
int(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^(1/
2)*(c*x^2+a)^(1/2))/x)-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/
2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1
/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(
-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))
-2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^
2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2
))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.68.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.68.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.68.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x}\,dx$$

input
```
integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x), x)
```

3.68.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.69 $\int \frac{1}{x^2\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.69.1 Optimal result

Integrand size = 27, antiderivative size = 367

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{a+cx^2}}{adx} - \frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} + \frac{e\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}$$

output
```
e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/d^2/a^(1/2)-(c*x^2+a)^(1/2)/a/d/x-1/2*f
*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2
*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e^2-2*d*f+e*(-4*d*f+e^2
)^(1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^
2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1
/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(e
^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(
e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.69.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 298, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx =$$
$$-\frac{d\sqrt{a+cx^2}+2\sqrt{a}ex\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)+ax\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{\right.}{}$$

input
```
Integrate[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-((d*Sqrt[a + c*x^2] + 2*Sqrt[a]*e*x*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])
/Sqrt[a]] + a*x*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2
 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1] + 2*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 2*Sqr
t[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e*f*Log[-(Sqrt[c]*x
) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqr
t[c]*e*#1^2 + 2*f*#1^3) & ])/(a*d^2*x))
```

3.69.3 Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{-df+e^2+efx}{d^2\sqrt{a+cx^2}\,(d+ex+fx^2)}-\frac{e}{d^2x\sqrt{a+cx^2}}+\frac{1}{dx^2\sqrt{a+cx^2}}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\operatorname{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+cx^2}}{adx}$$

input
```
Int[1/(x^2*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-(Sqrt[a + c*x^2]/(a*d*x)) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])*ArcTan
h[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2
*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*
d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + (f*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(
Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x
^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sq
rt[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^2)
```

3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.69.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(322) = 644$.

Time = 0.81 (sec) , antiderivative size = 733, normalized size of antiderivative = 2.00

method	result
risch	$-\frac{\sqrt{c x^2+a}}{a d x} - \frac{\frac{4 f e \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}} + \frac{f\left(-e+\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2} - \frac{c\left(e+\sqrt{-4df+e^2}\right)\left(x+\cdots\right.}{f}\cdots\right)}{\cdots}}{\cdots}$
default	$\frac{4f\sqrt{c x^2+a}}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)a x} + \frac{16 f^2 e \ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}} + \frac{4f^2\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e^2}{f^2}\cdots\right)}{\cdots}$

input
```
int(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+a)^(1/2)/a/d/x-1/d*(4*f*e/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^
(1/2))/a^(1/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)+f*(-e+(-4*d*f+e^2)^(1
/2))/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^
2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/
2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+
c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-f*(e+(-4*d*f+e^2)^(1/
2))/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

3.69.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.69.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x**2/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.69.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^2), x)
```

3.69.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/x^2/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x^2*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.70 $\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.70.1 Optimal result

Integrand size = 27, antiderivative size = 457

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx &= -\frac{\sqrt{a+cx^2}}{2adx^2}+\frac{e\sqrt{a+cx^2}}{ad^2x} \\
&+\frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} \\
&-\frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}} \\
&+\frac{c\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{(e^2-df)\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}
\end{aligned}$$

output
```
1/2*c*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d-(-d*f+e^2)*arctanh((c*x^2
+a)^(1/2)/a^(1/2))/d^3/a^(1/2)-1/2*(c*x^2+a)^(1/2)/a/d/x^2+e*(c*x^2+a)^(1/
2)/a/d^2/x+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c
*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e^3-4
*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(
2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f
-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f
+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(
1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2)))^(1/2)
```

3.70.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.61 (sec) , antiderivative size = 422, normalized size of antiderivative = 0.92

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\frac{d(-d+2ex)\sqrt{a+cx^2}}{ax^2} - \frac{2cd^2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{4(e^2-df)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}} + 2\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1\right.}{}$$

input
```
Integrate[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
((d*(-d + 2*e*x)*Sqrt[a + c*x^2])/(a*x^2) - (2*c*d^2*ArcTanh[(Sqrt[c]*x -
Sqrt[a + c*x^2])/Sqrt[a]])/a^(3/2) - (4*(e^2 - d*f)*ArcTanh[(-(Sqrt[c]*x)
+ Sqrt[a + c*x^2])/Sqrt[a]])/Sqrt[a] + 2*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1
+ 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*e^2*f*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1] - a*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2] - #1] + 2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - 4
*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - e^2*f*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2
+ 2*f*#1^3) & ])/(2*d^3)
```

3.70.3 Rubi [A] (verified)

Time = 1.59 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{e^2-df}{d^3x\sqrt{a+cx^2}}+\frac{-fx(e^2-df)-e(e^2-2df)}{d^3\sqrt{a+cx^2}\,(d+ex+fx^2)}-\frac{e}{d^2x^2\sqrt{a+cx^2}}+\frac{1}{dx^3\sqrt{a+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}-\frac{(e^2-df)\,\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}+\frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{e\sqrt{a+cx^2}}{ad^2x}-\frac{\sqrt{a+cx^2}}{2adx^2}$$

input
```
Int[1/(x^3*Sqrt[a + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-1/2*Sqrt[a + c*x^2]/(a*d*x^2) + (e*Sqrt[a + c*x^2])/(a*d^2*x) + (f*(2*e^3
 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
- 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2
+ c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(2*e^3 - 4*d*e*f - (e^2 - d
*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)
/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c
*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*
Sqrt[e^2 - 4*d*f])]) + (c*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(2*a^(3/2)*d)
- ((e^2 - d*f)*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(Sqrt[a]*d^3)
```

3.70.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.70.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 777, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{\sqrt{c\,x^2+a}\,(-2ex+d)}{2a\,d^2x^2}-\frac{\frac{4f\left(2adf-2e^2a+c\,d^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}-\frac{2fa\left(e\sqrt{-4df+e^2}+2df-e^2\right)\sqrt{2}\,\ln\left(\frac{\sqrt{-4df+e^2}\,ce+2a\,f^2-2cdf+c\,e}{f^2}\right.}{}}{}$
default	$-\frac{4f\left(-\frac{\sqrt{c\,x^2+a}}{2a\,x^2}+\frac{c\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}+\frac{16f^2e\sqrt{c\,x^2+a}}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2ax}-\frac{64f^3\left(df-e^2\right)\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{c\,x^2+a}}{x}\right.}{\left(-e+\sqrt{-4df+e^2}\right)^3\left(e+\sqrt{-4df+e^2}\right.}$

input
```
int(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(c*x^2+a)^(1/2)*(-2*e*x+d)/a/d^2/x^2-1/2/a/d^2*(4*f*(2*a*d*f-2*a*e^2+
c*d^2)/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+2*a^
(1/2)*(c*x^2+a)^(1/2))/x)-2*f*a*(e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/(-4*d*f+e
^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(
1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1
/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+2*f*a*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/
2))/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-2*c*d*f+c*e^2)/f^2-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2
)^(1/2)))+1/2*2^(1/2)*((-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-4*c*(e-(-4*d*f+e^2)^(1/2)
)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

3.70. $\int \frac{1}{x^3\sqrt{a+cx^2}(d+ex+fx^2)}\,dx$

3.70.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="fricas")
```

output
```
Timed out
```

3.70.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x**3/(f*x**2+e*x+d)/(c*x**2+a)**(1/2),x)
```

output
```
Integral(1/(x**3*sqrt(a + c*x**2)*(d + e*x + f*x**2)), x)
```

3.70.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+a}(fx^2+ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + a)*(f*x^2 + e*x + d)*x^3), x)
```

3.70.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(f*x^2+e*x+d)/(c*x^2+a)^(1/2),x, algorithm="giac")
```

output
```
Timed out
```

3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x^3*(a + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.71 $\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.71.1 Optimal result

Integrand size = 27, antiderivative size = 499

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{1}{cf\sqrt{a+cx^2}} - \frac{ex}{af^2\sqrt{a+cx^2}} + \frac{af(cd^2+a(e^2-df))+ce(cd^2+a(e^2-2df))x}{af^2(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$
$$-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$+\frac{\left(2adef-\left(e+\sqrt{e^2-4df}\right)(cd^2+a(e^2-df))\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
-1/c/f/(c*x^2+a)^(1/2)-e*x/a/f^2/(c*x^2+a)^(1/2)+(a*f*(c*d^2+a*(-d*f+e^2))
+c*e*(c*d^2+a*(-2*d*f+e^2))*x)/a/f^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2
)-1/2*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/
2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*d*e*f-(c*d^2+a
*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*
f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctan
h(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+
c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*d*e*f-(c*d^2+a*(-d*f+e^2))
*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)
/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.59 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.80

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-a(-cd+af+cex)-\sqrt{a}c\sqrt{a+cx^2}\text{RootSum}\Big[c^2d+2\sqrt{a}ce\#1-2cd}{}$$

input
```
Integrate[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-(a*(-(c*d) + a*f + c*e*x)) - Sqrt[a]*c*Sqrt[a + c*x^2]*RootSum[c^2*d + 2
*Sqrt[a]*c*e*#1 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & ,
(Sqrt[a]*c*d*e*Log[x] - Sqrt[a]*c*d*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#
1] + 2*c*d^2*Log[x]*#1 + 2*a*e^2*Log[x]*#1 - 2*a*d*f*Log[x]*#1 - 2*c*d^2*L
og[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - 2*a*e^2*Log[-Sqrt[a] + Sqrt[a +
 c*x^2] - x*#1]*#1 + 2*a*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 - S
qrt[a]*d*e*Log[x]*#1^2 + Sqrt[a]*d*e*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1
]*#1^2)/(Sqrt[a]*c*e - 2*c*d*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3)
& ])/(c*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sqrt[a + c*x^2])
```

3.71.3 Rubi [A] (verified)

Time = 1.79 (sec) , antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{x(e^2-df)+de}{f^2(a+cx^2)^{3/2}(d+ex+fx^2)}-\frac{e}{f^2(a+cx^2)^{3/2}}+\frac{x}{f(a+cx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\left(2adef-\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{\left(2adef-\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)+cd^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{cex\left(a\left(e^2-2df\right)+cd^2\right)+af\left(a\left(e^2-df\right)+cd^2\right)}{af^2\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}-\frac{ex}{af^2\sqrt{a+cx^2}}-\frac{1}{cf\sqrt{a+cx^2}}$$

input
```
Int[x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
-(1/(c*f*Sqrt[a + c*x^2])) - (e*x)/(a*f^2*Sqrt[a + c*x^2]) + (a*f*(c*d^2 +
 a*(e^2 - d*f)) + c*e*(c*d^2 + a*(e^2 - 2*d*f))*x)/(a*f^2*(a*c*e^2 + (c*d
- a*f)^2)*Sqrt[a + c*x^2]) - ((2*a*d*e*f - (e - Sqrt[e^2 - 4*d*f])*(c*d^2
+ a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*S
qrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])]])/(S
qrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 -
 2*d*f - e*Sqrt[e^2 - 4*d*f])]) + ((2*a*d*e*f - (e + Sqrt[e^2 - 4*d*f])*(c
*d^2 + a*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt
[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])
]])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])])
```

3.71.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1575 vs. $2(456) = 912$.

Time = 0.64 (sec) , antiderivative size = 1576, normalized size of antiderivative = 3.16

method	result	size
default	Expression too large to display	1576

input
```
int(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/c/f/(c*x^2+a)^(1/2)-e*x/a/f^2/(c*x^2+a)^(1/2)+1/2*(e^3-3*d*e*f+e^2*(-4*
d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4
*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^
(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2
)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1
/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/2*(-d*f*(-
4*d*f+e^2)^(1/2)+e^2*(-4*d*f+e^2)^(1/2)+3*d*e*f-e^3)/f^3/(-4*d*f+e^2)^(1/2
)*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4
*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e...
```

3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27621 vs. $2(454) = 908$.

Time = 121.37 (sec) , antiderivative size = 27621, normalized size of antiderivative = 55.35

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.71.6 Sympy [F]

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^3}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

input
```
integrate(x**3/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**3/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.71.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.71.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x^3/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.71.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^3}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^3/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.72 $\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.72.1 Optimal result

Integrand size = 27, antiderivative size = 410

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{ae+(cd-af)x}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2d(cd-af)+ae\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
(-a*e-(-a*f+c*d)*x)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctanh(1
/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(
e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*d*(-a*f+c*d)+a*e*(e-(-4*d*f+e^2
)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^
2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d
*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2)))^(1/2))*(2*d*(-a*f+c*d)+a*e*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a
*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2)))^(1/2)
```

3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.55 (sec) , antiderivative size = 385, normalized size of antiderivative = 0.94

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-ae-cdx+afx-\sqrt{a+cx^2}\text{RootSum}\left[c^2d+2\sqrt{a}ce\#1-2cd\#1^2+4\right.}{}$$

input
```
Integrate[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-(a*e) - c*d*x + a*f*x - Sqrt[a + c*x^2]*RootSum[c^2*d + 2*Sqrt[a]*c*e*#1
 - 2*c*d*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d^2*Log[
x]) + a*c*d*f*Log[x] + c^2*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] - a*
c*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1] + 2*a^(3/2)*e*f*Log[x]*#1 - 2
*a^(3/2)*e*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1 + c*d^2*Log[x]*#1^2
 - a*d*f*Log[x]*#1^2 - c*d^2*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2 +
 a*d*f*Log[-Sqrt[a] + Sqrt[a + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*c*e) + 2*c*
d*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1^2 - 2*d*#1^3) & ])/((c^2*d^2 + a^2*f^2 +
a*c*(e^2 - 2*d*f))*Sqrt[a + c*x^2])
```

3.72.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 393, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {2136, 27, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{2136}$$

$$\frac{\int \frac{2ac(d(cd-af)-aefx)}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{2ac\left((cd-af)^2+ace^2\right)}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{d(cd-af)-aefx}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{(cd-af)^2+ace^2}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$\big\downarrow$ 1367

$$\frac{\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}}{(cd-af)^2+ace^2}-\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$\big\downarrow$ 488

$$\frac{\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right.\right.}{}}{(cd-af)^2+ace^2}$$

$$\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$\big\downarrow$ 219

$$\frac{\frac{f\left(2d(cd-af)+ae\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2d(cd-af)+ae\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{}{\sqrt{2}\sqrt{}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\sqrt{e\right.}}}{(cd-af)^2+ace^2}$$

$$\frac{x(cd-af)+ae}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

input

```
Int[x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

3.72. $\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

output
```
-((a*e + (c*d - a*f)*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) + (-(
(f*(2*d*(c*d - a*f) + a*e*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e -
 Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2
 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (f*(2*d*(c*d - a*f) + a*e*(e + S
qrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*
Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(
Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d
*f])]))/(a*c*e^2 + (c*d - a*f)^2)
```

3.72.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

rule 2136
```
Int[(Px_)*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a
*c*e^2 + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c^2*
d - c*(2*a*f)) + c*(A*(2*c^2*d - c*(2*a*f)) - B*(-2*a*c*e) + C*(-2*a*(c*d -
 a*f)))*x), x] + Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[
(a + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*A*c - 2*a*C)*((c*d - a*f)^
2 - ((-a)*e)*(c*e))*(p + 1) + (2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*
f)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e) + ((-a)*B)*(2*c
^2*d - c*((Plus[2])*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a*c*e) + ((-a
)*B)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p + q + 2) - (2*(A*c*(c*d - a*f) -
a*(c*C*d - B*c*e - a*C*f)))*((-c)*e*(2*p + q + 4)))*x - c*f*(2*(A*c*(c*d -
a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; Free
Q[{a, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[a*c*e^2 +
(c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1524 vs. $2(372) = 744$.

Time = 0.65 (sec) , antiderivative size = 1525, normalized size of antiderivative = 3.72

method	result	size
default	Expression too large to display	1525

input
```
int(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/f*x/a/(c*x^2+a)^(1/2)+1/2*(-e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/f^2/(-4*d*f+
e^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+
e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/
2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*
e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*
c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(
1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2)
)/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*
f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(
(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2
))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c
-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)))+1/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f^2/(-4*d*f+e^2)^(1/2)*(2/
(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+
e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2
)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*...
```

3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26116 vs. $2(369) = 738$.

Time = 125.13 (sec) , antiderivative size = 26116, normalized size of antiderivative = 63.70

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.72.6 Sympy [F]

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{(a+cx^2)^{\frac{3}{2}}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**2/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.72.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.72.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.72.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^2}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^2/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.73 $\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.73.1 Optimal result

Integrand size = 25, antiderivative size = 411

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{cd-af-cex}{(ace^2+(cd-af)^2)\sqrt{a+cx^2}} + \frac{f\left(2cde-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} - \frac{f\left(2cde-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
(c*e*x+a*f-c*d)/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2*f*arctanh(1/2*(
2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-
2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*c*d*e-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1
/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*
d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e
^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(
1/2)))^(1/2))*(2*c*d*e-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/(a*c*e^2+(-a*f+c
*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/
2)))^(1/2)
```

3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 330, normalized size of antiderivative = 0.80

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-cd+af+cex-\sqrt{a+cx^2}\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2a\right.}{}$$

input
```
Integrate[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-(c*d) + a*f + c*e*x - Sqrt[a + c*x^2]*RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 +
 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*c*d*f*Log[-(
Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]) + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1] - 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 +
 c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]
*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*S
qrt[c]*e*#1^2 + 2*f*#1^3) & ])/((c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sq
rt[a + c*x^2])
```

3.73.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1350, 27, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \; 1350$$

$$\frac{\int -\frac{2ac(cde+f(cd-af)x)}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{2ac\left((cd-af)^2+ace^2\right)}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{cde+f(cd-af)x}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{(cd-af)^2+ace^2}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 1367}$$

$$-\frac{\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}}{(cd-af)^2+ace^2}-\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 488}$$

$$-\frac{\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}-\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-\ldots\right.}{\ldots}}{(cd-af)^2+ace^2}$$

$$\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\frac{f\left(2cde-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2cde-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\text{arctanh}\left(\ldots\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\ldots\right.}}}{(cd-af)^2+ace^2}$$

$$\frac{-af+cd-cex}{\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

input
```
Int[x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
-((c*d - a*f - c*e*x)/((a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2])) - (-((f
*(2*c*d*e - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e - S
qrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 -
 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(
e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (f*(2*c*d*e - (c*d - a*f)*(e + Sqr
t[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sq
rt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sq
rt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f
])]))/(a*c*e^2 + (c*d - a*f)^2)
```

3.73. $\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.73.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1350
```
Int[((g_.) + (h_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + c*x^2)^(p + 1)*((d + e*x + f*x^2)^
(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1)))*(g*c*(2*a*c*e) + ((-a
)*h)*(2*c^2*d - c*(2*a*f)) + c*(g*(2*c^2*d - c*(2*a*f)) - h*(-2*a*c*e))*x),
 x] + Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[(a + c*x^2)
^(p + 1)*(d + e*x + f*x^2)^q*Simp[(-2*g*c)*((c*d - a*f)^2 - ((-a)*e)*(c*e))
*(p + 1) + (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(a*f*(p + 1) - c*d*(p + 2))
 - e*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d - c*((Plus[2])*a*f)))*(p + q + 2)
 - (2*f*((g*c)*(2*a*c*e) + ((-a)*h)*(2*c^2*d + (-c)*((Plus[2])*a*f)))*(p +
q + 2) - (2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*((-c)*e*(2*p + q + 4)))*x - c
*f*(2*(g*c*(c*d - a*f) - a*((-h)*c*e)))*(2*p + 2*q + 5)*x^2, x], x], x] /;
FreeQ[{a, c, d, e, f, g, h, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] &&
NeQ[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1489 vs. $2(370) = 740$.

Time = 0.75 (sec) , antiderivative size = 1490, normalized size of antiderivative = 3.63

method	result	size
default	Expression too large to display	1490

input
```
int(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*(2/((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4
*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c
*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^
(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4
*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))+1/2*(-e+(-4*d*f+e^2)^(1/2)
)/(-4*d*f+e^2)^(1/2)/f*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*
f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1
/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f
+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(-4*d*f+e^2)^(1/2)*c*...
```

3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26234 vs. $2(368) = 736$.

Time = 105.28 (sec) , antiderivative size = 26234, normalized size of antiderivative = 63.83

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.73.6 Sympy [F]

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

input
```
integrate(x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.73.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.73.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.73.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.74 $\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.74.1 Optimal result

Integrand size = 24, antiderivative size = 416

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{c(ae+(cd-af)x)}{a(ace^2+(cd-af)^2)\sqrt{a+cx^2}} - \frac{f\left(2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}} + \frac{f\left(2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

output
```
c*(a*e+(-a*f+c*d)*x)/a/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)-1/2*f*arctan
h(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+
c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f
+e^2)^(1/2)))/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c
*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(
-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*
f+e^2)^(1/2)))^(1/2))*(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/(a*c*e^
2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f
+e^2)^(1/2)))^(1/2)
```

3.74.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 346, normalized size of antiderivative = 0.83

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{c(cdx+a(e-fx)) - a\sqrt{a+cx^2}\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2\right.}{}$$

input
```
Integrate[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(c*(c*d*x + a*(e - f*x)) - a*Sqrt[a + c*x^2]*RootSum[a^2*f + 2*a*Sqrt[c]*e
*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e*f*Log
[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^2*Log[-(Sqrt[c]*x) + S
qrt[a + c*x^2] - #1]*#1 - 2*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2]
 - #1]*#1 + 2*a*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 -
c*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*
#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/(a*(c^2*d^2 + a^2*f^2 +
a*c*(e^2 - 2*d*f))*Sqrt[a + c*x^2])
```

3.74.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1307, 27, 1367, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \; 1307$$

$$\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)} - \frac{\int -\frac{2ac(af^2+cexf+c(e^2-df))}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{2ac\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{af^2+cexf+c(e^2-df)}{\sqrt{cx^2+a}(fx^2+ex+d)}dx}{(cd-af)^2+ace^2} + \frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 1367}$$

$$\frac{\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}-\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+a}}dx}{\sqrt{e^2-4df}}}{(cd-af)^2+ace^2}+\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 488}$$

$$\frac{\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\int \frac{1}{4af^2+c\left(e+\sqrt{e^2-4df}\right)^2-\frac{\left(2af-c\left(e+\sqrt{e^2-4df}\right)x\right)^2}{cx^2+a}}d\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{cx^2+a}}}{\sqrt{e^2-4df}}-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+\cdots\right.\right.}{\cdots}}{(cd-af)^2+ace^2}$$

$$\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{f\left(2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)\right)\text{arctanh}\left(\cdots\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+c\left(-e\cdots\right)}}}{(cd-af)^2+ace^2}$$

$$\frac{c(x(cd-af)+ae)}{a\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}$$

input
```
Int[1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(c*(a*e + (c*d - a*f)*x))/(a*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) +
(-((f*(2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c
*(e - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqr
t[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])])) + (f*(2*a*f^2 + c*(e^2 - 2*d*
f - e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^
2 - 4*d*f])]))/(a*c*e^2 + (c*d - a*f)^2)
```

3.74. $\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1307
```
Int[((a_.) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(2*a*c^2*e + c*(2*c^2*d - c*(2*a*f))*x)*(a + c*x^2)^(p + 1
)*((d + e*x + f*x^2)^(q + 1)/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))),
 x] - Simp[1/((-4*a*c)*(a*c*e^2 + (c*d - a*f)^2)*(p + 1))   Int[(a + c*x^2)
^(p + 1)*(d + e*x + f*x^2)^q*Simp[2*c*((c*d - a*f)^2 - ((-a)*e)*(c*e))*(p +
 1) - (2*c^2*d - c*(2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(-2*a*c^2*e)*(p
 + q + 2) + (2*f*(2*a*c^2*e)*(p + q + 2) - (2*c^2*d - c*(2*a*f))*((-c)*e*(2
*p + q + 4)))*x + c*f*(2*c^2*d - c*(2*a*f))*(2*p + 2*q + 5)*x^2, x], x], x]
 /; FreeQ[{a, c, d, e, f, q}, x] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && Ne
Q[a*c*e^2 + (c*d - a*f)^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ
[q, 0]
```

rule 1367
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (f
_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(2*c*g - h*(
b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + f*x^2]), x], x] - Simp[(2*c*g -
 h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + f*x^2]), x], x]] /; FreeQ[{
a, b, c, d, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[b^2 - 4*a*c]
```

3.74.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1456 vs. 2(377) = 754.

Time = 0.79 (sec) , antiderivative size = 1457, normalized size of antiderivative = 3.50

method	result	size
default	Expression too large to display	1457

input
```
int(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^
2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*
(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^
2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1
/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4
*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f
+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*
d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*
f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+
2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c-c*(e-(-4*d*
f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*(-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e-(-4*d*f+e^2)^(1/2))*f/(-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x-1/2/f*(-e+(-4*d*f+e^2...
```

3.74.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27447 vs. 2(375) = 750.

Time = 74.11 (sec) , antiderivative size = 27447, normalized size of antiderivative = 65.98

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.74.6 Sympy [F]

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(a+cx^2)^{\frac{3}{2}}(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/((a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.74.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.74.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.74.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{1}{(c\,x^2+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.75 $\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.75.1 Optimal result

Integrand size = 27, antiderivative size = 526

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{1}{ad\sqrt{a+cx^2}} - \frac{a(af^2+c(e^2-df))+c^2dex}{ad\left(ace^2+(cd-af)^2\right)\sqrt{a+cx^2}}$$

$$+ \frac{f\left(2e(af^2+c(e^2-2df))-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e-\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$

$$- \frac{f\left(2e(af^2+c(e^2-2df))-\left(e+\sqrt{e^2-4df}\right)\left(af^2+c(e^2-df)\right)\right)\operatorname{arctanh}\left(\frac{2af-c\left(e+\sqrt{e^2-4df}\right)x}{\sqrt{2}\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}\sqrt{a+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left(ace^2+(cd-af)^2\right)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$

$$- \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

output
```
-arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d+1/a/d/(c*x^2+a)^(1/2)+(-a*(a*f
^2+c*(-d*f+e^2))-c^2*d*e*x)/a/d/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2)+1/2
*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/
(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*(a*f^2+c*(-2*d*f+
e^2))-(a*f^2+c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a*f+c*d)^2
)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^
(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+
a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2))*(2*e*(a*f^2+c
*(-2*d*f+e^2))-(a*f^2+c*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d/(a*c*e^2+(-a
*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)
^(1/2)))^(1/2)
```

3.75.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.82 (sec) , antiderivative size = 542, normalized size of antiderivative = 1.03

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx =$$

$$-\frac{c(-cd+af+cex)}{a\left(c^2d^2+a^2f^2+ac\left(e^2-2df\right)\right)\sqrt{a+cx^2}}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

$$+\frac{\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+4cd\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&,\frac{ace^2f\log\left(-\sqrt{c}x+\sqrt{a+cx^2}-\#1\right)-acdf^2\log}{}\right.}{}$$

input
```
Integrate[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
-((c*(-(c*d) + a*f + c*e*x))/(a*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f))*Sq
rt[a + c*x^2])) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[a]])/(a^(3
/2)*d) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2 - 2*Sq
rt[c]*e*#1^3 + f*#1^4 & , (a*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
#1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*f^3*Log[-(S
qrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[
a + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] -
 #1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 -
c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + c*d*f^2*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*
x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2
*f*#1^3) & ]/(d*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))
```

3.75.3 Rubi [A] (verified)

Time = 1.71 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d(a+cx^2)^{3/2}(d+ex+fx^2)}+\frac{1}{dx(a+cx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}+\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(e-\sqrt{e^2-4df}\right)\left(af^2+c\left(e^2-df\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(e-\sqrt{e^2-4df}\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}-\frac{f\left(2e\left(af^2+c\left(e^2-2df\right)\right)-\left(\sqrt{e^2-4df}+e\right)\left(af^2+c\left(e^2-df\right)\right)\right)\text{arctanh}\left(\frac{2af-cx\left(\sqrt{e^2-4df}+e\right)}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2+ace^2\right)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{a\left(af^2+c\left(e^2-df\right)\right)+c^2dex}{ad\sqrt{a+cx^2}\left((cd-af)^2+ace^2\right)}+\frac{1}{ad\sqrt{a+cx^2}}$$

input
```
Int[1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
1/(a*d*Sqrt[a + c*x^2]) - (a*(a*f^2 + c*(e^2 - d*f)) + c^2*d*e*x)/(a*d*(a*
c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(2*e*(a*f^2 + c*(e^2 - 2*d*f)
) - (e - Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - d*f)))*ArcTanh[(2*a*f - c*(e
 - Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e
^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (
c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*
(2*e*(a*f^2 + c*(e^2 - 2*d*f)) - (e + Sqrt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 -
 d*f)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*d*S
qrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f +
 e*Sqrt[e^2 - 4*d*f])]) - ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]]/(a^(3/2)*d)
```

3.75.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.75.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1564 vs. $2(480) = 960$.

Time = 0.68 (sec) , antiderivative size = 1565, normalized size of antiderivative = 2.98

method	result	size
default	Expression too large to display	1565

input
```
int(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/a/(c*x^2+a)^(1/2)-1
/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))
/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/
((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e
+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)
/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2
))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d
*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e
^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*
f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^
(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+
e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*
((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*(2/(-(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x-1/2/f*(-e+(-4*d*f+e^2)
^(1/2)))^2*c-c*(e-(-4*d*f+e^2)^(1/2))/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)...
```

3.75.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\,(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.75.6 Sympy [F]

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\left(a+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

input
```
integrate(1/x/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.75.7 Maxima [F]

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{\left(cx^2+a\right)^{\frac{3}{2}}\left(fx^2+ex+d\right)x}\,dx$$

input
```
integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

3.75.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.75.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\,{\left(c\,x^2+a\right)}^{3/2}\,\left(f\,x^2+e\,x+d\right)} \,d x$$

input
```
int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.76 $\int \frac{1}{x^2(a+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.76.1 Optimal result

Integrand size = 27, antiderivative size = 618

$$\int \frac{1}{x^2\,(a+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = -\frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$
$$-\frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{ae(af^2+c(e^2-2df))+cd(af^2+c(e^2-df))\,x}{ad^2\,(ace^2+(cd-af)^2)\sqrt{a+cx^2}}$$
$$+\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\operatorname{arctanh}\left(\frac{2af\cdots}{\sqrt{2}\sqrt{2af^2+c(\cdots}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df-e\sqrt{e^2-4df}\right)}}$$
$$-\frac{f\left(e\left(e+\sqrt{e^2-4df}\right)(af^2+c(e^2-2df))-2(af^2(e^2-df)+c(e^4-3de^2f+d^2f^2))\right)\operatorname{arctanh}\left(\frac{2af\cdots}{\sqrt{2}\sqrt{2af^2+c(\cdots}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\,(ace^2+(cd-af)^2)\sqrt{2af^2+c\left(e^2-2df+e\sqrt{e^2-4df}\right)}}$$
$$+\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

output
```
e*arctanh((c*x^2+a)^(1/2)/a^(1/2))/a^(3/2)/d^2-e/a/d^2/(c*x^2+a)^(1/2)-1/a
/d/x/(c*x^2+a)^(1/2)-2*c*x/a^2/d/(c*x^2+a)^(1/2)+(a*e*(a*f^2+c*(-2*d*f+e^2
))+c*d*(a*f^2+c*(-d*f+e^2))*x)/a/d^2/(a*c*e^2+(-a*f+c*d)^2)/(c*x^2+a)^(1/2
)+1/2*f*arctanh(1/2*(2*a*f-c*x*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(
1/2)/(2*a*f^2+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2))*(-2*a*f^2*(-d*f+e
^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f^2+c*(-2*d*f+e^2))*(e-(-4*d*f+e^2)^(
1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e
^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*f*arctanh(1/2*(2*a*f-c*x*(e+(-4*
d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+a)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e
^2)^(1/2)))^(1/2))*(-2*a*f^2*(-d*f+e^2)-2*c*(d^2*f^2-3*d*e^2*f+e^4)+e*(a*f
^2+c*(-2*d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d^2/(a*c*e^2+(-a*f+c*d)^2)*2^(1
/2)/(-4*d*f+e^2)^(1/2)/(2*a*f^2+c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.76.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.19 (sec) , antiderivative size = 684, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)}\,dx =$$

$$-\frac{\frac{d(a^3f^2+2c^3d^2x^2+ac^2(d^2+e^2x^2+dx(e-3fx))+a^2c(e^2+f(-2d+fx^2)))}{a^2(c^2d^2+a^2f^2+ac(e^2-2df))x\sqrt{a+cx^2}}+\frac{2e\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}}+\frac{\text{RootSum}\left[a^2f+2a\sqrt{c}e\#1+\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
-(((d*(a^3*f^2 + 2*c^3*d^2*x^2 + a*c^2*(d^2 + e^2*x^2 + d*x*(e - 3*f*x)) +
 a^2*c*(e^2 + f*(-2*d + f*x^2))))/(a^2*(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d
*f))*x*Sqrt[a + c*x^2]) + (2*e*ArcTanh[(Sqrt[c]*x - Sqrt[a + c*x^2])/Sqrt[
a]])/a^(3/2) + RootSum[a^2*f + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 - 2*a*f*#1^2
- 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (a*c*e^3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x
^2] - #1] - 2*a*c*d*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + a^2*e
*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1] + 2*c^(3/2)*e^4*Log[-(Sqrt[c
]*x) + Sqrt[a + c*x^2] - #1]*#1 - 6*c^(3/2)*d*e^2*f*Log[-(Sqrt[c]*x) + Sqr
t[a + c*x^2] - #1]*#1 + 2*c^(3/2)*d^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^
2] - #1]*#1 + 2*a*Sqrt[c]*e^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]
*#1 - 2*a*Sqrt[c]*d*f^3*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1 - c*e^
3*f*Log[-(Sqrt[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 + 2*c*d*e*f^2*Log[-(Sqrt
[c]*x) + Sqrt[a + c*x^2] - #1]*#1^2 - a*e*f^3*Log[-(Sqrt[c]*x) + Sqrt[a +
c*x^2] - #1]*#1^2)/(a*Sqrt[c]*e + 4*c*d*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 +
 2*f*#1^3) & ]/(c^2*d^2 + a^2*f^2 + a*c*(e^2 - 2*d*f)))/d^2)
```

3.76.3 Rubi [A] (verified)

Time = 1.88 (sec) , antiderivative size = 618, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{-df+e^2+efx}{d^2 (a+cx^2)^{3/2} (d+ex+fx^2)}-\frac{e}{d^2x (a+cx^2)^{3/2}}+\frac{1}{dx^2 (a+cx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e\operatorname{arctanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} +$$

$$\frac{f\left(e\left(e-\sqrt{e^2-4df}\right)(af^2+c(e^2-2df)) - 2(af^2(e^2-df)+c(d^2f^2-3de^2f+e^4))\right)\operatorname{arctanh}\left(\frac{2af-cx}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\,((cd-af)^2+ace^2)\sqrt{2af^2+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{f\left(e\left(\sqrt{e^2-4df}+e\right)(af^2+c(e^2-2df)) - 2(af^2(e^2-df)+c(d^2f^2-3de^2f+e^4))\right)\operatorname{arctanh}\left(\frac{2af-cx(}{\sqrt{2}\sqrt{a+cx^2}\sqrt{2af^2+}}\right.}{\sqrt{2}d^2\sqrt{e^2-4df}\,((cd-af)^2+ace^2)\sqrt{2af^2+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{cdx(af^2+c(e^2-df))+ae(af^2+c(e^2-2df))}{ad^2\sqrt{a+cx^2}\,((cd-af)^2+ace^2)} - \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$

input
```
Int[1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
-(e/(a*d^2*Sqrt[a + c*x^2])) - 1/(a*d*x*Sqrt[a + c*x^2]) - (2*c*x)/(a^2*d*
Sqrt[a + c*x^2]) + (a*e*(a*f^2 + c*(e^2 - 2*d*f)) + c*d*(a*f^2 + c*(e^2 -
d*f))*x)/(a*d^2*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[a + c*x^2]) + (f*(e*(e - Sq
rt[e^2 - 4*d*f])*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4
 - 3*d*e^2*f + d^2*f^2)))*ArcTanh[(2*a*f - c*(e - Sqrt[e^2 - 4*d*f])*x)/(S
qrt[2]*Sqrt[2*a*f^2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^
2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^
2 + c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])]) - (f*(e*(e + Sqrt[e^2 - 4*d*f]
)*(a*f^2 + c*(e^2 - 2*d*f)) - 2*(a*f^2*(e^2 - d*f) + c*(e^4 - 3*d*e^2*f +
d^2*f^2)))*ArcTanh[(2*a*f - c*(e + Sqrt[e^2 - 4*d*f])*x)/(Sqrt[2]*Sqrt[2*a
*f^2 + c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f])]*Sqrt[a + c*x^2])])/(Sqrt[2]*
d^2*Sqrt[e^2 - 4*d*f]*(a*c*e^2 + (c*d - a*f)^2)*Sqrt[2*a*f^2 + c*(e^2 - 2*
d*f + e*Sqrt[e^2 - 4*d*f])]) + (e*ArcTanh[Sqrt[a + c*x^2]/Sqrt[a]])/(a^(3/
2)*d^2)
```

3.76.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.76.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. $2(565) = 1130$.

Time = 0.79 (sec) , antiderivative size = 1639, normalized size of antiderivative = 2.65

method	result	size
default	Expression too large to display	1639
risch	Expression too large to display	1788

input
```
int(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(-1/a/x/(c*x^2+a)^(1/2
)-2*c/a^2*x/(c*x^2+a)^(1/2))-16*f^2*e/(-e+(-4*d*f+e^2)^(1/2))^2/(e+(-4*d*f
+e^2)^(1/2))^2*(1/a/(c*x^2+a)^(1/2)-1/a^(3/2)*ln((2*a+2*a^(1/2)*(c*x^2+a)^
(1/2))/x))-4*f^2/(e+(-4*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(2/((-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*((-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)+2*c*(e+(-4*d*f+e^2)^(
1/2))*f/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*
d*f+e^2)^(1/2))/f)-c*(e+(-4*d*f+e^2)^(1/2))/f)/(2*c*((-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-2*c*d*f+c*e^2)/f^2-c^2*(e+(-4*d*f+e^2)^(1/2))^2/f^2)/((x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)^2*c-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)
-2/((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)*f^2*2^(1/2)/(((-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/f^2)^(1/2)*ln((((-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-2*c*d*f+c*e^2)/f^2-c*(e+(-4*d*f+e^2)^(1/2))/f*(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)+1/2*2^(1/2)*(((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2*c*d*f+c*e^2)/
f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c-4*c*(e+(-4*d*f+e^2)^(1/
2))/f*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*((-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-2
*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+4*f^2/(-e+(-4
*d*f+e^2)^(1/2))^2/(-4*d*f+e^2)^(1/2)*(2/(-(-4*d*f+e^2)^(1/2)*c*e+2*a*f...
```

3.76.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.76.6 Sympy [F]

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \int \frac{1}{x^2 (a+cx^2)^{\frac{3}{2}} (d+ex+fx^2)} \, dx$$

input
```
integrate(1/x**2/(c*x**2+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x**2*(a + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.76.7 Maxima [F]

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \int \frac{1}{(cx^2+a)^{\frac{3}{2}} (fx^2+ex+d)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + a)^(3/2)*(f*x^2 + e*x + d)*x^2), x)
```

3.76.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/x^2/(c*x^2+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.76.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+cx^2)^{3/2} (d+ex+fx^2)} \, dx = \int \frac{1}{x^2 \left(c\,x^2+a\right)^{3/2} \left(f\,x^2+e\,x+d\right)} \, dx$$

input
```
int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x^2*(a + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.77 $\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.77.1 Optimal result

Integrand size = 28, antiderivative size = 392

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{d\sqrt{a+bx+cx^2}}{f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{(a+bx+cx^2)^{3/2}}{3cf} - \frac{bd\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}-\frac{b(b^2-4ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f} - \frac{d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

output
```
-1/3*(c*x^2+b*x+a)^(3/2)/c/f-1/16*b*(-4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f-1/2*b*d*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)-d*(c*x^2+b*x+a)^(1/2)/f^2+1/8*b*(2*c*x+b)
*(c*x^2+b*x+a)^(1/2)/c^2/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c
*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)
)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2
*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/
2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(5/2)
```

3.77.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.04

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$= \frac{-2\sqrt{c}\sqrt{a+x(b+cx)}(-3b^2f+2cf(4a+bx)+8c^2(3d+fx^2))+3b(8c^2d+b^2f-4acf)\log\left(b+2cx-2\right.}{}$$

input
```
Integrate[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(-3*b^2*f + 2*c*f*(4*a + b*x) + 8*c^2*(3
*d + f*x^2)) + 3*b*(8*c^2*d + b^2*f - 4*a*c*f)*Log[b + 2*c*x - 2*Sqrt[c]*S
qrt[a + x*(b + c*x)]] - 24*c^(5/2)*d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d
*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(48*c^
(5/2)*f^2)
```

3.77.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$\Big\downarrow \text{7276}$$
$$\int \left(\frac{dx\sqrt{a+bx+cx^2}}{f\left(d-fx^2\right)}-\frac{x\sqrt{a+bx+cx^2}}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b(b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2}f}-\frac{d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{5/2}}+\frac{d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{5/2}}-\frac{bd\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}+\frac{b(b+2cx)\sqrt{a+bx+cx^2}}{8c^2f}-\frac{d\sqrt{a+bx+cx^2}}{f^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3cf}$$

input
```
Int[(x^3*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
-((d*Sqrt[a + b*x + c*x^2])/f^2) + (b*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(
8*c^2*f) - (a + b*x + c*x^2)^(3/2)/(3*c*f) - (b*d*ArcTanh[(b + 2*c*x)/(2*S
qrt[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*f^2) - (b*(b^2 - 4*a*c)*ArcTanh
[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(5/2)*f) - (d*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqr
t[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
 c*x^2])])/(2*f^(5/2)) + (d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b
*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt
[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(5/2))
```

3.77.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.77.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 525, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{(8f c^2x^2+2bcfx+8acf-3b^2f+24c^2d)\sqrt{c x^2+bx+a}}{24c^2 f^2}+\frac{\frac{b\left(4acf-b^2f-8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{\sqrt{c}}+\frac{8c^2d(\sqrt{df}\,af+\sqrt{df}\,cd+bdf)\ln\left(\cdots\right.}{\cdots}}{\cdots}$
default	$-\frac{\frac{\left(c x^2+bx+a\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}\right)}{2c}}{f}-\frac{d\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}\cdots}{\cdots}}\right.}{\cdots}$

input
```
int(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(8*c^2*f*x^2+2*b*c*f*x+8*a*c*f-3*b^2*f+24*c^2*d)*(c*x^2+b*x+a)^(1/2)
/c^2/f^2+1/16/f^2/c^2*(b*(4*a*c*f-b^2*f-8*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))/c^(1/2)+8*c^2*d*((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)
/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+8*c^2*d*((d*f)^(1/2
)*a*f+(d*f)^(1/2)*c*d-b*d*f)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2))/(x+(d*f)^(1/2)/f)))
```

3.77.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.77.6 Sympy [F]

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate(x**3*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x**3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.77.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.77.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.77.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{x^3\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

input
```
int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

output
```
int((x^3*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.78 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.78.1 Optimal result

Integrand size = 28, antiderivative size = 316

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf} - \frac{(8c^2d-b^2f+4acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^2} + \frac{\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2} + \frac{\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2}$$

output
```
-1/8*(4*a*c*f-b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(3/2)/f^2-1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c/f+1/2*arctanh(1/2*(b
*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a
*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f^2
+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)
*f^(1/2))^(1/2)/f^2
```

3.78.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.54 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.01

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$= \frac{-2\sqrt{c}f(b+2cx)\sqrt{a+x(b+cx)}+(-8c^2d+b^2f-4acf)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)-4c^{3/2}d\text{RootSum}\left[b^2c\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
(-2*Sqrt[c]*f*(b + 2*c*x)*Sqrt[a + x*(b + c*x)] + (-8*c^2*d + b^2*f - 4*a*
c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 4*c^(3/2)*d*
RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^
4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])
/(8*c^(3/2)*f^2)
```

3.78.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$, Rules used = {2140, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$\Big\downarrow \; 2140$$
$$-\frac{\int -\frac{f(-fb^2+8c^2d+4acf)x^2+8bcdfx+(b^2+4ac)df}{4\sqrt{cx^2+bx+a}(d-fx^2)}dx}{2cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{f\left(-fb^2+8c^2d+4acf\right)x^2+8bcdfx+\left(b^2+4ac\right)df}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 2144

$$\frac{-\left(\left(4acf+b^2(-f)+8c^2d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)-\frac{\int -\frac{8cdf(cd+af+bfx)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f}}{8cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 27

$$\frac{8cd\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-\left(4acf+b^2(-f)+8c^2d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 1092

$$\frac{8cd\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-2\left(4acf+b^2(-f)+8c^2d\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 219

$$\frac{8cd\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(4acf+b^2(-f)+8c^2d\right)}{\sqrt{c}}}{8cf^2}-\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 1366

$$\frac{8cd\left(\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)-\text{ar}}{8cf^2}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 25

$$\frac{8cd\left(\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)-\text{arct}}{8cf^2}$$

$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$\big\downarrow$ 27

3.78. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

$$\frac{8cd\left(\frac{1}{2}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)-\frac{\text{arctanh}\left(\frac{b+2}{2\sqrt{c}\sqrt{a+}}\right.}{}}{8cf^2}$$
$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$$\Big\downarrow \; 1154$$

$$\frac{8cd\left(\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)-\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{\right.}{}$$
$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

$$\Big\downarrow \; 219$$

$$\frac{8cd\left(\frac{\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)-\frac{\text{arcta}}{}}{8cf^2}$$
$$\frac{(b+2cx)\sqrt{a+bx+cx^2}}{4cf}$$

input
```
Int[(x^2*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
-1/4*((b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(c*f) + (-(((8*c^2*d - b^2*f + 4*
a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]) +
8*c*d*(-1/2*((b*Sqrt[f] - (c*d + a*f)/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sq
rt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f
]*Sqrt[a + b*x + c*x^2])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((b*Sqrt[
f] + (c*d + a*f)/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])))/(8*c*f^2)
```

3.78. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.78.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2140
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)
*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   Int[(a + b*x + c
*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(
2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2
*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p +
2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*
f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p
 + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p +
q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a,
 b, c, d, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] &&
 NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.78.4 Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.53

method	result
risch	$-\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4cf}-\frac{\frac{\left(4acf-b^2f+8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{f\sqrt{c}}-\frac{4cd\left(b\sqrt{df}+fa+cd\right)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}}{}\right.}{\sqrt{df}}}{}$
default	$-\frac{\frac{(2cx+b)\sqrt{c x^2+bx+a}}{4c}+\frac{\left(4ac-b^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{8c^{\frac{3}{2}}}}{f}-\frac{d\left(\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{\left(2c\sqrt{df}+bf\right)}{}\right.}{}$

input
```
int(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c/f-1/8/f/c*(1/f*(4*a*c*f-b^2*f+8*c^2*d
)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-4*c*d*(b*(d*f)^(1/2)
+f*a+c*d)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(
1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2
)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f
)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-4*c*d*(b*(
d*f)^(1/2)-f*a-c*d)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln(
(2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+
(d*f)^(1/2)/f)))
```

3.78.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.78.6 Sympy [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.78.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.78.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

input
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

output
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.79 $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.79.1 Optimal result

Integrand size = 26, antiderivative size = 282

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{a+bx+cx^2}}{f} - \frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}}$$

output

```
-1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-(c*x^2
+b*x+a)^(1/2)/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*
d^(1/2)*f^(1/2))^(1/2)/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2
*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/
2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/f^(3/2)
```

3.79.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.24

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{2\sqrt{a+x(b+cx)}-\frac{b\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}}+\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#\right.}{}$$

input
```
Integrate[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
-1/2*(2*Sqrt[a + x*(b + c*x)] - (b*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x
*(b + c*x)])])/Sqrt[c] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*
#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]*#1^2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/f
```

3.79.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {1354, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}dx$$

$$\Big\downarrow \ 1354$$

$$\frac{\int \frac{bfx^2+2(cd+af)x+bd}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{bfx^2+2(cd+af)x+bd}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 2144$$

$$\frac{-\frac{\int -\frac{2f(bd+(cd+af)x)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f}-b\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{2\int \frac{bd+(cd+af)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-b\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 1092$$

$$\frac{2\int \frac{bd+(cd+af)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-2b\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{2\int \frac{bd+(cd+af)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2f}-\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 1366$$

$$\frac{2\left(\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)}{2f}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)}{2f}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}$$

$$\Big\downarrow \; 27$$

3.79. $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

$$\frac{2\left(\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}\right)-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}}{2f}-\cdots$$

$$\frac{\sqrt{a+bx+cx^2}}{f}$$

$\downarrow$ 1154

$$\frac{2\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \cdots}{\cdots}\cdots\right.}{2f}\cdots$$

$$\frac{\sqrt{a+bx+cx^2}}{f}$$

$\downarrow$ 219

$$\frac{2\left(\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}}-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}}\right)-\cdots}{2f}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}$$

input
```
Int[(x*Sqrt[a + b*x + c*x^2])/(d - f*x^2),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/f) + (-((b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
 b*x + c*x^2])])/Sqrt[c]) + 2*(-1/2*(Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/Sqrt[f] + (Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d]
+ b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[f])))/(2*f)
```

3.79. $\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.79.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1354
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))   Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(
c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q
+ 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.79.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(214) = 428$.

Time = 0.75 (sec) , antiderivative size = 460, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{f}-\frac{\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{\sqrt{c}}-\frac{(\sqrt{df}\,af+\sqrt{df}\,cd+bdf)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{(\cdots}}{x-\frac{\sqrt{df}}{f}}\right.}{\sqrt{df}\,f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}}{\cdots}$
default	$-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{(2c\sqrt{df}+bf)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\cdots\right.}{f}}\right.}{2f\sqrt{c}}}{\cdots}$

input
```
int(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/f-1/2/f*(b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)
)/c^(1/2)-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)/(d*f)^(1/2)/f/((b*(d*f)^
(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b
*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2
)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)
/f)^(1/2))/(x-(d*f)^(1/2)/f))-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d-b*d*f)/(d*f
)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a
+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+
f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.79.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(214) = 428.

Time = 186.76 (sec) , antiderivative size = 1192, normalized size of antiderivative = 4.23

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(c*f*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log((2*sqrt(c*x^2 +
b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) +
 2*b*c*d*x + b^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt((f
^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log(-(2*sqrt(c*x^2 + b*x + a)*f^4*sqr
t(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) - 2*b*c*d*x - b^2
*d - (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt(-(f^3*sqrt(b^2*d/f
^5) - c*d - a*f)/f^3)*log((2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqr
t(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f)/f^3) + 2*b*c*d*x + b^2*d - (b*f^3*x +
 2*a*f^3)*sqrt(b^2*d/f^5))/x) + c*f*sqrt(-(f^3*sqrt(b^2*d/f^5) - c*d - a*f
)/f^3)*log(-(2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt(-(f^3*sqrt(b
^2*d/f^5) - c*d - a*f)/f^3) - 2*b*c*d*x - b^2*d + (b*f^3*x + 2*a*f^3)*sqrt
(b^2*d/f^5))/x) + b*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2
+ b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*sqrt(c*x^2 + b*x + a)*c)/(c*f)
, 1/4*(c*f*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log((2*sqrt(c*x^2 +
 b*x + a)*f^4*sqrt(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)
+ 2*b*c*d*x + b^2*d + (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt((
f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3)*log(-(2*sqrt(c*x^2 + b*x + a)*f^4*sq
rt(b^2*d/f^5)*sqrt((f^3*sqrt(b^2*d/f^5) + c*d + a*f)/f^3) - 2*b*c*d*x - b^
2*d - (b*f^3*x + 2*a*f^3)*sqrt(b^2*d/f^5))/x) - c*f*sqrt(-(f^3*sqrt(b^2*d/
f^5) - c*d - a*f)/f^3)*log((2*sqrt(c*x^2 + b*x + a)*f^4*sqrt(b^2*d/f^5)...
```

3.79.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{x\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.79.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.79.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2),x)
```

output
```
int((x*(a + b*x + c*x^2)^(1/2))/(d - f*x^2), x)
```

3.80 $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

3.80.1 Optimal result

Integrand size = 25, antiderivative size = 266

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f}$$

output
```
-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f+1/2*arctanh(
1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/f/d^(
1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*
x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(
1/2))^(1/2)/f/d^(1/2)
```

3.80.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx =$$

$$-\frac{-4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx)}}\right)+\text{RootSum}\left[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-4af\#1^2+d\#1^4\&,\frac{-c^2}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

output
```
-1/2*(-4*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] +
RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^
4 & , (-(c^2*d*Log[x]) + b^2*f*Log[x] - a*c*f*Log[x] + c^2*d*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1] - b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^
2] - x*#1] + a*c*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a
]*b*f*Log[x]*#1 + 2*Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1]*#1 + c*d*Log[x]*#1^2 + a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*
x + c*x^2] - x*#1]*#1^2 - a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ])/f
```

3.80.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {1321, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}dx$$

$$\Big\downarrow \; 1321$$

$$\frac{\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{c\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}$$

$$\Big\downarrow \; 1092$$

$$\frac{\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{2c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}$$

$$\Big\downarrow \; 219$$

$$\frac{\int \frac{cd+af+bfx}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

$$\Big\downarrow \; 1366$$

$$\frac{\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{2}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{f}-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

$$\Big\downarrow \; 1154$$

$$\frac{\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)-\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int}{f}$$

$$-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

$$\Big\downarrow \; 219$$

3.80. $\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$

$$\frac{\frac{\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}}{f}-\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(d - f*x^2),x]
```

output
```
-((Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f) + (-
1/2*((b*Sqrt[f] - (c*d + a*f)/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] +
(2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + b*x + c*x^2])])/Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f] + ((b*Sqrt[f] + (c*
d + a*f)/Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt
[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2
*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]))/f
```

3.80.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1321
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f   Int[(c*d -
a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c,
 d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.80.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 771 vs. $2(202) = 404$.

Time = 0.68 (sec) , antiderivative size = 772, normalized size of antiderivative = 2.90

method	result
default	$-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}+\frac{(2c\sqrt{df}+bf)\ln\left(\frac{\frac{2c\sqrt{df}+bf}{2f}+c\left(x-\frac{\sqrt{df}}{f}\right)}{\sqrt{c}}+\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}}\right.}{2f\sqrt{c}}}{}$

input
```
int((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*f)^(1/2)*(((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((
1/2*(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f
)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)
^(1/2))/c^(1/2)-(b*(d*f)^(1/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1
/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1
/2)/f)))+1/2/(d*f)^(1/2)*(((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f
)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)+1/2/f*(-2*c*(d*f)^
(1/2)+b*f)*ln((1/2/f*(-2*c*(d*f)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2))/c^(1/2)-1/f*(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-
b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(
d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.80.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 568 vs. $2(202) = 404$.

Time = 65.90 (sec) , antiderivative size = 1139, normalized size of antiderivative = 4.28

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx$$
$$= \left[\frac{f\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}bf\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}+b^2+\left(bf^2x+2\,af^2\right)\sqrt{\frac{b^2}{df^3}}}{x}\right)-f\sqrt{\frac{df^2\sqrt{\frac{b^2}{df^3}}+cd+af}{df^2}}\log\left(\frac{2\cdots}{}\right.}{}\right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x +
 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d
*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) - f*sqrt((d*f^2*s
qrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c*x - 2*sqrt(c*x^2 + b*x +
 a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2
*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c
*d - a*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2
*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(
b^2/(d*f^3)))/x) - f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2))*
log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3))
- c*d - a*f)/(d*f^2)) + b^2 - (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) +
2*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + b*x + a)*(2*c*x
+ b)*sqrt(c) - 4*a*c))/f, 1/4*(f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f
)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2
/(d*f^3)) + c*d + a*f)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^
3)))/x) - f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f)/(d*f^2))*log((2*b*c
*x - 2*sqrt(c*x^2 + b*x + a)*b*f*sqrt((d*f^2*sqrt(b^2/(d*f^3)) + c*d + a*f
)/(d*f^2)) + b^2 + (b*f^2*x + 2*a*f^2)*sqrt(b^2/(d*f^3)))/x) + f*sqrt(-(d*
f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2))*log((2*b*c*x + 2*sqrt(c*x^2 +
b*x + a)*b*f*sqrt(-(d*f^2*sqrt(b^2/(d*f^3)) - c*d - a*f)/(d*f^2)) + b^2...
```

3.80.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = -\int \frac{\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(sqrt(a + b*x + c*x**2)/(-d + f*x**2), x)
```

3.80.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.80.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d-fx^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{d-f\,x^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d - f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d - f*x^2), x)
```

3.81 $\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx$

3.81.1 Optimal result

Integrand size = 28, antiderivative size = 267

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx = -\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{f}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d-1/2*arctanh(
1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d/f^(
1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*
x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(
1/2))^(1/2)/d/f^(1/2)
```

3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx = -\frac{-4\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{b^2}{}\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

output
```
-1/2*(-4*Sqrt[a]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + Ro
otSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4
& , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*
d*#1 - a*f*#1 + f*#1^3) & ])/d
```

3.81.3 Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d-fx^2)}\,dx$$

$$\Big\downarrow \ 7276$$

$$\int \left(\frac{\sqrt{a+bx+cx^2}}{dx}-\frac{fx\sqrt{a+bx+cx^2}}{d(fx^2-d)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{f}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{f}}-\frac{\sqrt{a}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x*(d - f*x^2)),x]
```

output
```
-((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - (S
qrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c
*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*d*Sqrt[f]) + (Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[f])
```

3.81.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(203) = 406$.

Time = 0.73 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.18

method	result
default	$\frac{\sqrt{c\,x^2+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d}-\frac{\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+c}{f}}}{}$

input
```
int((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*((c*x^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
/c^(1/2)-a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/2/d*(((x
-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/
2)+f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((1/2*(2*c*(d*f)^(1/2)+
b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/c^(1/2)-(b*(d
*f)^(1/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2
)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(
1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2/d*(((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2)+1/2/f*(-2*c*(d*f)^(1/2)+b*f)*ln((1/2/f*(-2*c*(d*f
)^(1/2)+b*f)+c*(x+(d*f)^(1/2)/f))/c^(1/2)+((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/c
^(1/2)-1/f*(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*l
n((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(
x+(d*f)^(1/2)/f)))
```

3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. $2(203) = 406$.

Time = 10.14 (sec) , antiderivative size = 1253, normalized size of antiderivative = 4.69

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*
x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d
 + a*f)/(d^2*f)) + 2*b*c*x + b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/
x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*
x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d
 + a*f)/(d^2*f)) - 2*b*c*x - b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/
x) - d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log((2*sqrt(c*
x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*
d - a*f)/(d^2*f)) + 2*b*c*x + b^2 - (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))
/x) + d*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) - c*d - a*f)/(d^2*f))*log(-(2*sqrt(
c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^3*f))*sqrt(-(d^2*f*sqrt(b^2/(d^3*f)) -
c*d - a*f)/(d^2*f)) - 2*b*c*x - b^2 + (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)
))/x) + 2*sqrt(a)*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c*x^2 + b*x +
 a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2))/d, 1/4*(d*sqrt((d^2*f*sqrt(b^2/(d^3
*f)) + c*d + a*f)/(d^2*f))*log((2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^
3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) + 2*b*c*x + b^2
+ (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt((d^2*f*sqrt(b^2/(d^3*
f)) + c*d + a*f)/(d^2*f))*log(-(2*sqrt(c*x^2 + b*x + a)*d^2*f*sqrt(b^2/(d^
3*f))*sqrt((d^2*f*sqrt(b^2/(d^3*f)) + c*d + a*f)/(d^2*f)) - 2*b*c*x - b^2
- (b*d*f*x + 2*a*d*f)*sqrt(b^2/(d^3*f)))/x) - d*sqrt(-(d^2*f*sqrt(b^2/(...
```

3.81.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\,(d-fx^2)}\,dx = -\int \frac{\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x/(-f*x**2+d),x)
```

output
```
-Integral(sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x)
```

3.81.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x), x)
```

3.81.8 Giac [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(-f*x^2+d),x, algorithm="giac")
```

output
```
sage2
```

3.81.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d-fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x\,\left(d-f\,x^2\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x*(d - f*x^2)), x)
```

3.82 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

3.82.1 Optimal result

Integrand size = 28, antiderivative size = 286

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d} + \frac{\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}} + \frac{\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}}$$

output
```
-1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-(c*x^2
+b*x+a)^(1/2)/d/x+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*
f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-
b*d^(1/2)*f^(1/2))^(1/2)/d^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*
(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(
1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^(3/2)
```

3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = -\frac{\frac{2\sqrt{a+x(b+cx)}}{x}-\frac{2b\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}+\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

output
```
-1/2*((2*Sqrt[a + x*(b + c*x)])/x - (2*b*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(
b + c*x)])/Sqrt[a]])/Sqrt[a] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 +
4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 - 2*a*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 +
 b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2
*c*d*#1 - a*f*#1 + f*#1^3) & ])/d
```

3.82.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{f\sqrt{a+bx+cx^2}}{d(d-fx^2)}+\frac{\sqrt{a+bx+cx^2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\,\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}}+\frac{\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\,\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}}-\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}-\frac{\sqrt{a+bx+cx^2}}{dx}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^2*(d - f*x^2)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
+ b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Arc
Tanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)) + (Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d
] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*
x^2])])/(2*d^(3/2))
```

3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 463 vs. $2(218) = 436$.

Time = 0.90 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.62

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}}{dx}-\frac{\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\sqrt{a}}-\frac{(\sqrt{df}\,af+\sqrt{df}\,cd+bdf)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+c\cdots}{f}}}{x-\sqrt{\cdots}}\right)}{df\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}\cdots}{\cdots}$
default	$\frac{-\frac{\left(c\,x^{2}+bx+a\right)^{\frac{3}{2}}}{ax}+\frac{b\left(\sqrt{c\,x^{2}+bx+a}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{2\sqrt{c}}-\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)\right)}{2a}+\frac{2c\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx+a}}{4c}+\frac{\left(4ac-b^{2}\right)\cdots}{\cdots}\right)}{a}\cdots}{d}$

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/d/x-1/2/d*(b/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x
+a)^(1/2))/x)-((d*f)^(1/2)*a*f+(d*f)^(1/2)*c*d+b*d*f)/d/f/((b*(d*f)^(1/2)+
f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*
(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2
*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1
/2))/(x-(d*f)^(1/2)/f))-(-(d*f)^(1/2)*a*f-(d*f)^(1/2)*c*d+b*d*f)/d/f/(1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(218) = 436$.

Time = 14.92 (sec) , antiderivative size = 1094, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$$
$$= \left[\frac{adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log\left(\frac{2\,bcx+2\,\sqrt{cx^2+bx+a}bd\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}+b^2+(bd^2x+2\,ad^2)\sqrt{\frac{b^2f}{d^5}}}{x}\right)-adx\sqrt{\frac{d^3\sqrt{\frac{b^2f}{d^5}}+cd+af}{d^3}}\log}{} \right.$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

output
```
[1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x + 2*s
qrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2
 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*sqrt(b^2*f/d^
5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*
sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d
^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x
+ 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)
 + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt(-(d^3*sqrt(b
^2*f/d^5) - c*d - a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b*d*sqr
t(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt
(b^2*f/d^5))/x) + sqrt(a)*b*x*log(-(8*a*b*x + (b^2 + 4*a*c)*x^2 - 4*sqrt(c
*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*sqrt(c*x^2 + b*x + a
)*a)/(a*d*x), 1/4*(a*d*x*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((
2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*
f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*sqrt((d^3*
sqrt(b^2*f/d^5) + c*d + a*f)/d^3)*log((2*b*c*x - 2*sqrt(c*x^2 + b*x + a)*b
*d*sqrt((d^3*sqrt(b^2*f/d^5) + c*d + a*f)/d^3) + b^2 + (b*d^2*x + 2*a*d^2)
*sqrt(b^2*f/d^5))/x) + a*d*x*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*d - a*f)/d^3)*
log((2*b*c*x + 2*sqrt(c*x^2 + b*x + a)*b*d*sqrt(-(d^3*sqrt(b^2*f/d^5) - c*
d - a*f)/d^3) + b^2 - (b*d^2*x + 2*a*d^2)*sqrt(b^2*f/d^5))/x) - a*d*x*s...
```

3.82. $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx$

3.82.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = -\int \frac{\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**2/(-f*x**2+d),x)
```

output
```
-Integral(sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x)
```

3.82.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^2), x)
```

3.82.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d-fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.82.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\,(d-fx^2)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(d-f\,x^2\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d - f*x^2)), x)
```

3.83 $\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$

3.83.1 Optimal result

Integrand size = 28, antiderivative size = 353

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = -\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2} + \frac{(b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d} - \frac{\sqrt{a}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} - \frac{\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2} + \frac{\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2}$$

output
```
1/8*(-4*a*c+b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2
)/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/4*(
b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2
)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2
))^(1/2))*f^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)/d^2+1/2*arctanh(1/2*(b
*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a
*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)/d^2
```

3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = \frac{-\frac{d(2a+bx)\sqrt{a+x(b+cx)}}{ax^2}+\frac{(b^2d-4a(cd+2af))\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}}-2f\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4c\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

output
```
(-((d*(2*a + b*x)*Sqrt[a + x*(b + c*x)])/(a*x^2)) + ((b^2*d - 4*a*(c*d + 2
*a*f))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) -
2*f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f
*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^
2 + a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d
- 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(4*d^2)
```

3.83.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{f^2x\sqrt{a+bx+cx^2}}{d^2(d-fx^2)}+\frac{f\sqrt{a+bx+cx^2}}{d^2x}+\frac{\sqrt{a+bx+cx^2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{3/2}d}-\frac{\sqrt{a}f\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2}+\frac{\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2}-\frac{(2a+bx)\sqrt{a+bx+cx^2}}{4adx^2}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^3*(d - f*x^2)),x]
```

output
```
-1/4*((2*a + b*x)*Sqrt[a + b*x + c*x^2])/(a*d*x^2) + ((b^2 - 4*a*c)*ArcTan
h[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(3/2)*d) - (Sqrt[a]
*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d^2 - (Sqrt[f]*
Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*
c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
b*x + c*x^2])])/(2*d^2) + (Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Arc
Tanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2)
```

3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.83.4 Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.36

method	result
risch	$-\frac{(bx+2a)\sqrt{c\,x^2+bx+a}}{4ad\,x^2}-\frac{-\frac{\left(-8a^2f-4acd+b^2d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}-\frac{4a\left(b\sqrt{df}+fa+cd\right)\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x \cdots\right.}{f}\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(b*x+2*a)*(c*x^2+b*x+a)^(1/2)/a/d/x^2-1/8/a/d*(-(-8*a^2*f-4*a*c*d+b^2
*d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-4*a*(b*(d*f)^(
1/2)+f*a+c*d)/d/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+4*a*(b*(d*f)^(1/2)
-f*a-c*d)/d/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f
*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2
)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. 2(275) = 550.

Time = 81.78 (sec) , antiderivative size = 1485, normalized size of antiderivative = 4.21

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

output
```
[1/16*(4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log
((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7
) + c*d*f + a*f^2)/d^4) + 2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*
sqrt(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f
+ a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^
4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4) - 2*b*c*f^2*x - b^2*f^2 - (b*d^3
*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) - 4*a^2*d^2*x^2*sqrt(-(d^4*sqrt(b^
2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x + a)*d^5*sqrt(b^2
*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4) + 2*b*c*f^2*x
 + b^2*f^2 - (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x) + 4*a^2*d^2*x^2
*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2)/d^4)*log(-(2*sqrt(c*x^2 + b
*x + a)*d^5*sqrt(b^2*f^3/d^7)*sqrt(-(d^4*sqrt(b^2*f^3/d^7) - c*d*f - a*f^2
)/d^4) - 2*b*c*f^2*x - b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7)
)/x) + (8*a^2*f - (b^2 - 4*a*c)*d)*sqrt(a)*x^2*log(-(8*a*b*x + (b^2 + 4*a*
c)*x^2 - 4*sqrt(c*x^2 + b*x + a)*(b*x + 2*a)*sqrt(a) + 8*a^2)/x^2) - 4*(a*
b*d*x + 2*a^2*d)*sqrt(c*x^2 + b*x + a))/(a^2*d^2*x^2), 1/8*(2*a^2*d^2*x^2*
sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log((2*sqrt(c*x^2 + b*x
+ a)*d^5*sqrt(b^2*f^3/d^7)*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^
4) + 2*b*c*f^2*x + b^2*f^2 + (b*d^3*f*x + 2*a*d^3*f)*sqrt(b^2*f^3/d^7))/x)
 - 2*a^2*d^2*x^2*sqrt((d^4*sqrt(b^2*f^3/d^7) + c*d*f + a*f^2)/d^4)*log(...
```

3.83.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3\left(d-fx^2\right)}\,dx=-\int \frac{\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**3/(-f*x**2+d),x)
```

output
```
-Integral(sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x)
```

3.83.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = \int -\frac{\sqrt{cx^2+bx+a}}{(fx^2-d)x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 - d)*x^3), x)
```

3.83.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.83.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^3(d-fx^2)}\,dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{x^3\,(d-f\,x^2)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^3*(d - f*x^2)), x)
```

3.84 $\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

3.84.1 Optimal result

Integrand size = 28, antiderivative size = 501

$$\begin{aligned}\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx &= -\frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f} \\ &- \frac{d(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^3} \\ &- \frac{d(a+bx+cx^2)^{3/2}}{3f^2} + \frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f} \\ &- \frac{(a+bx+cx^2)^{5/2}}{5cf} + \frac{3b(b^2-4ac)^2\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f} \\ &- \frac{bd(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^3} \\ &- \frac{d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}} \\ &+ \frac{d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{7/2}}\end{aligned}$$

output
```
-1/3*d*(c*x^2+b*x+a)^(3/2)/f^2+1/16*b*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c^2/f-
1/5*(c*x^2+b*x+a)^(5/2)/c/f+3/256*b*(-4*a*c+b^2)^2*arctanh(1/2*(2*c*x+b)/c
^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(7/2)/f-1/16*b*d*(12*a*c*f-b^2*f+24*c^2*d)*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/2*d*arctan
h(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(
7/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(
c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f
^(1/2))^(3/2)/f^(7/2)-3/128*b*(-4*a*c+b^2)*(2*c*x+b)*(c*x^2+b*x+a)^(1/2)/c
^3/f-1/8*d*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^3
```

3.84.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.02 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.47

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-2\sqrt{c}\sqrt{a+x(b+cx)}(45b^4f^2-30b^2cf^2(10a+bx)+16c^3f(160ad+70bdx+4}{}$$

input
```
Integrate[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(45*b^4*f^2 - 30*b^2*c*f^2*(10*a + b*x)
+ 16*c^3*f*(160*a*d + 70*b*d*x + 48*a*f*x^2 + 33*b*f*x^3) + 128*c^4*(15*d^
2 + 5*d*f*x^2 + 3*f^2*x^4) + 24*c^2*f*(16*a^2*f + 7*a*b*f*x + b^2*(10*d +
f*x^2))) - 15*b*(-384*c^4*d^2 - 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2
 + 16*c^2*f*(b^2*d + 3*a^2*f))*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c
*x)]] - 1920*c^(7/2)*d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1
^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a
*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 -
a*f*#1 + f*#1^3) & ])/(3840*c^(7/2)*f^3)
```

3.84.3 Rubi [A] (verified)

Time = 1.62 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{dx(a+bx+cx^2)^{3/2}}{f(d-fx^2)}-\frac{x(a+bx+cx^2)^{3/2}}{f}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b(b^2-4ac)^2\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{256c^{7/2}f}-\frac{bd\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+b^2(-f)+24c^2d)}{16c^{3/2}f^3}-\frac{d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{7/2}}+\frac{d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{7/2}}-\frac{3b(b^2-4ac)(b+2cx)\sqrt{a+bx+cx^2}}{128c^3f}-\frac{d\sqrt{a+bx+cx^2}(8acf+b^2f+2bcfx+8c^2d)}{8cf^3}+\frac{b(b+2cx)(a+bx+cx^2)^{3/2}}{16c^2f}-\frac{d(a+bx+cx^2)^{3/2}}{3f^2}-\frac{(a+bx+cx^2)^{5/2}}{5cf}$$

input
```
Int[(x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

output
```
(-3*b*(b^2 - 4*a*c)*(b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(128*c^3*f) - (d*(8
*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*f^3) - (
d*(a + b*x + c*x^2)^(3/2))/(3*f^2) + (b*(b + 2*c*x)*(a + b*x + c*x^2)^(3/2
))/(16*c^2*f) - (a + b*x + c*x^2)^(5/2)/(5*c*f) + (3*b*(b^2 - 4*a*c)^2*Arc
Tanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(256*c^(7/2)*f) - (b*
d*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*
x + c*x^2])])/(16*c^(3/2)*f^3) - (d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*
ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(7/2)) + (d*(c*
d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c
*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b
*x + c*x^2])])/(2*f^(7/2))
```

3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.84.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{(384c^4 f^2 x^4+528b f^2 c^3 x^3+768a c^3 f^2 x^2+24b^2 c^2 f^2 x^2+640c^4 df x^2+168ab c^2 f^2 x-30b^3 c f^2 x+1120b c^3 df x+384a^2 c^2 f^2-300a b^2 c f}{1920c^3 f^3}$
default	Expression too large to display

input
```
int(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(384*c^4*f^2*x^4+528*b*c^3*f^2*x^3+768*a*c^3*f^2*x^2+24*b^2*c^2*f^
2*x^2+640*c^4*d*f*x^2+168*a*b*c^2*f^2*x-30*b^3*c*f^2*x+1120*b*c^3*d*f*x+38
4*a^2*c^2*f^2-300*a*b^2*c*f^2+2560*a*c^3*d*f+45*b^4*f^2+240*b^2*c^2*d*f+19
20*c^4*d^2)*(c*x^2+b*x+a)^(1/2)/c^3/f^3+1/256/f^3/c^3*(b*(48*a^2*c^2*f^2-2
4*a*b^2*c*f^2-192*a*c^3*d*f+3*b^4*f^2+16*b^2*c^2*d*f-384*c^4*d^2)*ln((1/2*
b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+128*c^3*d*((d*f)^(1/2)*a^2*f^2
+2*(d*f)^(1/2)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2
+2*b*c*d^2*f)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*
f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^
(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-
(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+128*c^
3*d*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(
1/2)*c^2*d^2-2*a*b*d*f^2-2*b*c*d^2*f)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.84.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx=\text{Timed out}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.84.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx=\text{Timed out}$$

input
```
integrate(x**3*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.84.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.84.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x^3\,(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

input
```
int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

output
```
int((x^3*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.85 $\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

3.85.1 Optimal result

Integrand size = 28, antiderivative size = 417

$$\begin{aligned}\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = & -\frac{(b(80c^2d-3b^2f+12acf)+2c(16c^2d-3b^2f+12acf)\,x)\sqrt{a+bx+cx^2}}{64c^2f^2} \\ & -\frac{(b+2cx)\,(a+bx+cx^2)^{3/2}}{8cf} \\ & -\frac{(128c^4d^2+192ac^3df+3b^4f^2-24ab^2cf^2+48c^2f(b^2d+a^2f))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{5/2}f^3} \\ & +\frac{\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3} \\ & +\frac{\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^3}\end{aligned}$$

output
```
-1/8*(2*c*x+b)*(c*x^2+b*x+a)^(3/2)/c/f-1/128*(128*c^4*d^2+192*a*c^3*d*f+3*
b^4*f^2-24*a*b^2*c*f^2+48*c^2*f*(a^2*f+b^2*d))*arctanh(1/2*(2*c*x+b)/c^(1/
2)/(c*x^2+b*x+a)^(1/2))/c^(5/2)/f^3+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)
+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2)
)^(1/2))*d^(1/2)*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^3+1/2*arctanh(1/2*(b*
d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*
f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^3-
1/64*(b*(12*a*c*f-3*b^2*f+80*c^2*d)+2*c*(12*a*c*f-3*b^2*f+16*c^2*d)*x)*(c*
x^2+b*x+a)^(1/2)/c^2/f^2
```

3.85.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.77 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.46

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-2\sqrt{c}f\sqrt{a+x(b+cx)}(-3b^3f+2b^2cfx+8c^2x(4cd+5af+2cfx^2)+4bc(20c}{}$$

input
```
Integrate[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

output
```
(-2*Sqrt[c]*f*Sqrt[a + x*(b + c*x)]*(-3*b^3*f + 2*b^2*c*f*x + 8*c^2*x*(4*c
*d + 5*a*f + 2*c*f*x^2) + 4*b*c*(20*c*d + 5*a*f + 6*c*f*x^2)) + (128*c^4*d
^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2 + 48*c^2*f*(b^2*d + a^2*f)
)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]] - 64*c^(5/2)*d*RootSum[
b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b
*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^3*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*b*
f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*
c*d*#1 - a*f*#1 + f*#1^3) & ])/(128*c^(5/2)*f^3)
```

3.85.3 Rubi [A] (verified)

Time = 1.42 (sec) , antiderivative size = 433, normalized size of antiderivative = 1.04, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.464, Rules used = {2140, 27, 2140, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$$

$$\Big\downarrow \; 2140$$

$$-\frac{\int -\frac{3\sqrt{cx^2+bx+a}\left(f\left(-3fb^2+16c^2d+12acf\right)x^2+16bcdfx+\left(3b^2+4ac\right)df\right)}{4\left(d-fx^2\right)}dx}{12cf^2}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{cx^2+bx+a}\left(f\left(-3fb^2+16c^2d+12acf\right)x^2+16bcdfx+\left(3b^2+4ac\right)df\right)}{d-fx^2}dx}{16cf^2}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$$\Big\downarrow \; 2140$$

$$-\frac{\frac{\int \frac{-\left(\left(3f^2b^4-24acf^2b^2+128c^4d^2+192ac^3df+48c^2f\left(fa^2+b^2d\right)\right)x^2f^2\right)+d\left(3fb^4-8c(10cd+3af)b^2-16ac^2(4cd+5af)\right)f^2-256bc^2d(cd+af)xf^2}{4\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{2cf^2}-\sqrt{a+bx\ldots}}{16cf^2}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\int \frac{-\left(\left(3f^2b^4-24acf^2b^2+128c^4d^2+192ac^3df+48c^2f\left(fa^2+b^2d\right)\right)x^2f^2\right)+d\left(3fb^4-8c(10cd+3af)b^2-16ac^2(4cd+5af)\right)f^2-256bc^2d(cd+af)xf^2}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}-\sqrt{a+bx\ldots}}{16cf^2}-\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$$\Big\downarrow \; 2144$$

$$-\frac{\frac{f\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx-\frac{\int\frac{128c^2df^2\left(c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x\right)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f}}{8cf^2}}{16cf^2}-\frac{\sqrt{a+bx}\cdots}{}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 27

$$-\frac{\frac{f\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\int\frac{1}{\sqrt{cx^2+bx+a}}dx-128c^2df\int\frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}}{16cf^2}-\frac{\sqrt{a+bx+}\cdots}{}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 1092

$$-\frac{\frac{2f\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)\int\frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}-128c^2df\int\frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}}{16cf^2}\cdots$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 219

$$-\frac{\frac{\frac{f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\int\frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}}{16cf^2}-\frac{\sqrt{}\cdots}{}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 1366

$$-\frac{\frac{\frac{f\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\left(\frac{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int\frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+c}\cdots}}{2\sqrt{d}}\cdots\right.}{8cf^2}}{16cf^2}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 25

3.85. $\displaystyle\int\frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

$$-\frac{\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\left(\frac{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+}}}{2\sqrt{d}}\right.}{8cf^2}}{16cf^2}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 27

$$-\frac{\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int\frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}d}{2\sqrt{d}}\right.}{8cf^2}}{16cf^2}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 1154

$$-\frac{\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\left(-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int\frac{}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-(}}{}\right.}{}}{}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

$\downarrow$ 219

$$-\frac{\frac{f\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(48c^2f\left(a^2f+b^2d\right)-24ab^2cf^2+192ac^3df+3b^4f^2+128c^4d^2\right)}{\sqrt{c}}-128c^2df\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+}{2\sqrt{a+bx+cx}}\right.}{2\sqrt{d}}\right.}{8cf^2}}{}$$

$$\frac{(b+2cx)\left(a+bx+cx^2\right)^{3/2}}{8cf}$$

input
```
Int[(x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

3.85. $\int \frac{x^2\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

output
```
-1/8*((b + 2*c*x)*(a + b*x + c*x^2)^(3/2))/(c*f) + (-1/4*((b*(80*c^2*d - 3
*b^2*f + 12*a*c*f) + 2*c*(16*c^2*d - 3*b^2*f + 12*a*c*f)*x)*Sqrt[a + b*x +
 c*x^2])/c - ((f*(128*c^4*d^2 + 192*a*c^3*d*f + 3*b^4*f^2 - 24*a*b^2*c*f^2
 + 48*c^2*f*(b^2*d + a^2*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
 c*x^2])])/Sqrt[c] - 128*c^2*d*f*(((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*A
rcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]) + ((c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sq
rt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*Sqrt[d])))/(8*c*f^2))/(16*c*f^2)
```

3.85.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2140
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)
*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   Int[(a + b*x + c
*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(
2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2
*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p +
2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*
f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p
 + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p +
q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a,
 b, c, d, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] &&
 NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.85.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 648, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{\left(16 f\,c^{3} x^{3}+24 b\,c^{2} f\,x^{2}+40 a\,c^{2} f x+2 b^{2} c f x+32 c^{3} d x+20 a b c f-3 b^{3} f+80 b d\,c^{2}\right) \sqrt{c\,x^{2}+b x+a}}{64 c^{2} f^{2}}-\frac{\left(48 a^{2} c^{2} f^{2}-24 a\,b^{2} c\,f^{2}+192 a\,c^{3} d f+3 b^{4}\right.}{}$
default	Expression too large to display

input
```
int(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/64*(16*c^3*f*x^3+24*b*c^2*f*x^2+40*a*c^2*f*x+2*b^2*c*f*x+32*c^3*d*x+20*
a*b*c*f-3*b^3*f+80*b*c^2*d)*(c*x^2+b*x+a)^(1/2)/c^2/f^2-1/128/c^2/f^2*(1/f
*(48*a^2*c^2*f^2-24*a*b^2*c*f^2+192*a*c^3*d*f+3*b^4*f^2+48*b^2*c^2*d*f+128
*c^4*d^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-64*c^2*d*(2*
(d*f)^(1/2)*a*b*f+2*(d*f)^(1/2)*b*c*d-a^2*f^2-2*a*c*d*f-b^2*d*f-c^2*d^2)/(
d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+
f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/
2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d
*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-64*c^
2*d*(2*(d*f)^(1/2)*a*b*f+2*(d*f)^(1/2)*b*c*d+a^2*f^2+2*a*c*d*f+b^2*d*f+c^2
*d^2)/(d*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)
+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*
a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1
/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.85.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx=\text{Timed out}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.85.6 Sympy [F]

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx=-\int \frac{ax^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^4\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
-Integral(a*x**2*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**
3*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**4*sqrt(a + b*x
+ c*x**2)/(-d + f*x**2), x)
```

3.85.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.85.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x^2\,(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

input
```
int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

output
```
int((x^2*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.86 $\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx$

3.86.1 Optimal result

Integrand size = 26, antiderivative size = 349

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2} dx = -\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cf^2} - \frac{(a+bx+cx^2)^{3/2}}{3f} - \frac{b(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}f^2} - \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}} + \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{5/2}}$$

output
```
-1/3*(c*x^2+b*x+a)^(3/2)/f-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2
*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^2-1/2*arctanh(1/2*(b*d^(1/2
)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^
(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/f^(5/2)+1/2*arctan
h(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2
)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^(
5/2)-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x+a)^(1/2)/c/f^2
```

3.86.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 619, normalized size of antiderivative = 1.77

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-2\sqrt{c}\sqrt{a+x(b+cx)}(3b^2f+2cf(16a+7bx)+8c^2(3d+fx^2))+3b(-24c^2d+}{}$$

input
```
Integrate[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

output
```
(-2*Sqrt[c]*Sqrt[a + x*(b + c*x)]*(3*b^2*f + 2*c*f*(16*a + 7*b*x) + 8*c^2*
(3*d + f*x^2)) + 3*b*(-24*c^2*d + b^2*f - 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2
*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 24*c^(3/2)*RootSum[b^2*d - a^2*f - 4*b*
Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[
a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sq
rt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*
Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(48*c^(3/2)*f^2)
```

3.86.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {1354, 27, 2140, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$$

$$\Big\downarrow \text{1354}$$

$$\frac{\int \frac{3\sqrt{cx^2+bx+a}\left(bfx^2+2(cd+af)x+bd\right)}{2(d-fx^2)}dx}{3f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 27

$$\frac{\int \frac{\sqrt{cx^2+bx+a}\left(bfx^2+2(cd+af)x+bd\right)}{d-fx^2}dx}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 2140

$$\frac{-\frac{\int -\frac{bf^2\left(-fb^2+24c^2d+12acf\right)x^2+16cf\left(dfb^2+(cd+af)^2\right)x+bdf\left(fb^2+8c^2d+20acf\right)}{4\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{2cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 27

$$\frac{\frac{\int \frac{bf^2\left(-fb^2+24c^2d+12acf\right)x^2+16cf\left(dfb^2+(cd+af)^2\right)x+bdf\left(fb^2+8c^2d+20acf\right)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{8cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 2144

$$\frac{\frac{-bf\left(12acf+b^2(-f)+24c^2d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx-\frac{\int -\frac{16cf^2\left(2bd(cd+af)+\left(dfb^2+(cd+af)^2\right)x\right)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 27

$$\frac{\frac{16cf\int \frac{2bd(cd+af)+\left(dfb^2+(cd+af)^2\right)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-bf\left(12acf+b^2(-f)+24c^2d\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{8cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\Big\downarrow$ 1092

3.86. $\displaystyle\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}dx$

$$\frac{\frac{16cf\int \frac{2bd(cd+af)+\left(dfb^2+(cd+af)^2\right)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-2bf\left(12acf+b^2(-f)+24c^2d\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\downarrow$ 219

$$\frac{\frac{16cf\int \frac{2bd(cd+af)+\left(dfb^2+(cd+af)^2\right)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx-\frac{bf\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(12acf+b^2(-f)+24c^2d\right)}{\sqrt{c}}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{4cf}}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\downarrow$ 1366

$$\frac{\frac{16cf\left(\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)-\frac{bf\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\ldots}{\ldots}}{8cf^2}\ldots}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\downarrow$ 25

$$\frac{\frac{16cf\left(\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)-\frac{bf\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+\ldots}}\right)\ldots}{\ldots}}{8cf^2}\ldots}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\downarrow$ 27

$$\frac{\frac{16cf\left(\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}\right)-\frac{bf\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(12acf+\ldots\right)}{\sqrt{c}}}{8cf^2}\ldots}{2f}-\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$\downarrow$ 1154

3.86. $\int \frac{x\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

$$16cf\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int\frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int\frac{}{4\left(\sqrt{d}\sqrt{f}\right.}}{}\right.$$

$$8cf^2$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

$$\Big\downarrow \; 219$$

$$16cf\left(\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}}\right)-$$

$$8cf^2$$

$$2f$$

$$\frac{\left(a+bx+cx^2\right)^{3/2}}{3f}$$

input
```
Int[(x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x]
```

output
```
-1/3*(a + b*x + c*x^2)^(3/2)/f + (-1/4*((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c
*f*x)*Sqrt[a + b*x + c*x^2])/(c*f) + (-((b*f*(24*c^2*d - b^2*f + 12*a*c*f)
*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]) + 16*c*f
*(-1/2*((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqr
t[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]
*Sqrt[a + b*x + c*x^2])])/Sqrt[f] + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f])))/(8*c*
f^2))/(2*f)
```

3.86. $\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

3.86.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1354
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d + f*x^2)^(q +
 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))   Int[(a + b*x + c*x^
2)^(p - 1)*(d + f*x^2)^q*Simp[h*p*(b*d) + a*(-2*g*f)*(p + q + 1) + (2*h*p*(
c*d - a*f) + b*(-2*g*f)*(p + q + 1))*x + (h*p*((-b)*f) + c*(-2*g*f)*(p + q
+ 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4*
a*c, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.86. $\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

rule 2140
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p) + 2*c*C*f*(p + q + 1)*x)
*(a + b*x + c*x^2)^p*((d + f*x^2)^(q + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q +
 3))), x] - Simp[1/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))   Int[(a + b*x + c
*x^2)^(p - 1)*(d + f*x^2)^q*Simp[p*(b*d)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(
2*p + 2*q + 3)) + (p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f) + f*(-2*A*f)*(2
*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p +
2*q + 3)) + (p + q + 1)*((-b)*c*(C*(-4*d*f)*(2*p + q + 2) + f*(2*C*d + 2*A*
f)*(2*p + 2*q + 3))))*x + (p*((-b)*f)*(C*((-b)*f)*(q + 1) - c*((-B)*f)*(2*p
 + 2*q + 3)) + (p + q + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(-4*d*f)*(2*p +
q + 2) + f*(2*C*d + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a,
 b, c, d, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] &&
 NeQ[2*p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.86.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(275) = 550$.

Time = 0.73 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{\left(8 f\,c^{2} x^{2}+14 b c f x+32 a c f+3 b^{2} f+24 c^{2} d\right) \sqrt{c\,x^{2}+b x+a}}{24 c\,f^{2}}-\frac{\frac{b\left(12 a c f-b^{2} f+24 c^{2} d\right) \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x+a}\right)}{\sqrt{c}}-\frac{8 c\left(\sqrt{d f}\,a^{2} f^{2}+2 \sqrt{d f}\,a c d f+\right.}{}}{}$
default	Expression too large to display

input
```
int(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(8*c^2*f*x^2+14*b*c*f*x+32*a*c*f+3*b^2*f+24*c^2*d)*(c*x^2+b*x+a)^(1/
2)/c/f^2-1/16/f^2/c*(b*(12*a*c*f-b^2*f+24*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))/c^(1/2)-8*c*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+
(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/(d*f)^(1/
2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-8*c*((d*f)^(1/2)*a^2*f^2+2*(d
*f)^(1/2)*a*c*d*f+(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2-2*a*b*d*f^2-2*b*
c*d^2*f)/(d*f)^(1/2)/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d
*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)
/f))
```

3.86.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.86.6 Sympy [F]

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\int \frac{ax\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^3\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
-Integral(a*x*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x**2*s
qrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**3*sqrt(a + b*x + c
*x**2)/(-d + f*x**2), x)
```

3.86.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.86.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{x\,(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2),x)
```

output
```
int((x*(a + b*x + c*x^2)^(3/2))/(d - f*x^2), x)
```

3.87 $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

3.87.1 Optimal result

Integrand size = 25, antiderivative size = 315

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx &= -\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f} \\ &- \frac{(8c^2d+3b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}f^2} \\ &+ \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2} \\ &+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}f^2}\end{aligned}$$

output
```
-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a
)^(1/2))/f^2/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)
-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a
*f-b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(
1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(
1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/f^2/d^(1/2)-1/4*(2*c*x+5*b)
*(c*x^2+b*x+a)^(1/2)/f
```

3.87.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.00 (sec) , antiderivative size = 729, normalized size of antiderivative = 2.31

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \frac{-\sqrt{c}f(5b+2cx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f+12acf)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{a}-\sqrt{a+x(b+cx}}\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

output
```
(-(Sqrt[c]*f*(5*b + 2*c*x)*Sqrt[a + x*(b + c*x)]) + (8*c^2*d + 3*b^2*f + 1
2*a*c*f)*ArcTanh[(Sqrt[c]*x)/(Sqrt[a] - Sqrt[a + x*(b + c*x)])] - 2*Sqrt[c
]*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#
1^4 & , (-(c^3*d^2*Log[x]) + b^2*c*d*f*Log[x] - 2*a*c^2*d*f*Log[x] + 2*a*b
^2*f^2*Log[x] - a^2*c*f^2*Log[x] + c^3*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c
*x^2] - x*#1] - b^2*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + 2
*a*c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*a*b^2*f^2*Log[
-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a^2*c*f^2*Log[-Sqrt[a] + Sqrt[a
 + b*x + c*x^2] - x*#1] - 4*Sqrt[a]*b*c*d*f*Log[x]*#1 - 4*a^(3/2)*b*f^2*Lo
g[x]*#1 + 4*Sqrt[a]*b*c*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#
1 + 4*a^(3/2)*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c^2*
d^2*Log[x]*#1^2 + b^2*d*f*Log[x]*#1^2 + 2*a*c*d*f*Log[x]*#1^2 + a^2*f^2*Lo
g[x]*#1^2 - c^2*d^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - b^
2*d*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - 2*a*c*d*f*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a^2*f^2*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^
3) & ])/(4*Sqrt[c]*f^2)
```

3.87.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {1309, 27, 2144, 27, 1092, 219, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$$

$$\Big\downarrow \text{1309}$$

$$\frac{\int \frac{5db^2+16(cd+af)xb+(3fb^2+8c^2d+12acf)x^2+4a(cd+2af)}{4\sqrt{cx^2+bx+a}(d-fx^2)}dx}{2f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5db^2+16(cd+af)xb+(3fb^2+8c^2d+12acf)x^2+4a(cd+2af)}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{8f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{2144}$$

$$\frac{-\frac{\int -\frac{8\left(c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x\right)}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f} - \frac{(12acf+3b^2f+8c^2d)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{8\int \frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f} - \frac{(12acf+3b^2f+8c^2d)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{1092}$$

$$\frac{\frac{8\int \frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f} - \frac{2(12acf+3b^2f+8c^2d)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}}{8f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{8\int \frac{c^2d^2+2acfd+f\left(fa^2+b^2d\right)+2bf(cd+af)x}{\sqrt{cx^2+bx+a}\left(d-fx^2\right)}dx}{f} - \frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+3b^2f+8c^2d)}{\sqrt{c}f}}{8f} - \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$$\Big\downarrow \text{1366}$$

3.87. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

$$\frac{\frac{8\left(\frac{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}-\frac{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right.}{\sqrt{c}f}}{8f}$$
$$\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$\downarrow$ 25

$$\frac{\frac{8\left(\frac{\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}+\frac{\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}}{8f}$$
$$\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$\downarrow$ 27

$$\frac{\frac{8\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+3b}{\sqrt{c}f}}{8f}$$
$$\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$\downarrow$ 1154

$$\frac{\frac{8\left(-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^2\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}}-\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^2\int}{4\left(\sqrt{d}\sqrt{f}b+\right.}\right.}{f}}{8f}$$
$$\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

$\downarrow$ 219

3.87. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{d-fx^2}\,dx$

$$\frac{\frac{8\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}}\right)}{f}-\ \mathrm{ar}}{8f}$$

$$\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4f}$$

input

```
Int[(a + b*x + c*x^2)^(3/2)/(d - f*x^2),x]
```

output

```
-1/4*((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/f + (-(((8*c^2*d + 3*b^2*f + 12
*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f
)) + (8*(((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*S
qrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*
f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*
Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[d])))
/f)/(8*f)
```

3.87.3.1 Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1309
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Simp[1/(2*f*(p +
q)*(2*p + 2*q + 1))   Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*
d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q +
1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) -
(c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p
- 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1
, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.87. $\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx$

3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(245) = 490$.

Time = 0.84 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.73

method	result
risch	$-\frac{(2cx+5b)\sqrt{c\,x^2+bx+a}}{4f}-\frac{\frac{\left(12acf+3b^2f+8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}-\frac{\left(8\sqrt{df}\,abf+8\sqrt{df}\,bcd-4a^2f^2-8acdf-4b^2df-4c^2d^2\right)\ln\left(\frac{-2b\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)/f-1/8/f*(1/f*(12*a*c*f+3*b^2*f+8*c^2*
d)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-(8*(d*f)^(1/2)*a*b*
f+8*(d*f)^(1/2)*b*c*d-4*a^2*f^2-8*a*c*d*f-4*b^2*d*f-4*c^2*d^2)/(d*f)^(1/2)
/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(8*(d*f)^(1/2)*
a*b*f+8*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/(d*f)^(
1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(
2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

3.87.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.87.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-d+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
-Integral(a*sqrt(a + b*x + c*x**2)/(-d + f*x**2), x) - Integral(b*x*sqrt(a
 + b*x + c*x**2)/(-d + f*x**2), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2
)/(-d + f*x**2), x)
```

3.87.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.87.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{d-fx^2}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{d-f\,x^2}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(d - f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(d - f*x^2), x)
```

3.88 $\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx$

3.88.1 Optimal result

Integrand size = 28, antiderivative size = 469

$$\begin{aligned}
\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx &= \frac{(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd} \\
&- \frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cdf} \\
&- \frac{a^{3/2}\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d} - \frac{b(b^2-12ac)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d} \\
&- \frac{b(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}df} \\
&- \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}} \\
&+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2df^{3/2}}
\end{aligned}$$

output
```
-a^(3/2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d-1/16*b*(-12*
a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d-1/16
*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(
1/2))/c^(3/2)/d/f-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*
f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-
b*d^(1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+
x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))
^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d/f^(3/2)+1/8*(2*b*c*x+8*a*c+b^2
)*(c*x^2+b*x+a)^(1/2)/c/d-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*x^2+b*x
+a)^(1/2)/c/d/f
```

3.88.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.75 (sec) , antiderivative size = 601, normalized size of antiderivative = 1.28

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx =$$
$$-\frac{2cd\sqrt{a+x(b+cx)}-4a^{3/2}f\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)-3b\sqrt{c}d\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

output
```
-1/2*(2*c*d*Sqrt[a + x*(b + c*x)] - 4*a^(3/2)*f*ArcTanh[(Sqrt[c]*x - Sqrt[
a + x*(b + c*x)])/Sqrt[a]] - 3*b*Sqrt[c]*d*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])] + RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#
1^2 + 2*a*f*#1^2 - f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
 + a*b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x)
+ Sqrt[a + b*x + c*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1^2 + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*
a*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 -
 a*f*#1 + f*#1^3) & ])/(d*f)
```

3.88.3 Rubi [A] (verified)

Time = 1.55 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x\left(d-fx^2\right)}dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{\left(a+bx+cx^2\right)^{3/2}}{dx}-\frac{fx\left(a+bx+cx^2\right)^{3/2}}{d\left(fx^2-d\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{b\left(b^2-12ac\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d}-\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(12acf+b^2(-f)+24c^2d\right)}{16c^{3/2}df}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2df^{3/2}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2df^{3/2}}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cdf}+\frac{\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x]
```

output
```
((b^2 + 8*a*c + 2*b*c*x)*Sqrt[a + b*x + c*x^2])/(8*c*d) - ((8*c^2*d + b^2*
f + 8*a*c*f + 2*b*c*f*x)*Sqrt[a + b*x + c*x^2])/(8*c*d*f) - (a^(3/2)*ArcTa
nh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d - (b*(b^2 - 12*a*c)*A
rcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d) - (b
*(24*c^2*d - b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x
 + c*x^2])])/(16*c^(3/2)*d*f) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*Arc
Tanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*f^(3/2)) + ((c*d +
 b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sq
rt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d*f^(3/2))
```

3.88.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. $2(377) = 754$.

Time = 0.89 (sec) , antiderivative size = 1639, normalized size of antiderivative = 3.49

method	result	size
default	Expression too large to display	1639

input
```
int((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*(c*x^2+b*x+a)^(3/2)+1/2*b*(1/4*(2*c*x+b)/c*(c*x^2+b*x+a)^(1/2)+1/
8*(4*a*c-b^2)/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2)))+a*((c*x
^2+b*x+a)^(1/2)+1/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-
a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)))-1/2/d*(1/3*((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(3/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*(1/4*(2*c*(x-(d*f)^(1/2)/f)+(
2*c*(d*f)^(1/2)+b*f)/f)/c*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+1/8*(4*c*(b*(d*f)^(1/2)+
f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/c^(3/2)*ln((1/2*(2*c*(d*f)^(1/2)+b
*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)
+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)))+(b*(d*f)^(1/2
)+f*a+c*d)/f*(((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2
)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)+1/2*(2*c*(d*f)^(1/2)+b*f)/f*ln((1/2*
(2*c*(d*f)^(1/2)+b*f)/f+c*(x-(d*f)^(1/2)/f))/c^(1/2)+((x-(d*f)^(1/2)/f)^2*
c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/
2))/c^(1/2)-(b*(d*f)^(1/2)+f*a+c*d)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln
((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*
((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+
b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/
f))))-1/2/d*(1/3*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(...
```

3.88.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx=\text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.88.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx+fx^3}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x/(-f*x**2+d),x)
```

output
```
-Integral(a*sqrt(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(b*x*sqrt
(a + b*x + c*x**2)/(-d*x + f*x**3), x) - Integral(c*x**2*sqrt(a + b*x + c*
x**2)/(-d*x + f*x**3), x)
```

3.88.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x), x)
```

3.88.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.88.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x(d-fx^2)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x\,(d-f\,x^2)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x*(d - f*x^2)), x)
```

3.89 $\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx$

3.89.1 Optimal result

Integrand size = 28, antiderivative size = 463

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx &= \frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d} \\ &- \frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d} - \frac{(a+bx+cx^2)^{3/2}}{dx} \\ &- \frac{3\sqrt{a}b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d} + \frac{3(b^2+4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d} \\ &- \frac{(8c^2d+3b^2f+12acf)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}df} \\ &+ \frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f} \\ &+ \frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}f}\end{aligned}$$

output
```
-(c*x^2+b*x+a)^(3/2)/d/x-3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)
^(1/2))*a^(1/2)/d+3/8*(4*a*c+b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x
+a)^(1/2))/d/c^(1/2)-1/8*(12*a*c*f+3*b^2*f+8*c^2*d)*arctanh(1/2*(2*c*x+b)/
c^(1/2)/(c*x^2+b*x+a)^(1/2))/d/f/c^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^
(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^
(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f+1/2*arctanh(1/2*
(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d
+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)/d^(3/2)/f
+3/4*(2*c*x+3*b)*(c*x^2+b*x+a)^(1/2)/d-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)
/d
```

3.89.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.65 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \frac{-2af\sqrt{a+x(b+cx)}+6\sqrt{a}bfx\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)+2c^{3/2}dx\log\left(f\left(b+\right.\right.}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

output
```
(-2*a*f*Sqrt[a + x*(b + c*x)] + 6*Sqrt[a]*b*f*x*ArcTanh[(Sqrt[c]*x - Sqrt[
a + x*(b + c*x)])/Sqrt[a]] + 2*c^(3/2)*d*x*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sq
rt[a + x*(b + c*x)])] - x*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d
*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] + b^3*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
 a^2*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(5/2)*d^2*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b^2*Sqrt[c]*d*f*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 4*a*c^(3/2)*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a^2*Sqrt[c]*f^2*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*b*c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2 + 2*a*b*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*d*f*x)
```

3.89.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{f(a+bx+cx^2)^{3/2}}{d(d-fx^2)}+\frac{(a+bx+cx^2)^{3/2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(12acf+3b^2f+8c^2d)}{8\sqrt{c}df}+\frac{3(4ac+b^2)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}d}+\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}f}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}f}-\frac{3\sqrt{a}b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{(a+bx+cx^2)^{3/2}}{dx}+\frac{3(3b+2cx)\sqrt{a+bx+cx^2}}{4d}-\frac{(5b+2cx)\sqrt{a+bx+cx^2}}{4d}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x]
```

output
```
(3*(3*b + 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d) - ((5*b + 2*c*x)*Sqrt[a + b*
x + c*x^2])/(4*d) - (a + b*x + c*x^2)^(3/2)/(d*x) - (3*Sqrt[a]*b*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*d) + (3*(b^2 + 4*a*c)*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(8*Sqrt[c]*d) - ((8*
c^2*d + 3*b^2*f + 12*a*c*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(8*Sqrt[c]*d*f) + ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh
[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*f) + ((c*d + b*S
qrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d
] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*
x^2])])/(2*d^(3/2)*f)
```

3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.89.4 Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 585, normalized size of antiderivative = 1.26

method	result
risch	$-\frac{a\sqrt{c\,x^2+bx+a}}{dx}-\frac{\frac{2c^{\frac{3}{2}}\,d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f}+3b\sqrt{a}\,\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)-\frac{\left(-\sqrt{df}\,a^2f^2-2\sqrt{df}\,acdf-\sqrt{df}\,b^2df-\sqrt{df}\,c^2d^2\right.}{}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-a/d*(c*x^2+b*x+a)^(1/2)/x-1/2/d*(2*c^(3/2)*d/f*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2))+3*b*a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x
)-(-(d*f)^(1/2)*a^2*f^2-2*(d*f)^(1/2)*a*c*d*f-(d*f)^(1/2)*b^2*d*f-(d*f)^(1
/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/d/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)
^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(
1/2))/(x+(d*f)^(1/2)/f))-1/f^2*((d*f)^(1/2)*a^2*f^2+2*(d*f)^(1/2)*a*c*d*f+
(d*f)^(1/2)*b^2*d*f+(d*f)^(1/2)*c^2*d^2+2*a*b*d*f^2+2*b*c*d^2*f)/d/((b*(d*
f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2
)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(
1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.89.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.89.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^2+fx^4}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**2/(-f*x**2+d),x)
```

output
```
-Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(b*x*s
qrt(a + b*x + c*x**2)/(-d*x**2 + f*x**4), x) - Integral(c*x**2*sqrt(a + b*
x + c*x**2)/(-d*x**2 + f*x**4), x)
```

3.89.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^2), x)
```

3.89.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^2/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.89.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d-fx^2)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^2\,(d-f\,x^2)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^2*(d - f*x^2)), x)
```

3.90 $\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$

3.90.1 Optimal result

Integrand size = 28, antiderivative size = 614

$$\begin{aligned}\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = &-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx} \\ &+\frac{f(b^2+8ac+2bcx)\sqrt{a+bx+cx^2}}{8cd^2} \\ &-\frac{(8c^2d+b^2f+8acf+2bcfx)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{(a+bx+cx^2)^{3/2}}{2dx^2} \\ &-\frac{3(b^2+4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}-\frac{a^{3/2}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} \\ &+\frac{3b\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{b(b^2-12ac)f\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\ &-\frac{b(24c^2d-b^2f+12acf)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2} \\ &-\frac{\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}} \\ &+\frac{\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{f}}\end{aligned}$$

output
```
-1/2*(c*x^2+b*x+a)^(3/2)/d/x^2-a^(3/2)*f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*
x^2+b*x+a)^(1/2))/d^2-1/16*b*(-12*a*c+b^2)*f*arctanh(1/2*(2*c*x+b)/c^(1/2)
/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-1/16*b*(12*a*c*f-b^2*f+24*c^2*d)*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/d^2-3/8*(4*a*c+b^2)*ar
ctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+3/2*b*arctanh(1
/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-1/2*arctanh(1/2*(b*d^(
1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b
*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f-b*d^(1/2)*f^(1/2))^(3/2)/d^2/f^(1/2)+1/2
*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+
a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*(c*d+a*f+b*d^(1/2)*f^(1/2))^(3
/2)/d^2/f^(1/2)-3/4*(-2*c*x+b)*(c*x^2+b*x+a)^(1/2)/d/x+1/8*f*(2*b*c*x+8*a*
c+b^2)*(c*x^2+b*x+a)^(1/2)/c/d^2-1/8*(2*b*c*f*x+8*a*c*f+b^2*f+8*c^2*d)*(c*
x^2+b*x+a)^(1/2)/c/d^2
```

3.90.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.98 (sec) , antiderivative size = 587, normalized size of antiderivative = 0.96

$$\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}\,dx =$$

$$-\frac{\frac{d(2a+5bx)\sqrt{a+x(b+cx)}}{x^2}+\frac{(3b^2d+4a(3cd+2af))\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}}+2\text{RootSum}\Big[b^2d-a^2f-4b\sqrt{c}d\#1+4cd}{}$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

output
```
-1/4*((d*(2*a + 5*b*x)*Sqrt[a + x*(b + c*x)])/x^2 + ((3*b^2*d + 4*a*(3*c*d
 + 2*a*f))*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a
] + 2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
 f*#1^4 & , (2*b^2*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] -
a*c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*b^2*d*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a^2*c*d*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] - a^3*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] - 4*b*c^(3/2)*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1 - 4*a*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1 + c^2*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b^2*d*f*
Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*a*c*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a^2*f^2*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3)
& ])/d^2
```

3.90.3 Rubi [A] (verified)

Time = 1.73 (sec) , antiderivative size = 614, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{f^2x(a+bx+cx^2)^{3/2}}{d^2(d-fx^2)}+\frac{f(a+bx+cx^2)^{3/2}}{d^2x}+\frac{(a+bx+cx^2)^{3/2}}{dx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^{3/2}f\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{bf\left(b^2-12ac\right)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}d^2}-\frac{b\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)\left(12acf+b^2(-f)+24c^2d\right)}{16c^{3/2}d^2}-\frac{3\left(4ac+b^2\right)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{a}d}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}+\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{f}}+\frac{3b\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2d}-\frac{\sqrt{a+bx+cx^2}\left(8acf+b^2f+2bcfx+8c^2d\right)}{8cd^2}+\frac{f\left(8ac+b^2+2bcx\right)\sqrt{a+bx+cx^2}}{8cd^2}-\frac{\left(a+bx+cx^2\right)^{3/2}}{2dx^2}-\frac{3(b-2cx)\sqrt{a+bx+cx^2}}{4dx}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x]
```

output
```
(-3*(b - 2*c*x)*Sqrt[a + b*x + c*x^2])/(4*d*x) + (f*(b^2 + 8*a*c + 2*b*c*x
)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - ((8*c^2*d + b^2*f + 8*a*c*f + 2*b*c*f
*x)*Sqrt[a + b*x + c*x^2])/(8*c*d^2) - (a + b*x + c*x^2)^(3/2)/(2*d*x^2) -
 (3*(b^2 + 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/
(8*Sqrt[a]*d) - (a^(3/2)*f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c
*x^2])])/d^2 + (3*b*Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x +
c*x^2])])/(2*d) - (b*(b^2 - 12*a*c)*f*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[
a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - (b*(24*c^2*d - b^2*f + 12*a*c*f)*Ar
cTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(16*c^(3/2)*d^2) - (
(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (
2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
+ b*x + c*x^2])])/(2*d^2*Sqrt[f]) + ((c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[f])
```

3.90. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{x^3\left(d-fx^2\right)}\,dx$

3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.90.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 549, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}\,(5bx+2a)}{4d\,x^{2}}-\frac{-\frac{\left(-8a^{2}f-12acd-3b^{2}d\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{d\sqrt{a}}-\frac{\left(-8\sqrt{df}\,abf-8\sqrt{df}\,bcd+4a^{2}f^{2}+8acdf+4b^{2}df+4c^{2}d^{2}\ldots\right.}{\ldots}}{\ldots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(5*b*x+2*a)/d/x^2-1/8/d*(-(-8*a^2*f-12*a*c*d-3*b^
2*d)/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-(-8*(d*f)^(1/
2)*a*b*f-8*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/d/f/
(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^
(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-(8*(d*f)^(1/2)*a*b
*f+8*(d*f)^(1/2)*b*c*d+4*a^2*f^2+8*a*c*d*f+4*b^2*d*f+4*c^2*d^2)/d/f/((b*(d
*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^
(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+
c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.90.5 Fricas [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.90.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{-dx^3+fx^5}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/x**3/(-f*x**2+d),x)
```

output
```
-Integral(a*sqrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(b*x*s
qrt(a + b*x + c*x**2)/(-d*x**3 + f*x**5), x) - Integral(c*x**2*sqrt(a + b*
x + c*x**2)/(-d*x**3 + f*x**5), x)
```

3.90.7 Maxima [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \int -\frac{(cx^2+bx+a)^{\frac{3}{2}}}{(fx^2-d)x^3}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate((c*x^2 + b*x + a)^(3/2)/((f*x^2 - d)*x^3), x)
```

3.90.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/x^3/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.90.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{x^3(d-fx^2)}\,dx = \int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^3\,(d-f\,x^2)}\,dx$$

input
```
int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(3/2)/(x^3*(d - f*x^2)), x)
```

3.91 $\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx$

3.91.1 Optimal result

Integrand size = 24, antiderivative size = 189

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} dx = -\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2} - \frac{1}{2}(a-b+c)^{3/2}\operatorname{arctanh}\left(\frac{2a-b+(b-2c)x}{2\sqrt{a-b+c}\sqrt{a+bx+cx^2}}\right) - \frac{(3b^2+12ac+8c^2)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}} + \frac{1}{2}(a+b+c)^{3/2}\operatorname{arctanh}\left(\frac{2a+b+(b+2c)x}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)$$

output
```
-1/2*(a-b+c)^(3/2)*arctanh(1/2*(2*a-b+(b-2*c)*x)/(a-b+c)^(1/2)/(c*x^2+b*x+
a)^(1/2))+1/2*(a+b+c)^(3/2)*arctanh(1/2*(2*a+b+(b+2*c)*x)/(a+b+c)^(1/2)/(c
*x^2+b*x+a)^(1/2))-1/8*(12*a*c+3*b^2+8*c^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/
(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)
```

3.91.2 Mathematica [A] (verified)

Time = 1.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \frac{1}{4}\left(-\left((5b+2cx)\sqrt{a+x(b+cx)}\right)\right.$$
$$-4(-a+b-c)^{3/2}\arctan\left(\frac{\sqrt{-a+b-c}x}{\sqrt{a}(1+x)-\sqrt{a+x(b+cx)}}\right)+4(-a-b-c)^{3/2}\arctan\left(\frac{\sqrt{-a-b-cx}}{\sqrt{a}(-1+x)+\sqrt{a+x}}\right.$$

input
```
Integrate[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

output
```
(-((5*b + 2*c*x)*Sqrt[a + x*(b + c*x)]) - 4*(-a + b - c)^(3/2)*ArcTan[(Sqr
t[-a + b - c]*x)/(Sqrt[a]*(1 + x) - Sqrt[a + x*(b + c*x)])] + 4*(-a - b -
c)^(3/2)*ArcTan[(Sqrt[-a - b - c]*x)/(Sqrt[a]*(-1 + x) + Sqrt[a + x*(b + c
*x)])] - ((3*b^2 + 4*c*(3*a + 2*c))*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a
 + x*(b + c*x)])])/Sqrt[c])/4
```

3.91.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {1309, 27, 2144, 27, 1092, 219, 1366, 25, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx$$
$$\Big\downarrow \text{1309}$$
$$\frac{1}{2}\int \frac{8a^2+4ca+5b^2+(3b^2+8c^2+12ac)x^2+16b(a+c)x}{4(1-x^2)\sqrt{cx^2+bx+a}}dx-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{8}\int \frac{8a^2+4ca+5b^2+(3b^2+8c^2+12ac)x^2+16b(a+c)x}{(1-x^2)\sqrt{cx^2+bx+a}}dx-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$
$$\Big\downarrow \text{2144}$$

$$\frac{1}{8}\left(-\int -\frac{8\left(a^2+2ca+b^2+c^2+2b(a+c)x\right)}{\left(1-x^2\right)\sqrt{cx^2+bx+a}}dx-\left(\left(12ac+3b^2+8c^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)\right)-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 27

$$\frac{1}{8}\left(8\int \frac{a^2+2ca+b^2+c^2+2b(a+c)x}{\left(1-x^2\right)\sqrt{cx^2+bx+a}}dx-\left(12ac+3b^2+8c^2\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx\right)-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 1092

$$\frac{1}{8}\left(8\int \frac{a^2+2ca+b^2+c^2+2b(a+c)x}{\left(1-x^2\right)\sqrt{cx^2+bx+a}}dx-2\left(12ac+3b^2+8c^2\right)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}\right)-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 219

$$\frac{1}{8}\left(8\int \frac{a^2+2ca+b^2+c^2+2b(a+c)x}{\left(1-x^2\right)\sqrt{cx^2+bx+a}}dx-\frac{\left(12ac+3b^2+8c^2\right)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}}\right)-\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 1366

$$\frac{1}{8}\left(8\left(\frac{1}{2}(a+b+c)^2\int \frac{1}{(1-x)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}(a-b+c)^2\int -\frac{1}{(x+1)\sqrt{cx^2+bx+a}}dx\right)-\frac{\left(12ac+3b^2+\right.}{}\right.$$
$$\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 25

$$\frac{1}{8}\left(8\left(\frac{1}{2}(a-b+c)^2\int \frac{1}{(x+1)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}(a+b+c)^2\int \frac{1}{(1-x)\sqrt{cx^2+bx+a}}dx\right)-\frac{\left(12ac+3b^2+8\right.}{}\right.$$
$$\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$\big\downarrow$ 1154

3.91. $\int \frac{\left(a+bx+cx^2\right)^{3/2}}{1-x^2}\,dx$

$$\frac{1}{8}\left(8\left((a-b+c)^2\left(-\int \frac{1}{4(a-b+c)-\frac{(2a-b+(b-2c)x)^2}{cx^2+bx+a}}d\frac{2a-b+(b-2c)x}{\sqrt{cx^2+bx+a}}\right)-(a+b+c)^2\int \frac{1}{4(a+b+c)-\frac{(2a}{}}\right.\right.$$

$$\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(8\left(\frac{1}{2}(a+b+c)^{3/2}\text{arctanh}\left(\frac{2a+x(b+2c)+b}{2\sqrt{a+b+c}\sqrt{a+bx+cx^2}}\right)-\frac{1}{2}(a-b+c)^{3/2}\text{arctanh}\left(\frac{2a+x(b-2c)-b}{2\sqrt{a-b+c}\sqrt{a+bx+c}}\right.\right.\right.$$

$$\frac{1}{4}(5b+2cx)\sqrt{a+bx+cx^2}$$

input
```
Int[(a + b*x + c*x^2)^(3/2)/(1 - x^2),x]
```

output
```
-1/4*((5*b + 2*c*x)*Sqrt[a + b*x + c*x^2]) + (-(((3*b^2 + 12*a*c + 8*c^2)*
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/Sqrt[c]) + 8*(-1/2
*((a - b + c)^(3/2)*ArcTanh[(2*a - b + (b - 2*c)*x)/(2*Sqrt[a - b + c]*Sqr
t[a + b*x + c*x^2])]) + ((a + b + c)^(3/2)*ArcTanh[(2*a + b + (b + 2*c)*x)
/(2*Sqrt[a + b + c]*Sqrt[a + b*x + c*x^2])])/2))/8
```

3.91.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1309
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Simp[1/(2*f*(p +
q)*(2*p + 2*q + 1))   Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*
d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q +
1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) -
(c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p
- 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1
, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.91. $\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2} \, dx$

3.91.4 Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.63

method	result
risch	$-\frac{(2cx+5b)\sqrt{c\,x^2+bx+a}}{4}-\frac{3b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{8\sqrt{c}}-c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)-\frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c}\right.}{2}$
default	$-\frac{\left((-1+x)^2c+(b+2c)(-1+x)+a+b+c\right)^{\frac{3}{2}}}{6}-\frac{(b+2c)\left(\frac{(2c(-1+x)+b+2c)\sqrt{(-1+x)^2c+(b+2c)(-1+x)+a+b+c}}{4c}+\frac{\left(4c(a+b+c)-(b+2c)^2\right)\ln}{}\right.}{4}$

input
```
int((c*x^2+b*x+a)^(3/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*c*x+5*b)*(c*x^2+b*x+a)^(1/2)-3/8*b^2*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))/c^(1/2)-c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))
-3/2*a*c^(1/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/8*(-4*a^2+8*a
*b-8*a*c-4*b^2+8*b*c-4*c^2)/(a-b+c)^(1/2)*ln((2*a-2*b+2*c+(b-2*c)*(1+x)+2*
(a-b+c)^(1/2)*((1+x)^2*c+(b-2*c)*(1+x)+a-b+c)^(1/2))/(1+x))+1/8*(4*a^2+8*a
*b+8*a*c+4*b^2+8*b*c+4*c^2)/(a+b+c)^(1/2)*ln((2*a+2*b+2*c+(b+2*c)*(-1+x)+2
*(a+b+c)^(1/2)*((-1+x)^2*c+(b+2*c)*(-1+x)+a+b+c)^(1/2))/(-1+x))
```

3.91.5 Fricas [A] (verification not implemented)

Time = 116.63 (sec) , antiderivative size = 2579, normalized size of antiderivative = 13.65

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
[1/16*((3*b^2 + 12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4
*sqrt(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) + 4*((a - b)*c + c^2)*
sqrt(a - b + c)*log(-((b^2 + 4*(a - 2*b)*c + 8*c^2)*x^2 - 4*sqrt(c*x^2 + b
*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(a - b + c) + 8*a^2 - 8*a*b + b^2 + 4*
a*c + 2*(4*a*b - 3*b^2 - 4*(a - b)*c)*x)/(x^2 + 2*x + 1)) + 4*((a + b)*c +
 c^2)*sqrt(a + b + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x
^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^
2 + 4*a*c + 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) - 4*(2*c^2
*x + 5*b*c)*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*((a - b)*c + c^2)*sqrt(-a +
 b - c)*arctan(-1/2*sqrt(c*x^2 + b*x + a)*((b - 2*c)*x + 2*a - b)*sqrt(-a
+ b - c)/(((a - b)*c + c^2)*x^2 + a^2 - a*b + a*c + (a*b - b^2 + b*c)*x))
- (3*b^2 + 12*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt
(c*x^2 + b*x + a)*(2*c*x + b)*sqrt(c) - 4*a*c) - 4*((a + b)*c + c^2)*sqrt(
a + b + c)*log(-((b^2 + 4*(a + 2*b)*c + 8*c^2)*x^2 + 4*sqrt(c*x^2 + b*x +
a)*((b + 2*c)*x + 2*a + b)*sqrt(a + b + c) + 8*a^2 + 8*a*b + b^2 + 4*a*c +
 2*(4*a*b + 3*b^2 + 4*(a + b)*c)*x)/(x^2 - 2*x + 1)) + 4*(2*c^2*x + 5*b*c)
*sqrt(c*x^2 + b*x + a))/c, -1/16*(8*((a + b)*c + c^2)*sqrt(-a - b - c)*arc
tan(1/2*sqrt(c*x^2 + b*x + a)*((b + 2*c)*x + 2*a + b)*sqrt(-a - b - c)/(((
a + b)*c + c^2)*x^2 + a^2 + a*b + a*c + (a*b + b^2 + b*c)*x)) - (3*b^2 + 1
2*a*c + 8*c^2)*sqrt(c)*log(-8*c^2*x^2 - 8*b*c*x - b^2 + 4*sqrt(c*x^2 + ...
```

3.91.6 Sympy [F]

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = -\int \frac{a\sqrt{a+bx+cx^2}}{x^2-1}\,dx - \int \frac{bx\sqrt{a+bx+cx^2}}{x^2-1}\,dx - \int \frac{cx^2\sqrt{a+bx+cx^2}}{x^2-1}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(3/2)/(-x**2+1),x)
```

output
```
-Integral(a*sqrt(a + b*x + c*x**2)/(x**2 - 1), x) - Integral(b*x*sqrt(a +
b*x + c*x**2)/(x**2 - 1), x) - Integral(c*x**2*sqrt(a + b*x + c*x**2)/(x**
2 - 1), x)
```

3.91.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for
 more deta
```

3.91.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(3/2)/(-x^2+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.91.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx+cx^2)^{3/2}}{1-x^2}\,dx = -\int \frac{(c\,x^2+b\,x+a)^{3/2}}{x^2-1}\,dx$$

input
```
int(-(a + b*x + c*x^2)^(3/2)/(x^2 - 1),x)
```

output
```
-int((a + b*x + c*x^2)^(3/2)/(x^2 - 1), x)
```

3.92 $\int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx$

3.92.1 Optimal result

Integrand size = 22, antiderivative size = 75

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx = -\frac{1}{2}\arctan\left(\frac{3-x}{2\sqrt{-1-x+x^2}}\right)+\operatorname{arctanh}\left(\frac{1-2x}{2\sqrt{-1-x+x^2}}\right)+\frac{1}{2}\operatorname{arctanh}\left(\frac{1+3x}{2\sqrt{-1-x+x^2}}\right)$$

output
```
-1/2*arctan(1/2*(3-x)/(x^2-x-1)^(1/2))+arctanh(1/2*(1-2*x)/(x^2-x-1)^(1/2)
)+1/2*arctanh(1/2*(1+3*x)/(x^2-x-1)^(1/2))
```

3.92.2 Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx = \arctan\left(1-x+\sqrt{-1-x+x^2}\right)+\operatorname{arctanh}\left(1+x-\sqrt{-1-x+x^2}\right)+\log\left(1-2x+2\sqrt{-1-x+x^2}\right)$$

input
```
Integrate[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

output
```
ArcTan[1 - x + Sqrt[-1 - x + x^2]] + ArcTanh[1 + x - Sqrt[-1 - x + x^2]] +
 Log[1 - 2*x + 2*Sqrt[-1 - x + x^2]]
```

3.92.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.409, Rules used = {1321, 25, 1092, 219, 1366, 25, 1154, 217, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2-x-1}}{1-x^2}\,dx$$

$$\Big\downarrow \text{1321}$$

$$\int -\frac{x}{(1-x^2)\sqrt{x^2-x-1}}dx-\int \frac{1}{\sqrt{x^2-x-1}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sqrt{x^2-x-1}}dx-\int \frac{x}{(1-x^2)\sqrt{x^2-x-1}}dx$$

$$\Big\downarrow \text{1092}$$

$$-\int \frac{x}{(1-x^2)\sqrt{x^2-x-1}}dx-2\int \frac{1}{4-\frac{(1-2x)^2}{x^2-x-1}}d\left(-\frac{1-2x}{\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow \text{219}$$

$$\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)-\int \frac{x}{(1-x^2)\sqrt{x^2-x-1}}dx$$

$$\Big\downarrow \text{1366}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2-x-1}}dx-\frac{1}{2}\int -\frac{1}{(x+1)\sqrt{x^2-x-1}}dx+\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{2}\int \frac{1}{(1-x)\sqrt{x^2-x-1}}dx+\frac{1}{2}\int \frac{1}{(x+1)\sqrt{x^2-x-1}}dx+\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow \text{1154}$$

$$\int \frac{1}{-\frac{(3-x)^2}{x^2-x-1}-4}d\frac{3-x}{\sqrt{x^2-x-1}}-\int \frac{1}{4-\frac{(3x+1)^2}{x^2-x-1}}d\left(-\frac{3x+1}{\sqrt{x^2-x-1}}\right)+\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow \text{217}$$

$$-\int \frac{1}{4-\frac{(3x+1)^2}{x^2-x-1}}d\left(-\frac{3x+1}{\sqrt{x^2-x-1}}\right)-\frac{1}{2}\arctan\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\text{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)$$

$$\Big\downarrow 219$$

$$-\frac{1}{2}\arctan\left(\frac{3-x}{2\sqrt{x^2-x-1}}\right)+\operatorname{arctanh}\left(\frac{1-2x}{2\sqrt{x^2-x-1}}\right)+\frac{1}{2}\operatorname{arctanh}\left(\frac{3x+1}{2\sqrt{x^2-x-1}}\right)$$

input
```
Int[Sqrt[-1 - x + x^2]/(1 - x^2),x]
```

output
```
-1/2*ArcTan[(3 - x)/(2*Sqrt[-1 - x + x^2])] + ArcTanh[(1 - 2*x)/(2*Sqrt[-1
 - x + x^2])] + ArcTanh[(1 + 3*x)/(2*Sqrt[-1 - x + x^2])]/2
```

3.92.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1321
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (f_.)*(x_)^2), x_Symbol]
 :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f   Int[(c*d -
a*f - b*f*x)/(Sqrt[a + b*x + c*x^2]*(d + f*x^2)), x], x] /; FreeQ[{a, b, c,
 d, f}, x] && NeQ[b^2 - 4*a*c, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.92.4 Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.36

method	result
default	$-\frac{\sqrt{(-1+x)^2-2+x}}{2}-\frac{\ln\left(-\frac{1}{2}+x+\sqrt{(-1+x)^2-2+x}\right)}{4}+\frac{\arctan\left(\frac{-3+x}{2\sqrt{(-1+x)^2-2+x}}\right)}{2}+\frac{\sqrt{(1+x)^2-2-3x}}{2}-\frac{3\ln(-\frac{1}{2}+x+\sqrt{(}}{4}$
trager	$-\frac{\ln\left(\frac{8\sqrt{x^2-x-1}\,x^2+8x^3+12\sqrt{x^2-x-1}\,x+8x^2+2\sqrt{x^2-x-1}-9x-11}{1+x}\right)}{2}+\frac{\operatorname{RootOf}(\_Z^2+1)\ln(-\frac{-\operatorname{RootOf}(\_Z^2+1)x+2\sqrt{x^2-x-1}+3}{-1+x}}{2}$

input
```
int((x^2-x-1)^(1/2)/(-x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((-1+x)^2-2+x)^(1/2)-1/4*ln(-1/2+x+((-1+x)^2-2+x)^(1/2))+1/2*arctan(1
/2*(-3+x)/((-1+x)^2-2+x)^(1/2))+1/2*((1+x)^2-2-3*x)^(1/2)-3/4*ln(-1/2+x+((
1+x)^2-2-3*x)^(1/2))-1/2*arctanh(1/2*(-1-3*x)/((1+x)^2-2-3*x)^(1/2))
```

3.92.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \arctan\left(-x+\sqrt{x^2-x-1}+1\right)-\frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}\right) + \frac{1}{2}\log\left(-x+\sqrt{x^2-x-1}-2\right)+\log\left(-2\,x+2\,\sqrt{x^2-x-1}+1\right)$$

input
```
integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="fricas")
```

output
```
arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(-x + sqrt(x^2 - x - 1)) + 1/2
*log(-x + sqrt(x^2 - x - 1) - 2) + log(-2*x + 2*sqrt(x^2 - x - 1) + 1)
```

3.92.6 Sympy [F]

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = -\int \frac{\sqrt{x^2-x-1}}{x^2-1}\,dx$$

input
```
integrate((x**2-x-1)**(1/2)/(-x**2+1),x)
```

output
```
-Integral(sqrt(x**2 - x - 1)/(x**2 - 1), x)
```

3.92.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \frac{1}{2}\arcsin\left(\frac{2\sqrt{5}x}{5\,|2\,x-2|} - \frac{6\sqrt{5}}{5\,|2\,x-2|}\right) - \log\left(x+\sqrt{x^2-x-1}-\frac{1}{2}\right) - \frac{1}{2}\log\left(\frac{2\sqrt{x^2-x-1}}{|2\,x+2|} + \frac{2}{|2\,x+2|} - \frac{3}{2}\right)$$

input
```
integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="maxima")
```

output
```
1/2*arcsin(2/5*sqrt(5)*x/abs(2*x - 2) - 6/5*sqrt(5)/abs(2*x - 2)) - log(x
+ sqrt(x^2 - x - 1) - 1/2) - 1/2*log(2*sqrt(x^2 - x - 1)/abs(2*x + 2) + 2/
abs(2*x + 2) - 3/2)
```

3.92.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2}\,dx = \arctan\left(-x+\sqrt{x^2-x-1}+1\right) - \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}\right|\right) + \frac{1}{2}\log\left(\left|-x+\sqrt{x^2-x-1}-2\right|\right) + \log\left(\left|-2\,x+2\sqrt{x^2-x-1}+1\right|\right)$$

input
```
integrate((x^2-x-1)^(1/2)/(-x^2+1),x, algorithm="giac")
```

output
```
arctan(-x + sqrt(x^2 - x - 1) + 1) - 1/2*log(abs(-x + sqrt(x^2 - x - 1)))
+ 1/2*log(abs(-x + sqrt(x^2 - x - 1) - 2)) + log(abs(-2*x + 2*sqrt(x^2 - x
 - 1) + 1))
```

3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{-1-x+x^2}}{1-x^2} dx = -\int \frac{\sqrt{x^2-x-1}}{x^2-1} dx$$

input
```
int(-(x^2 - x - 1)^(1/2)/(x^2 - 1),x)
```

output
```
-int((x^2 - x - 1)^(1/2)/(x^2 - 1), x)
```

3.93 $\int \frac{(x+x^2)^{3/2}}{1+x^2} dx$

3.93.1 Optimal result

Integrand size = 17, antiderivative size = 130

$$\int \frac{(x+x^2)^{3/2}}{1+x^2} dx = \frac{1}{4}(5+2x)\sqrt{x+x^2} + \sqrt{1+\sqrt{2}}\arctan\left(\frac{1+\sqrt{2}-x}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x+x^2}}\right) - \sqrt{-1+\sqrt{2}}\text{arctanh}\left(\frac{1-\sqrt{2}-x}{\sqrt{2\left(-1+\sqrt{2}\right)}\sqrt{x+x^2}}\right) - \frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{x+x^2}}\right)$$

output
```
-5/4*arctanh(x/(x^2+x)^(1/2))+1/4*(5+2*x)*(x^2+x)^(1/2)-arctanh((1-x-2^(1/
2))/(x^2+x)^(1/2)/(-2+2*2^(1/2))^(1/2))*(2^(1/2)-1)^(1/2)+arctan((1-x+2^(1
/2))/(x^2+x)^(1/2)/(2+2*2^(1/2))^(1/2))*(1+2^(1/2))^(1/2)
```

3.93.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.93

$$\int \frac{(x+x^2)^{3/2}}{1+x^2} dx = \frac{\sqrt{x}\sqrt{1+x}\left(\sqrt{x}\sqrt{1+x}(5+2x) + 5\log\left(-\sqrt{x}+\sqrt{1+x}\right) + 8\text{RootSum}\left[16+32\#1+1\right.\right.}{4\sqrt{x(1+x)}}$$

input
```
Integrate[(x + x^2)^(3/2)/(1 + x^2),x]
```

output
```
(Sqrt[x]*Sqrt[1 + x]*(Sqrt[x]*Sqrt[1 + x]*(5 + 2*x) + 5*Log[-Sqrt[x] + Sqr
t[1 + x]] + 8*RootSum[16 + 32*#1 + 16*#1^2 + #1^4 & , (Log[-2*x + 2*Sqrt[x
]*Sqrt[1 + x] + #1]*#1^2)/(8 + 8*#1 + #1^3) & ]))/(4*Sqrt[x*(1 + x)])
```

3.93.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.15, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {1309, 27, 2144, 27, 1091, 219, 1369, 25, 1363, 216, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(x^2+x)^{3/2}}{x^2+1}dx$$

$$\Big\downarrow \text{1309}$$

$$\frac{1}{4}(2x+5)\sqrt{x^2+x}-\frac{1}{2}\int \frac{5x^2+16x+5}{4(x^2+1)\sqrt{x^2+x}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}(2x+5)\sqrt{x^2+x}-\frac{1}{8}\int \frac{5x^2+16x+5}{(x^2+1)\sqrt{x^2+x}}dx$$

$$\Big\downarrow \text{2144}$$

$$\frac{1}{8}\left(-5\int \frac{1}{\sqrt{x^2+x}}dx-\int \frac{16x}{(x^2+1)\sqrt{x^2+x}}dx\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{8}\left(-5\int \frac{1}{\sqrt{x^2+x}}dx-16\int \frac{x}{(x^2+1)\sqrt{x^2+x}}dx\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \text{1091}$$

$$\frac{1}{8}\left(-16\int \frac{x}{(x^2+1)\sqrt{x^2+x}}dx-10\int \frac{1}{1-\frac{x^2}{x^2+x}}d\frac{x}{\sqrt{x^2+x}}\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{8}\left(-16\int \frac{x}{(x^2+1)\sqrt{x^2+x}}dx-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \; 1369$$

$$\frac{1}{8}\left(-16\left(\frac{\int -\frac{\left(1-\sqrt{2}\right)x+1}{(x^2+1)\sqrt{x^2+x}}dx}{2\sqrt{2}}-\frac{\int -\frac{\left(1+\sqrt{2}\right)x+1}{(x^2+1)\sqrt{x^2+x}}dx}{2\sqrt{2}}\right)-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{8}\left(-16\left(\frac{\int \frac{\left(1+\sqrt{2}\right)x+1}{(x^2+1)\sqrt{x^2+x}}dx}{2\sqrt{2}}-\frac{\int \frac{\left(1-\sqrt{2}\right)x+1}{(x^2+1)\sqrt{x^2+x}}dx}{2\sqrt{2}}\right)-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \; 1363$$

$$\frac{1}{8}\left(-16\left(\frac{\left(1-\sqrt{2}\right)\int \frac{1}{\frac{\left(-x-\sqrt{2}+1\right)^2}{x^2+x}+2\left(1-\sqrt{2}\right)}d\frac{-x-\sqrt{2}+1}{\sqrt{x^2+x}}}{\sqrt{2}}-\frac{\left(1+\sqrt{2}\right)\int \frac{1}{\frac{\left(-x+\sqrt{2}+1\right)^2}{x^2+x}+2\left(1+\sqrt{2}\right)}d\frac{-x+\sqrt{2}+1}{\sqrt{x^2+x}}}{\sqrt{2}}\right)-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \; 216$$

$$\frac{1}{8}\left(-16\left(\frac{\left(1-\sqrt{2}\right)\int \frac{1}{\frac{\left(-x-\sqrt{2}+1\right)^2}{x^2+x}+2\left(1-\sqrt{2}\right)}d\frac{-x-\sqrt{2}+1}{\sqrt{x^2+x}}}{\sqrt{2}}-\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)\right)-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

$$\Big\downarrow \; 220$$

$$\frac{1}{8}\left(-16\left(-\frac{1}{2}\sqrt{1+\sqrt{2}}\arctan\left(\frac{-x+\sqrt{2}+1}{\sqrt{2\left(1+\sqrt{2}\right)}\sqrt{x^2+x}}\right)-\frac{\left(1-\sqrt{2}\right)\text{arctanh}\left(\frac{-x-\sqrt{2}+1}{\sqrt{2\left(\sqrt{2}-1\right)}\sqrt{x^2+x}}\right)}{2\sqrt{\sqrt{2}-1}}\right)-10\text{arctanh}\left(\frac{x}{\sqrt{x^2+x}}\right)\right)+\frac{1}{4}\sqrt{x^2+x}(2x+5)$$

input
```
Int[(x + x^2)^(3/2)/(1 + x^2),x]
```

3.93. $\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx$

output
```
((5 + 2*x)*Sqrt[x + x^2])/4 + (-16*(-1/2*(Sqrt[1 + Sqrt[2]]*ArcTan[(1 + Sq
rt[2] - x)/(Sqrt[2*(1 + Sqrt[2])]*Sqrt[x + x^2])]) - ((1 - Sqrt[2])*ArcTan
h[(1 - Sqrt[2] - x)/(Sqrt[2*(-1 + Sqrt[2])]*Sqrt[x + x^2])])/(2*Sqrt[-1 +
Sqrt[2]])) - 10*ArcTanh[x/Sqrt[x + x^2]])/8
```

3.93.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1309
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b*(3*p + 2*q) + 2*c*(p + q)*x)*(a + b*x + c*x^2)^(p - 1)*
((d + f*x^2)^(q + 1)/(2*f*(p + q)*(2*p + 2*q + 1))), x] - Simp[1/(2*f*(p +
q)*(2*p + 2*q + 1))   Int[(a + b*x + c*x^2)^(p - 2)*(d + f*x^2)^q*Simp[b^2*
d*(p - 1)*(2*p + q) - (p + q)*(b^2*d*(1 - p) - 2*a*(c*d - a*f*(2*p + 2*q +
1))) - (2*b*(c*d - a*f)*(1 - p)*(2*p + q) - 2*(p + q)*b*(2*c*d*(2*p + q) -
(c*d + a*f)*(2*p + 2*q + 1)))*x + (b^2*f*p*(1 - p) + 2*c*(p + q)*(c*d*(2*p
- 1) - a*f*(4*p + 2*q - 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 1] && NeQ[p + q, 0] && NeQ[2*p + 2*q + 1
, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 1363
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f
_.)*(x_)^2]), x_Symbol] :> Simp[-2*a*g*h   Subst[Int[1/Simp[2*a^2*g*h*c + a
*e*x^2, x], x], x, Simp[a*h - g*c*x, x]/Sqrt[d + e*x + f*x^2]], x] /; FreeQ
[{a, c, d, e, f, g, h}, x] && EqQ[a*h^2*e + 2*g*h*(c*d - a*f) - g^2*c*e, 0]
```

rule 1369
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 + a*c*e^2, 2]}, Simp
[1/(2*q)   Int[Simp[(-a)*h*e - g*(c*d - a*f - q) + (h*(c*d - a*f + q) - g*c
*e)*x, x]/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[
Simp[(-a)*h*e - g*(c*d - a*f + q) + (h*(c*d - a*f - q) - g*c*e)*x, x]/((a +
 c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d, e, f, g, h}, x]
&& NeQ[e^2 - 4*d*f, 0] && NegQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.93. $\int \frac{(x+x^2)^{3/2}}{1+x^2} \, dx$

3.93.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(98) = 196$.

Time = 4.60 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.05

method	result
pseudoelliptic	$-\frac{\left(\left(-\frac{\sqrt{2}}{2}+1\right)\ln\left(\frac{\sqrt{2}\,x-\sqrt{(1+x)x}\,\sqrt{2+2\sqrt{2}}+x+1}{x}\right)+\left(\frac{\sqrt{2}}{2}-1\right)\ln\left(\frac{\sqrt{2}\,x+\sqrt{(1+x)x}\,\sqrt{2+2\sqrt{2}}+x+1}{x}\right)-\frac{5\sqrt{-2+2\sqrt{2}}\,\ln\left(\frac{\sqrt{(1+x)x}-x \cdots}{x}\right)}{8} \cdots\right)}{\sqrt{-2+2\sqrt{2}}\,(x \cdots}$
trager	$\left(\frac{5}{4}+\frac{x}{2}\right)\sqrt{x^2+x}-\frac{5\ln\left(1+2x+2\sqrt{x^2+x}\right)}{8}-\frac{\mathrm{RootOf}\left(\mathrm{RootOf}\left(\_Z^4+16\_Z^2+128\right)^2+\_Z^2+16\right)\ln\left(-\frac{3\,\mathrm{RootO} \cdots}{\cdots}\right)}{\cdots}$
risch	$\frac{(5+2x)(1+x)x}{4\sqrt{(1+x)x}}-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{\sqrt{\frac{4\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}-\frac{3\sqrt{2}\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}+4+3\sqrt{2}}\,\sqrt{2}\left(\sqrt{-2+2\sqrt{2}}\,\arctan\left(\frac{\sqrt{-2 \cdots}}{\cdots}\right) \cdots\right)}{\cdots}$
default	$-\frac{5\ln\left(x+\frac{1}{2}+\sqrt{x^2+x}\right)}{8}+\frac{x\sqrt{x^2+x}}{2}+\frac{5\sqrt{x^2+x}}{4}+\frac{\sqrt{\frac{4\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}-\frac{3\sqrt{2}\left(-\sqrt{2}-1+x\right)^2}{\left(-\sqrt{2}+1-x\right)^2}+4+3\sqrt{2}}\,\sqrt{2}\left(\sqrt{-2+2\sqrt{2}}\,\mathrm{arct} \cdots\right)}{\cdots}$

input
```
int((x^2+x)^(3/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
-1/(-2+2*2^(1/2))^(1/2)*((-1/2*2^(1/2)+1)*ln((2^(1/2)*x-((1+x)*x)^(1/2)*(2
+2*2^(1/2))^(1/2)+x+1)/x)+(1/2*2^(1/2)-1)*ln((2^(1/2)*x+((1+x)*x)^(1/2)*(2
+2*2^(1/2))^(1/2)+x+1)/x)-5/8*(-2+2*2^(1/2))^(1/2)*ln(((((1+x)*x)^(1/2)-x)/
x)+5/8*(-2+2*2^(1/2))^(1/2)*ln((x+((1+x)*x)^(1/2))/x)-1/2*(-2+2*2^(1/2))^(
1/2)*(x+5/2)*((1+x)*x)^(1/2)+2^(1/2)*(arctan(((2+2*2^(1/2))^(1/2)*x-2*((1+
x)*x)^(1/2))/x/(-2+2*2^(1/2))^(1/2))-arctan(((2+2*2^(1/2))^(1/2)*x+2*((1+x
)*x)^(1/2))/x/(-2+2*2^(1/2))^(1/2))))*x^2/(x+((1+x)*x)^(1/2))^2/(-((1+x)*x
)^(1/2)+x)^2
```

3.93.5 Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \frac{1}{4}\sqrt{x^2+x}(2x+5) - \frac{1}{2}\sqrt{2i-2}\log\left(-2x+(i+1)\sqrt{2i-2}+2\sqrt{x^2+x}-2i\right) + \frac{1}{2}\sqrt{2i-2}\log\left(-2x-(i+1)\sqrt{2i-2}+2\sqrt{x^2+x}-2i\right) - \frac{1}{2}\sqrt{-2i-2}\log\left(-2x-(i-1)\sqrt{-2i-2}+2\sqrt{x^2+x}+2i\right) + \frac{1}{2}\sqrt{-2i-2}\log\left(-2x+(i-1)\sqrt{-2i-2}+2\sqrt{x^2+x}+2i\right) + \frac{5}{8}\log\left(-2x+2\sqrt{x^2+x}-1\right)$$

input
```
integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 + x)*(2*x + 5) - 1/2*sqrt(2*I - 2)*log(-2*x + (I + 1)*sqrt(2*
I - 2) + 2*sqrt(x^2 + x) - 2*I) + 1/2*sqrt(2*I - 2)*log(-2*x - (I + 1)*sqr
t(2*I - 2) + 2*sqrt(x^2 + x) - 2*I) - 1/2*sqrt(-2*I - 2)*log(-2*x - (I - 1
)*sqrt(-2*I - 2) + 2*sqrt(x^2 + x) + 2*I) + 1/2*sqrt(-2*I - 2)*log(-2*x +
(I - 1)*sqrt(-2*I - 2) + 2*sqrt(x^2 + x) + 2*I) + 5/8*log(-2*x + 2*sqrt(x^
2 + x) - 1)
```

3.93.6 Sympy [F]

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \int \frac{(x(x+1))^{\frac{3}{2}}}{x^2+1}\,dx$$

input
```
integrate((x**2+x)**(3/2)/(x**2+1),x)
```

output
```
Integral((x*(x + 1))**(3/2)/(x**2 + 1), x)
```

3.93.7 Maxima [F]

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \int \frac{(x^2+x)^{\frac{3}{2}}}{x^2+1}\,dx$$

input
```
integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate((x^2 + x)^(3/2)/(x^2 + 1), x)
```

3.93.8 Giac [F(-2)]

Exception generated.

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((x^2+x)^(3/2)/(x^2+1),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%{poly1[29378258633931653019799718485334854911359614337
236965430
```

3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(x+x^2)^{3/2}}{1+x^2}\,dx = \int \frac{(x^2+x)^{3/2}}{x^2+1}\,dx$$

input
```
int((x + x^2)^(3/2)/(x^2 + 1),x)
```

output
```
int((x + x^2)^(3/2)/(x^2 + 1), x)
```

3.94 $\int \frac{x^4}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.94.1 Optimal result

Integrand size = 28, antiderivative size = 369

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf}-\frac{d\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{d^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\frac{d^{3/2}\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-1/8*(-4*a*c+3*b^2)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(
5/2)/f-d*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+3/
4*b*(c*x^2+b*x+a)^(1/2)/c^2/f-1/2*x*(c*x^2+b*x+a)^(1/2)/c/f+1/2*d^(3/2)*ar
ctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^
(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
1/2)+1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^2/(c*d+a*f+b
*d^(1/2)*f^(1/2))^(1/2)
```

3.94.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.68

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$

$$= \frac{2\sqrt{c}f(3b-2cx)\sqrt{a+x(b+cx)}+(8c^2d+3b^2f-4acf)\log\left(c^2f^2\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)-4}{}$$

input
```
Integrate[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
(2*Sqrt[c]*f*(3*b - 2*c*x)*Sqrt[a + x*(b + c*x)] + (8*c^2*d + 3*b^2*f - 4*
a*c*f)*Log[c^2*f^2*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])] - 4*c^(5
/2)*d^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2
 - f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c
]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*
#1 - a*f*#1 + f*#1^3) & ])/(8*c^(5/2)*f^2)
```

3.94.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d-fx^2)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{d^2}{f^2(d-fx^2)\sqrt{a+bx+cx^2}}-\frac{d}{f^2\sqrt{a+bx+cx^2}}-\frac{x^2}{f\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{5/2}f}+\frac{d^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{d^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{d\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{3b\sqrt{a+bx+cx^2}}{4c^2f}-\frac{x\sqrt{a+bx+cx^2}}{2cf}$$

input
```
Int[x^4/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
(3*b*Sqrt[a + b*x + c*x^2])/(4*c^2*f) - (x*Sqrt[a + b*x + c*x^2])/(2*c*f)
- (d*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(Sqrt[c]*f^2)
 - ((3*b^2 - 4*a*c)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]
)/(8*c^(5/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
 - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x
^2])])/(2*f^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d^(3/2)*ArcTanh[(b*S
qrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^2*Sqrt[c*d + b*Sqrt[d]*Sqrt
[f] + a*f])
```

3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.94.4 Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.25

method	result
risch	$\frac{(-2cx+3b)\sqrt{c\,x^2+bx+a}}{4c^2 f}+\frac{\frac{\left(4acf-3b^2f-8c^2d\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}+\frac{4c^2d^2\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}}{}}}{}\right)}{\sqrt{df}\,f\sqrt{\frac{b}{}}}}{}$
default	$-\frac{\frac{x\sqrt{c\,x^2+bx+a}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}\right)}{4c}-\frac{a\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f}-\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f^2\sqrt{c}}+\frac{d^2}{}$

input
```
int(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(-2*c*x+3*b)*(c*x^2+b*x+a)^(1/2)/c^2/f+1/8/c^2/f*(1/f*(4*a*c*f-3*b^2*f
-8*c^2*d)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)+4*c^2*d^2/(d
*f)^(1/2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d
)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(
b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-4*c^2*d^2/(d*f)^(1/2)/
f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/
f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d)
)^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f
)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))
```

3.94.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.94.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^4}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**4/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x**4/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

3.94.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.94.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{x^4}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.95 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.95.1 Optimal result

Integrand size = 28, antiderivative size = 287

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{cf}+\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-(c*x^2+
b*x+a)^(1/2)/c/f-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(3/2)/
(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)
+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2)
)^(1/2))/f^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.95.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.35 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.73

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\frac{2\sqrt{a+x(b+cx)}}{c}+\frac{b\log\left(cf\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}}+d\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-\right.}{2f}$$

input
```
Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*((2*Sqrt[a + x*(b + c*x)])/c + (b*Log[c*f*(b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)])])/c^(3/2) + d*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
+ 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b
*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#1^3) & ])/f
```

3.95.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{dx}{f(d-fx^2)\sqrt{a+bx+cx^2}}-\frac{x}{f\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f} - \frac{d\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}} + \frac{d\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}} - \frac{\sqrt{a+bx+cx^2}}{cf}$$

input
```
Int[x^3/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(c*f)) + (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a
+ b*x + c*x^2])])/(2*c^(3/2)*f) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2
*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a +
 b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (d*ArcT
anh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f^(3/2)*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f])
```

3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.95.4 Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.43

method	result
default	$-\frac{\frac{\sqrt{c x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f}+\frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-}{}}}{x+\frac{\sqrt{df}}{f}}\right.}{2f^2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$
risch	$-\frac{\sqrt{c x^2+bx+a}}{cf}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx+a}\right)}{2c^{\frac{3}{2}}f}+\frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2}}{x+\frac{\sqrt{df}}{f}}\right.}{2f^2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

input
```
int(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(1/c*(c*x^2+b*x+a)^(1/2)-1/2*b/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+
b*x+a)^(1/2)))+1/2*d/f^2/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*
(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(
-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2
)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/
2)/f))+1/2*d/f^2/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*
a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)
/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

3.95.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.95.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^3}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x**3/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

3.95.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.95.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.95.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{x^3}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.96 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.96.1 Optimal result

Integrand size = 28, antiderivative size = 266

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{d}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)+1/2*arctanh(
1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*
x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/f/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2)
```

3.96.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.26 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.73

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \frac{\frac{2\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}} - d\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{b\log\left(-\sqrt{\right.}}{}\right.}{2f}$$

input
```
Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
((2*Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/Sqrt[c] - d*Root
Sum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
, (b*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1
+ f*#1^3) & ])/(2*f)
```

3.96.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {2144, 25, 27, 1092, 219, 1316, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \text{2144}$$

$$-\frac{\int -\frac{d}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{d}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\int \frac{1}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}$$

$$\Big\downarrow \text{1092}$$

$$\frac{d\int \frac{1}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\int \frac{1}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

$$\Big\downarrow \text{1316}$$

$$\frac{d\left(\frac{1}{2}\int \frac{1}{\sqrt{d}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\int \frac{1}{\sqrt{d}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

$$\Big\downarrow \text{27}$$

$$\frac{d\left(\frac{\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}\right)}{f}-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

$$\Big\downarrow \text{1154}$$

$$\frac{d\Bigg(-\frac{\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}}-\frac{\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{}\right.}{cx^2+bx+a}}}{\sqrt{d}}}{f}$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{f}-$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}$$

3.96. $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

input
```
Int[x^2/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-(ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f)) + (d
*(ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[
c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqr
t[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x +
 c*x^2])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])))/f
```

3.96.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1316
```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Simp[1/2   Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x]
, x] + Simp[1/2   Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2144
```
Int[(Px_)/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]),
x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[Px,
x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[1/c   Int[(A*
c - a*C + B*c*x)/((a + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.96.4 Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.50

method	result
default	$-\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx+a}\right)}{f\sqrt{c}}-\frac{d\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{(-2c\sqrt{df}+bf)\left(x-\right.}{f}}}{x+\frac{\sqrt{df}}{f}}\right.}{2\sqrt{df}\,f\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

input
```
int(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*d/(d*f)^(1/2)
/f/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1
/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d
))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/
f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2*d/(d*f)^(1/
2)/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)
^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))
```

3.96.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.96.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x^2}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x**2/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2))
, x)
```

3.96.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.96.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.96.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{x^2}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.97 $\int \frac{x}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.97.1 Optimal result

Integrand size = 26, antiderivative size = 220

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/2)
*f^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.97.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.20 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.68

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\Bigg[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2 -f\#1^4\&,\frac{a\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)-\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2}{-b\sqrt{c}d+2cd\#1+af\#1-f\#1^3}\&\Bigg]$$

input
```
Integrate[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 -
f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f
*#1 - f*#1^3) & ]
```

3.97.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d-fx^2)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{1366}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}$$

input
```
Int[x/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sq
rt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(Sqrt[f]*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sq
rt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])]/(2*Sqrt[f]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

3.97.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 353 vs. $2(164) = 328$.

Time = 0.82 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.61

method	result
default	$\frac{\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2f\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}+\frac{\ln\left(\frac{\frac{-2b\sqrt{df}+}{f}$

input
```
int(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2
)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f
)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))+1/2/f/(1/f*(-b*(d*f)^(1/2)+f
*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)
*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/
f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*
a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

3.97.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2753 vs. $2(164) = 328$.

Time = 0.65 (sec) , antiderivative size = 2753, normalized size of antiderivative = 12.51

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2
*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^
3 - (b^2 - 2*a*c)*d*f^2))*log((2*b*c*d*x + b^2*d + 2*(b^2*d*f - (c^3*d^3*f
 + a^3*f^4 - (b^2*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2
*d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c
 + 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))*sqrt(c*x^2 + b*x +
a)*sqrt((c*d + a*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*
d/(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c
+ 6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3
 - (b^2 - 2*a*c)*d*f^2)) - (2*a*c^2*d^2*f + 2*a^3*f^3 - 2*(a*b^2 - 2*a^2*c
)*d*f^2 + (b*c^2*d^2*f + a^2*b*f^3 - (b^3 - 2*a*b*c)*d*f^2)*x)*sqrt(b^2*d/
(c^4*d^4*f + a^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c +
6*a^2*c^2)*d^2*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/x) - 1/4*sqrt((c*d + a
*f + (c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)*d*f^2)*sqrt(b^2*d/(c^4*d^4*f + a
^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^2
*f^3 - 2*(a^2*b^2 - 2*a^3*c)*d*f^4)))/(c^2*d^2*f + a^2*f^3 - (b^2 - 2*a*c)
*d*f^2))*log((2*b*c*d*x + b^2*d - 2*(b^2*d*f - (c^3*d^3*f + a^3*f^4 - (b^2
*c - 3*a*c^2)*d^2*f^2 - (a*b^2 - 3*a^2*c)*d*f^3)*sqrt(b^2*d/(c^4*d^4*f + a
^4*f^5 - 2*(b^2*c^2 - 2*a*c^3)*d^3*f^2 + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*...
```

3.97.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{x}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(x/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x
)
```

3.97.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.97.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

3.97.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\int \frac{x}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(x/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.98 $\int \frac{1}{\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.98.1 Optimal result

Integrand size = 25, antiderivative size = 220

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx=\frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}}+\frac{\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2)*
f^(1/2))^(1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(
1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d
+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.98.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{1}{2}\text{RootSum}\Big[c^2d-b^2f+4\sqrt{a}bf\#1-2cd\#1^2-4af\#1^2$$
$$+d\#1^4\&, \frac{-c\log(x)+c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)+\log(x)\#1^2-\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}\right.}{-\sqrt{a}bf+cd\#1+2af\#1-d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 - 4*a*f*#1^2 +
d*#1^4 & , (-(c*Log[x]) + c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] +
 Log[x]*#1^2 - Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[
a]*b*f) + c*d*#1 + 2*a*f*#1 - d*#1^3) & ]
```

3.98.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1316, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1316$$

$$\frac{1}{2}\int \frac{1}{\sqrt{d}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx+\frac{1}{2}\int \frac{1}{\sqrt{d}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}+\frac{\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{d}}$$

$$\Big\downarrow \; 1154$$

$$-\frac{\int \frac{1}{4\left(-\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{-2\sqrt{f}a+\left(2c\sqrt{d}-b\sqrt{f}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}}-\frac{\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{d}}$$

$$\Big\downarrow \ 219$$

$$\frac{\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{d}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*
d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]/(2*Sqrt[d]*Sqrt[c*d -
 b*Sqrt[d]*Sqrt[f] + a*f]) + ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[
d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c
*x^2])]/(2*Sqrt[d]*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

3.98.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1316
```
Int[1/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(x_)^2]), x_Sy
mbol] :> Simp[1/2   Int[1/((a - Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x]
, x] + Simp[1/2   Int[1/((a + Rt[(-a)*c, 2]*x)*Sqrt[d + e*x + f*x^2]), x],
x] /; FreeQ[{a, c, d, e, f}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(164) = 328$.

Time = 0.79 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.63

method	result
default	$\frac{\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}+fa+cd}{f}}}{x-\frac{\sqrt{df}}{f}}\right)}{2\sqrt{df}\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}} - \frac{\ln\left(\frac{\frac{-2b\sqrt{df}+\cdots}{f}}{\cdots}\right)}{\cdots}$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a
+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/
f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f))-1/2/(d*f)^(1/2)/(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))
```

3.98.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2641 vs. $2(164) = 328$.

Time = 0.72 (sec) , antiderivative size = 2641, normalized size of antiderivative = 12.00

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2
*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c +
 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^
2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 + 2*(b*c*d + a*b*f - (b*c^2*d
^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4
- 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*
(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*x + a)*sqrt((c*d + a*f + (c^
2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 -
 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(
a^2*b^2 - 2*a^3*c)*d^2*f^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))
 - (2*a*c^2*d^2 + 2*a^3*f^2 - 2*(a*b^2 - 2*a^2*c)*d*f + (b*c^2*d^2 + a^2*b
*f^2 - (b^3 - 2*a*b*c)*d*f)*x)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^
2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 -
2*a^3*c)*d^2*f^3)))/x) - 1/4*sqrt((c*d + a*f + (c^2*d^3 + a^2*d*f^2 - (b^2
 - 2*a*c)*d^2*f)*sqrt(b^2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d
^4*f + (b^4 - 4*a*b^2*c + 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f
^3)))/(c^2*d^3 + a^2*d*f^2 - (b^2 - 2*a*c)*d^2*f))*log((2*b*c*x + b^2 - 2*
(b*c*d + a*b*f - (b*c^2*d^3 + a^2*b*d*f^2 - (b^3 - 2*a*b*c)*d^2*f)*sqrt(b^
2*f/(c^4*d^5 + a^4*d*f^4 - 2*(b^2*c^2 - 2*a*c^3)*d^4*f + (b^4 - 4*a*b^2*c
+ 6*a^2*c^2)*d^3*f^2 - 2*(a^2*b^2 - 2*a^3*c)*d^2*f^3)))*sqrt(c*x^2 + b*...
```

3.98.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-d\sqrt{a+bx+cx^2}+fx^2\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-d*sqrt(a + b*x + c*x**2) + f*x**2*sqrt(a + b*x + c*x**2)), x
)
```

3.98.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.98.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueBad Argument Type
```

3.98.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{(d-f\,x^2)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/((d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.99 $\int \frac{1}{x\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.99.1 Optimal result

Integrand size = 28, antiderivative size = 267

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} - \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{\sqrt{f}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)-1/2*arctanh(
1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/
(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
1/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*
x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/d/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(1/2)
```

3.99.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.72

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{\frac{4\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - f\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&, \frac{a\log\left(-\sqrt{c}x+\right.}{}\right.}{2d}$$

input
```
Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
((4*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - f*Root
Sum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 &
, (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1 + a*f*#1 - f*#
1^3) & ])/(2*d)
```

3.99.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{ 7276}$$
$$\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}}-\frac{fx}{d(fx^2-d)\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sqrt{f}\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{\sqrt{f}\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(x*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) - (S
qrt[f]*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*
Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d*Sqrt[c*d
 - b*Sqrt[d]*Sqrt[f] + a*f]) + (Sqrt[f]*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] +
 (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*d*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f])
```

3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.99.4 Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.46

method	result
default	$-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{d\sqrt{a}}+\frac{\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(-2c\sqrt{df}+bf\right)\cdots}{f}\cdots}}{x+\frac{\sqrt{df}}{f}}\cdots\right)}{2d\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$

input
```
int(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/d/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)+1/2/d/(1/f*(-b*
(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x
+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(
d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))+1/2/d/((b*(d*f)^(1/2)+f*a+c
*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/
(x-(d*f)^(1/2)/f))
```

3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2993 vs. $2(203) = 406$.

Time = 47.36 (sec) , antiderivative size = 5995, normalized size of antiderivative = 22.45

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.99.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx\sqrt{a+bx+cx^2}+fx^3\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-d*x*sqrt(a + b*x + c*x**2) + f*x**3*sqrt(a + b*x + c*x**2)),
 x)
```

3.99.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x), x)
```

3.99.8 Giac [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
sage2
```

3.99.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.100 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)} \, dx$

3.100.1 Optimal result

Integrand size = 28, antiderivative size = 291

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d-fx^2)} \, dx = -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d} + \frac{f\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-(c*x^2+
b*x+a)^(1/2)/a/d/x+1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)
-b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2
)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/
2)+x*(2*c*d^(1/2)+b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/
2))^(1/2))/d^(3/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.100.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\frac{2\sqrt{a+x(b+cx)}}{ax}+\frac{2b\,\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}}+f\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\right.}{2d}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*((2*Sqrt[a + x*(b + c*x)])/(a*x) + (2*b*ArcTanh[(Sqrt[c]*x - Sqrt[a +
 x*(b + c*x)])/Sqrt[a]])/a^(3/2) + f*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d
*#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - 2*Sqrt[c]*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/d
```

3.100.3 Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(d-fx^2)\sqrt{a+bx+cx^2}}dx$$

$$\Big\downarrow \text{ 7276}$$

$$\int \left(\frac{f}{d(d-fx^2)\sqrt{a+bx+cx^2}}+\frac{1}{dx^2\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{f\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

input
```
Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + b*x + c*x^2])])/(2*a^(3/2)*d) + (f*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] +
(2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a
 + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f*Ar
cTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d
+ b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*Sqrt[c*d +
b*Sqrt[d]*Sqrt[f] + a*f])
```

3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.100.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.46

method	result
default	$\frac{-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{f\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\,\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c}}{x+\frac{\sqrt{df}}{f}}\right)}{2d\sqrt{df}\,\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}$
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{adx}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}d}+\frac{f\ln\left(\frac{\frac{2b\sqrt{df}+2fa+2cd}{f}+\frac{\left(2c\sqrt{df}+bf\right)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}\,\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c}}{x-\frac{\sqrt{df}}{f}}\right)}{2d\sqrt{df}\,\sqrt{\frac{b\sqrt{df}+fa+cd}{f}}}$

input
```
int(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/x*(c*x^2+b*x+a)^(1/2)+1/2*b/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2
+b*x+a)^(1/2))/x))-1/2*f/d/(d*f)^(1/2)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/
2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)
)/(x+(d*f)^(1/2)/f))+1/2*f/d/(d*f)^(1/2)/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)
*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)
+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/
2)/f))
```

3.100.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3005 vs. 2(223) = 446.

Time = 209.66 (sec) , antiderivative size = 6018, normalized size of antiderivative = 20.68

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.100.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx^2\sqrt{a+bx+cx^2}+fx^4\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-d*x**2*sqrt(a + b*x + c*x**2) + f*x**4*sqrt(a + b*x + c*x**2
)), x)
```

3.100.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^2), x)
```

3.100.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x^2\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.101 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d-fx^2)}\,dx$

3.101.1 Optimal result

Integrand size = 28, antiderivative size = 376

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} - \frac{(3b^2-4ac)\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} - \frac{f\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2} - \frac{f^{3/2}\mathrm{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}} + \frac{f^{3/2}\mathrm{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}}$$

output
```
-1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(
5/2)/d-f*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-1/
2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x-1/2*f^(3/2
)*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x
+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a*f-b*d^(1/2)*f^(1/2
))^(1/2)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f
^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^2/(c*d+a
*f+b*d^(1/2)*f^(1/2))^(1/2)
```

3.101.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.67 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx$$
$$= \frac{\frac{d(-2a+3bx)\sqrt{a+x(b+cx)}}{a^2x^2} - \frac{(3b^2d-4acd+8a^2f)\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}} - 2f^2\operatorname{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4\right.}{4d^2}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
((d*(-2*a + 3*b*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) - ((3*b^2*d - 4*a*c*d
+ 8*a^2*f)*ArcTanh[(-(Sqrt[c]*x) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2
) - 2*f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1^2 + 2*a*f*#1
^2 - f*#1^4 & , (a*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-(b*Sqrt[c]*d) + 2*c*d*#1
+ a*f*#1 - f*#1^3) & ])/(4*d^2)
```

3.101.3 Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3 (d-fx^2)\sqrt{a+bx+cx^2}}\,dx$$
$$\Big\downarrow \text{ 7276}$$
$$\int \left(\frac{f^2 x}{d^2 (d-fx^2)\sqrt{a+bx+cx^2}}+\frac{f}{d^2 x\sqrt{a+bx+cx^2}}+\frac{1}{dx^3\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{(3b^2-4ac)\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{f^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}+\frac{f^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}-\frac{f\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d - f*x^2)),x]
```

output
```
-1/2*Sqrt[a + b*x + c*x^2]/(a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*
d*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^
2])])/(8*a^(5/2)*d) - (f*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[a]*d^2) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sq
rt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x
+ c*x^2])])/(2*d^2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) + (f^(3/2)*ArcTanh
[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*S
qrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^2*Sqrt[c*d + b*Sqrt[d]
*Sqrt[f] + a*f])
```

3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.101.4 Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 457, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-3bx+2a)}{4a^2 d\,x^2}-\frac{-\frac{4f\,a^2\ln\left(\frac{\frac{-2b\sqrt{df}+2fa+2cd}{f}+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+2\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{(-2c\sqrt{df}+bf)}{f}\cdots}}{x+\frac{\sqrt{df}}{f}}\cdots\right)}{d\sqrt{\frac{-b\sqrt{df}+fa+cd}{f}}}\cdots}{\cdots}$
default	$\frac{-\frac{\sqrt{c\,x^2+bx+a}}{2a\,x^2}-\frac{3b\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{4a}+\frac{c\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}}{d}-\frac{f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx\cdots}}{x}\cdots\right)}{d^2\sqrt{a}}\cdots$

input
```
int(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-3*b*x+2*a)/a^2/d/x^2-1/8/a^2/d*(-4*f*a^2/d/(1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f))-4*f*a^2/d/((b*(d*f)^(1
/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f
)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/
f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f
)^(1/2))/(x-(d*f)^(1/2)/f))-(-8*a^2*f+4*a*c*d-3*b^2*d)/d/a^(1/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))
```

3.101.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.101.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = -\int \frac{1}{-dx^3\sqrt{a+bx+cx^2}+fx^5\sqrt{a+bx+cx^2}}\,dx$$

input
```
integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-d*x**3*sqrt(a + b*x + c*x**2) + f*x**5*sqrt(a + b*x + c*x**2
)), x)
```

3.101.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int -\frac{1}{\sqrt{cx^2+bx+a}(fx^2-d)x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 - d)*x^3), x)
```

3.101.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d-fx^2)}\,dx = \int \frac{1}{x^3\,\left(d-f\,x^2\right)\,\sqrt{c\,x^2+b\,x+a}}\,dx$$

input
```
int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)),x)
```

output
```
int(1/(x^3*(d - f*x^2)*(a + b*x + c*x^2)^(1/2)), x)
```

3.102 $\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.102.1 Optimal result

Integrand size = 28, antiderivative size = 466

$$\begin{aligned}\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = &-\frac{2x(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}}\\
&+\frac{2d(b+2cx)}{(b^2-4ac)f^2\sqrt{a+bx+cx^2}}-\frac{2d^2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)x)}{(b^2-4ac)f^2(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}}\\
&+\frac{2b\sqrt{a+bx+cx^2}}{c(b^2-4ac)f}-\frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}\\
&+\frac{d^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}}+\frac{d^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2f\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}\end{aligned}$$

output
```
-arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f+1/2*d^(3/2)*
arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a
)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
3/2)+1/2*d^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(3/2)-2*x*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*d
*(2*c*x+b)/(-4*a*c+b^2)/f^2/(c*x^2+b*x+a)^(1/2)-2*d^2*(b*(b^2*f-c*(3*a*f+c
*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/(-4*a*c+b^2)/f^2/(b^2*d*f-(a*f+c*d)^2)/(
c*x^2+b*x+a)^(1/2)+2*b*(c*x^2+b*x+a)^(1/2)/c/(-4*a*c+b^2)/f
```

3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.15 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.97

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(b^4dx+ab^2d(b-4cx)-a^3f(b-2cx)-a^2(3bcd-2c^2dx+b^2fx))}{c(-b^2+4ac)(c^2d^2+2acdf+f(-b^2d+a^2f))\sqrt{a+x(b+cx)}}$$
$$+\frac{\log\left(cf\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{c^{3/2}f}$$
$$-\frac{d^2\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{bcd\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+2abf\log\left(-\sqrt{\ldots}\right.}{\ldots}\right.}{2f\left(c^2d^2+\ldots\right.}$$

input
```
Integrate[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(2*(b^4*d*x + a*b^2*d*(b - 4*c*x) - a^3*f*(b - 2*c*x) - a^2*(3*b*c*d - 2*c
^2*d*x + b^2*f*x)))/(c*(-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) +
 a^2*f))*Sqrt[a + x*(b + c*x)]) + Log[c*f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a +
x*(b + c*x)])]/(c^(3/2)*f) - (d^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*S
qrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*
#1 + f*#1^3) & ])/(2*f*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a^2*f)))
```

3.102.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{(d-fx^2)(a+bx+cx^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{d^2}{f^2 (d-fx^2) (a+bx+cx^2)^{3/2}}-\frac{d}{f^2 (a+bx+cx^2)^{3/2}}-\frac{x^2}{f (a+bx+cx^2)^{3/2}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}f}+\frac{d^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2f\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2f\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{2d^2(b(b^2f-c(3af+cd))-cx(2acf+b^2(-f)+2c^2d))}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)}+\frac{2d(b+2cx)}{f^2(b^2-4ac)\sqrt{a+bx+cx^2}}+\frac{2b\sqrt{a+bx+cx^2}}{cf(b^2-4ac)}-\frac{2x(2a+bx)}{f(b^2-4ac)\sqrt{a+bx+cx^2}}$$

input
```
Int[x^4/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-2*x*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*d*(b + 2*c
*x))/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) - (2*d^2*(b*(b^2*f - c*(c*d
 + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2 - 4*a*c)*f^2*(b^2*d*f
 - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (2*b*Sqrt[a + b*x + c*x^2])/(c*
(b^2 - 4*a*c)*f) - ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/
(c^(3/2)*f) + (d^(3/2)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b
*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])
])/(2*f*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d^(3/2)*ArcTanh[(b*Sqrt[
d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*f*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)
^(3/2))
```

3.102.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. $2(394) = 788$.

Time = 0.83 (sec) , antiderivative size = 1064, normalized size of antiderivative = 2.28

method	result	size
default	Expression too large to display	1064

input
```
int(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(-x/c/(c*x^2+b*x+a)^(1/2)-1/2*b/c*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))+1/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(c*
x^2+b*x+a)^(1/2)))-2*d/f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2/f
^2*d^2/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*
(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/
2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+
1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d
*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*
f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)
/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f
*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))
^(1/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)
+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2/f^2*d^2/(d*f
)^(1/2)*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1
/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b
*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*
f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f
*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(
1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(...
```

3.102.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.102.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.102.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.102.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^4/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.102.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{x^4}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^4/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.103 $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.103.1 Optimal result

Integrand size = 28, antiderivative size = 341

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(2a+bx)}{(b^2-4ac)f\sqrt{a+bx+cx^2}} - \frac{2d(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)f(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{d\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{f}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
-1/2*d*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^
2+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)/(c*d+a*f-b*d^(1/
2)*f^(1/2))^(3/2)+1/2*d*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+
b*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/f^(1/2)
/(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a
)^(1/2)-2*d*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/f/(b
^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.74 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.20

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{8a^3f-4b^3dx-4abd(b-3cx)+4a^2(2cd+bfx)-(b^2-4ac)\,d\sqrt{a+x}}{}$$

input
```
Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(8*a^3*f - 4*b^3*d*x - 4*a*b*d*(b - 3*c*x) + 4*a^2*(2*c*d + b*f*x) - (b^2
- 4*a*c)*d*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
+ 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
+ a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log
[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(b^2 -
4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)])
```

3.103.3 Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(d-fx^2)(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{dx}{f(d-fx^2)(a+bx+cx^2)^{3/2}}-\frac{x}{f(a+bx+cx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{d\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{2d\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}-\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[x^3/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) - (2*d*(a*(2*c^2*
d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*f*(b^2*d*f - (c*
d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] +
 (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[
a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)) + (d
*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*(c*d + b
*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.103.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 959 vs. $2(281) = 562$.

Time = 0.87 (sec) , antiderivative size = 960, normalized size of antiderivative = 2.82

method	result
default	$-\frac{-\frac{1}{c\sqrt{cx^2+bx+a}}-\frac{b(2cx+b)}{c\left(4ac-b^2\right)\sqrt{cx^2+bx+a}}}{f} - \frac{d\left(\frac{f}{\left(-b\sqrt{df}+fa+cd\right)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2c+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}} - \frac{}{\left(-b\sqrt{df}+f\right.}\right.}{}$

input
```
int(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/f*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/
2))-1/2*d/f^2*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c
*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-
2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(
-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(
1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1
/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f
*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*
c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2
)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*
(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2*d/f^2*(1/(b*(d*f)
^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^
(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(
1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f
)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/
(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)
^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1
/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d
*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.103.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17339 vs. $2(281) = 562$.

Time = 22.37 (sec) , antiderivative size = 17339, normalized size of antiderivative = 50.85

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.103.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.103.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.103.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.103.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{x^3}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^3/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.104 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.104.1 Optimal result

Integrand size = 28, antiderivative size = 297

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(ab(cd-af)+c(b^2d-2a(cd+af))\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\sqrt{d}\,\mathrm{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b
*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d+a*f-b*d^(1/2)*
f^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(
1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*d^(1/2)/(c*d
+a*f+b*d^(1/2)*f^(1/2))^(3/2)+2*(a*b*(-a*f+c*d)+c*(b^2*d-2*a*(a*f+c*d))*x)
/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.104.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.25

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{4b^2cdx+4acd(b-2cx)-4a^2f(b+2cx)+(b^2-4ac)\,d\sqrt{a+x(b+cx)}}{}$$

input
```
Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(4*b^2*c*d*x + 4*a*c*d*(b - 2*c*x) - 4*a^2*f*(b + 2*c*x) + (b^2 - 4*a*c)*d
*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*d*#1
^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^
2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3
/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*f*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3
) & ])/(2*(b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[
a + x*(b + c*x)])
```

3.104.3 Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2137, 27, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(d-fx^2)(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \; 2137$$

$$\frac{2\int -\frac{(b^2-4ac)d(cd+af-bfx)}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{(b^2-4ac)(b^2df-(af+cd)^2)}+\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\int\frac{cd+af-bfx}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{b^2df-(af+cd)^2}$$

$$\Big\downarrow \text{ 1366}$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\left(-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int\frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int-\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\left(\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int\frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\left(\frac{1}{2}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int\frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int\frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\left(\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int\frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)-\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int -\cdots\right)}{b^2df-(af+cd)^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\left(cx\left(b^2d-2a(af+cd)\right)+ab(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}-\frac{d\left(\frac{\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{b^2df-(af+cd)^2}$$

3.104. $\int\frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

input
```
Int[x^2/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(2*(a*b*(c*d - a*f) + c*(b^2*d - 2*a*(c*d + a*f))*x))/((b^2 - 4*a*c)*(b^2*
d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (d*(((b*Sqrt[f] + (c*d + a*f
)/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)
/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c
*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((b*Sqrt[f] - (c*d + a*f)/Sqrt[d])*ArcTan
h[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c*d + b*
Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*d + b*Sqrt[d]*Sq
rt[f] + a*f])))/(b^2*d*f - (c*d + a*f)^2)
```

3.104.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

rule 2137
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f_.)*(x_)^2)^(q_
), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C = Coeff[P
x, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*
c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((A*c - a*C)*((-b)*(c*d + a*f)) + (A
*b - a*B)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(2*a*f)
) - B*(b*c*d + a*b*f) + C*(b^2*d - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2
- 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)
*(d + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2 - (b*d)*((-b)*f))*
(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*
C*d - a*C*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*((A*c - a*C)*((-b)*(c*d +
 a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p + q + 2) - (b^2*(C*d
 + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - a*C*f)))*(b*f
*(p + 1)))*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + a*B*f) + 2*(A*c*(c*d - a*f
) - a*(c*C*d - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; FreeQ[{a, b, c,
 d, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)
^2, 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(239) = 478$.

Time = 0.77 (sec) , antiderivative size = 946, normalized size of antiderivative = 3.19

method	result
default	$-\frac{2(2cx+b)}{f(4ac-b^2)\sqrt{c\,x^2+bx+a}}+\frac{d\left(\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}}-\frac{}{(-b\sqrt{df}+fa+cd)\left(\frac{4c(-b\sqrt{d}}{}\right.}\right.}{}$

input
```
int(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*d/(d*f)^(1/2)/f*(f/(-b*
(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+
(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/
(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f)
)/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f
)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^
(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+
c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x
+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1/2)/f)^
2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c
*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2*d/(d*f)^(1/2)/f*(1/(b*(d*f)^(1/2)+f*a+
c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*
d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a
+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1
/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.104.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17285 vs. 2(239) = 478.

Time = 13.62 (sec) , antiderivative size = 17285, normalized size of antiderivative = 58.20

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.104.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.104.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.104.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.104.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{x^2}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x^2/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.105 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.105.1 Optimal result

Integrand size = 26, antiderivative size = 299

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\sqrt{f}\,\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{\sqrt{f}\,\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
-1/2*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+
b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*d+a*f-b*d^(1/2)
*f^(1/2))^(3/2)+1/2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))*f^(1/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)
*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.105.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.36

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{-8a^2cf-4bc^2dx+4a(-2c^2d+b^2f+bcfx)-(b^2-4ac)\,f\sqrt{a+x(b+}}{}$$

input
```
Integrate[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-8*a^2*c*f - 4*b*c^2*d*x + 4*a*(-2*c^2*d + b^2*f + b*c*f*x) - (b^2 - 4*a*
c)*f*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1 + 4*c*
d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(Sqr
t[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*(b^2 - 4*a*c)
*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*Sqrt[a + x*(b + c*x)])
```

3.105.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1351, 27, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(d-fx^2)(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \; 1351$$

$$\frac{2\int \frac{(b^2-4ac)f(bd-(cd+af)x)}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{(b^2-4ac)(b^2df-(af+cd)^2)}-\frac{2(a(2acf+b^2(-f)+2c^2d)+bcx(cd-af))}{(b^2-4ac)\sqrt{a+bx+cx^2}(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{f\int \frac{bd-(cd+af)x}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{b^2df-(af+cd)^2}-\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\downarrow$ 1366

$$\frac{f\left(-\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}d\right.}{b^2df-(af+cd)^2}$$

$$\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\downarrow$ 25

$$\frac{f\left(\frac{1}{2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}$$

$$\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\downarrow$ 27

$$\frac{f\left(\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx}{2\sqrt{f}}\right)}{b^2df-(af+cd)^2}-$$

$$\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\downarrow$ 1154

$$\frac{f\left(\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)}{\sqrt{f}}-\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)\int \frac{}{4\left(-\sqrt{d}\sqrt{\right.}}}{}\right.}{b^2df-(af+cd)^2}$$

$$\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$\downarrow$ 219

3.105. $\int \frac{x}{\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx$

$$\frac{f\left(\frac{\left(af+b\sqrt{d}\sqrt{f}+cd\right)\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{f}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{b^2df-(af+cd)^2}-$$

$$\frac{2\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

input
```
Int[x/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-2*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*(b
^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) + (f*(((c*d + b*Sqrt[d]*Sqr
t[f] + a*f)*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x
)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[
f]*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((c*d - b*Sqrt[d]*Sqrt[f] + a*f)
*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(2*Sqrt[c
*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[f]*Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f])))/(b^2*d*f - (c*d + a*f)^2)
```

3.105.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1351
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (f
_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^
(q + 1)/((b^2 - 4*a*c)*(b^2*d*f + (c*d - a*f)^2)*(p + 1)))*((g*c)*((-b)*(c*
d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)) + c*(g*(2*c^2*d + b^2
*f - c*(2*a*f)) - h*(b*c*d + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*S
imp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d)*((-b)*f))*(p + 1) + (b^2*(g*f) - b
*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(a*f*(p + 1) - c*d*(p + 2)) - (2*f*
((g*c)*((-b)*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(2*a*f)))*(p +
 q + 2) - (b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(b*f*(p + 1
)))*x - c*f*(b^2*(g*f) - b*(h*c*d + a*h*f) + 2*(g*c*(c*d - a*f)))*(2*p + 2*
q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, g, h, q}, x] && NeQ[b^2 - 4
*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ[
p] && ILtQ[q, -1])
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(241) = 482$.

Time = 0.68 (sec) , antiderivative size = 899, normalized size of antiderivative = 3.01

method	result
default	$-\frac{\frac{f}{\left(-b\sqrt{df}+fa+cd\right)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^{2}c+\frac{\left(-2c\sqrt{df}+bf\right)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}}-\frac{\left(-2c\sqrt{df}+bf\right)\left(2c\left(x+\frac{\sqrt{df}}{f}\right.\right.}{\left(-b\sqrt{df}+fa+cd\right)\left(\frac{4c\left(-b\sqrt{df}+fa+cd\right)}{f}-\frac{\left(-2c\sqrt{df}+bf\right)^{2}}{f^{2}}\right)\sqrt{\left(x\right.}}}{}$

input
```
int(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2/f*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^
(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*
f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d
*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*
f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+
1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d
*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)
^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(
d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*
f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2/f*(1/(b*(d*f)^(1/2)+f*a+
c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b
*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*
d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a
+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/
2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1
/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+
c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d
)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f
)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.105.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17258 vs. $2(241) = 482$.

Time = 15.82 (sec) , antiderivative size = 17258, normalized size of antiderivative = 57.72

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.105.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.105.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.105.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{x}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(x/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.106 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.106.1 Optimal result

Integrand size = 25, antiderivative size = 310

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = -\frac{2(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} + \frac{f\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{d}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
1/2*f*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2
+b*x+a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/(c*d+a*f-b*d^(1/2
)*f^(1/2))^(3/2)+1/2*f*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b
*f^(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(1/2)/
(c*d+a*f+b*d^(1/2)*f^(1/2))^(3/2)-2*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^
2*f+2*c^2*d)*x)/(-4*a*c+b^2)/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.106.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.76 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.50

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{-4b^3f+4bc(cd+3af)-4b^2cfx+8c^2(cd+af)x+(b^2-4ac)\,f\sqrt{a+}}{}$$

input
```
Integrate[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-4*b^3*f + 4*b*c*(c*d + 3*a*f) - 4*b^2*c*f*x + 8*c^2*(c*d + a*f)*x + (b^2
 - 4*a*c)*f*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b^2*f + 4*Sqrt[a]*b*f*#1
 - 2*c*d*#1^2 - 4*a*f*#1^2 + d*#1^4 & , (-(c^2*d*Log[x]) - b^2*f*Log[x] -
a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + b^2*f*
Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f*Log[-Sqrt[a] + Sqrt[a
 + b*x + c*x^2] - x*#1] + 2*Sqrt[a]*b*f*Log[x]*#1 - 2*Sqrt[a]*b*f*Log[-Sqr
t[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*Log[x]*#1^2 + a*f*Log[x]*#1^
2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*f*Log[-Sqrt[
a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sqrt[a]*b*f) + c*d*#1 + 2*a*f*
#1 - d*#1^3) & ])/(2*(b^2 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^
2*f))*Sqrt[a + x*(b + c*x)])
```

3.106.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {1306, 27, 1366, 25, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(d-fx^2)(a+bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1306}$$

$$-\frac{2\int \frac{(b^2-4ac)f(cd+af-bfx)}{2\sqrt{cx^2+bx+a}(d-fx^2)}dx}{(b^2-4ac)(b^2df-(af+cd)^2)}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\,(b^2df-(af+cd)^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{f\int \frac{cd+af-bfx}{\sqrt{cx^2+bx+a}(d-fx^2)}dx}{b^2df-(af+cd)^2}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 1366

$$-\frac{f\left(-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int -\frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 25

$$-\frac{f\left(\frac{1}{2}\sqrt{f}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\sqrt{f}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\sqrt{f}\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 27

$$-\frac{f\left(\frac{1}{2}\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int \frac{1}{\left(\sqrt{f}x+\sqrt{d}\right)\sqrt{cx^2+bx+a}}dx-\frac{1}{2}\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{\left(\sqrt{d}-\sqrt{f}x\right)\sqrt{cx^2+bx+a}}dx\right)}{b^2df-(af+cd)^2}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 1154

$$-\frac{f\left(\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\int \frac{1}{4\left(\sqrt{d}\sqrt{f}b+cd+af\right)-\frac{\left(2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}\right)^2}{cx^2+bx+a}}d\left(-\frac{2\sqrt{f}a+\left(\sqrt{f}b+2c\sqrt{d}\right)x+b\sqrt{d}}{\sqrt{cx^2+bx+a}}\right)-\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\int\right.}{b^2df-(af+cd)^2}$$

$$\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

$\downarrow$ 219

3.106. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

$$-\frac{f\left(\frac{\left(\frac{af+cd}{\sqrt{d}}+b\sqrt{f}\right)\operatorname{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}-\frac{\left(b\sqrt{f}-\frac{af+cd}{\sqrt{d}}\right)\operatorname{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{b^2df-(af+cd)^2}-\frac{2\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}$$

input
```
Int[1/((a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-2*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
 - 4*a*c)*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - (f*(((b*Sqrt[
f] + (c*d + a*f)/Sqrt[d])*ArcTanh[(b*Sqrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d]
- b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^
2])])/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f] + a*f]) - ((b*Sqrt[f] - (c*d + a*f)/
Sqrt[d])*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*x)/(
2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c*d
 + b*Sqrt[d]*Sqrt[f] + a*f])))/(b^2*d*f - (c*d + a*f)^2)
```

3.106.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1306
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (f_.)*(x_)^2)^(q_), x
_Symbol] :> Simp[(b^3*f + b*c*(c*d - 3*a*f) + c*(2*c^2*d + b^2*f - c*(2*a*f
))*x)*(a + b*x + c*x^2)^(p + 1)*((d + f*x^2)^(q + 1)/((b^2 - 4*a*c)*(b^2*d*
f + (c*d - a*f)^2)*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*(b^2*d*f + (c*d -
a*f)^2)*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + f*x^2)^q*Simp[2*c*(b^
2*d*f + (c*d - a*f)^2)*(p + 1) - (2*c^2*d + b^2*f - c*(2*a*f))*(a*f*(p + 1)
 - c*d*(p + 2)) + (2*f*(b^3*f + b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d +
 b^2*f - c*(2*a*f))*(b*f*(p + 1)))*x + c*f*(2*c^2*d + b^2*f - c*(2*a*f))*(2
*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, f, q}, x] && NeQ[b^2 -
4*a*c, 0] && LtQ[p, -1] && NeQ[b^2*d*f + (c*d - a*f)^2, 0] &&  !( !IntegerQ
[p] && ILtQ[q, -1]) &&  !IGtQ[q, 0]
```

rule 1366
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (
f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[(-a)*c, 2]}, Simp[(h/2 + c*(g/(2*q
)))   Int[1/((-q + c*x)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[(h/2 - c*(g/(
2*q)))   Int[1/((q + c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, c, d
, e, f, g, h}, x] && NeQ[e^2 - 4*d*f, 0] && PosQ[(-a)*c]
```

3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(252) = 504$.

Time = 0.77 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.91

method	result
default	$\frac{f}{(-b\sqrt{df}+fa+cd)\sqrt{\left(x+\frac{\sqrt{df}}{f}\right)^2 c+\frac{(-2c\sqrt{df}+bf)\left(x+\frac{\sqrt{df}}{f}\right)}{f}+\frac{-b\sqrt{df}+fa+cd}{f}}} - \frac{(-2c\sqrt{df}+bf)\left(2c\left(x+\frac{\sqrt{df}}{f}\right)-\cdots\right.}{(-b\sqrt{df}+fa+cd)\left(\frac{4c(-b\sqrt{df}+fa+cd)}{f}-\frac{(-2c\sqrt{df}+bf)^2}{f^2}\right)\sqrt{\left(x+\cdots\right.}}$

input
```
int(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/2/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2
*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-
(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f
*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)
^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^
(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1
/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-
2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1
/2)*((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/
f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2))/(x+(d*f)^(1/2)/f)))-1/2/(d*f)^(1/2)*(1/
(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(
x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b
*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c
*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)
^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^
(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*
(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*
(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)
/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))
```

3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 17397 vs. $2(252) = 504$.

Time = 14.10 (sec) , antiderivative size = 17397, normalized size of antiderivative = 56.12

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
Timed out
```

3.106.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(((c*sqrt(4*d*f))/(2*f^2)>0)', se
e `assume?
```

3.106.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.106.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \int \frac{1}{(d-f\,x^2)\,(c\,x^2+b\,x+a)^{3/2}}\,dx$$

input
```
int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/((d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.107 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.107.1 Optimal result

Integrand size = 28, antiderivative size = 394

$$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,d\sqrt{a+bx+cx^2}} - \frac{2f(a(2c^2d-b^2f+2acf)+bc(cd-af)x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} - \frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d} - \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^{3/2}\operatorname{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d-1/2*f^(3/2)*
arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+a
)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f-b*d^(1/2)*f^(1/2))^(
3/2)+1/2*f^(3/2)*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^(1/
2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d/(c*d+a*f+b*d
^(1/2)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(
1/2)-2*f*(a*(2*a*c*f-b^2*f+2*c^2*d)+b*c*(-a*f+c*d)*x)/(-4*a*c+b^2)/d/(b^2*
d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)
```

3.107.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.23 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.23

$$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx = \frac{2(b^4f+2ac^2(cd+af)-b^2c(cd+4af)+b^3cfx-bc^2(cd+3af)x)}{a(-b^2+4ac)(c^2d^2+2acdf+f(-b^2d+a^2f))\sqrt{a+x(b+cx)}} + \frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}d} + \frac{f^2\text{RootSum}\left[b^2d-a^2f-4b\sqrt{c}d\#1+4cd\#1^2+2af\#1^2-f\#1^4\&,\frac{b^2d\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+acd\log\left(-\sqrt{c}\ldots\right.}{\ldots}\right.}{\ldots}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(2*(b^4*f + 2*a*c^2*(c*d + a*f) - b^2*c*(c*d + 4*a*f) + b^3*c*f*x - b*c^2*
(c*d + 3*a*f)*x))/(a*(-b^2 + 4*a*c)*(c^2*d^2 + 2*a*c*d*f + f*(-(b^2*d) + a
^2*f))*Sqrt[a + x*(b + c*x)]) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*
x)])/Sqrt[a]])/(a^(3/2)*d) + (f^2*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*#1
 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b^2*d*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1] + a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
 + a^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Lo
g[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*d*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*
x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*f*#1 + f*#1^3) & ])/(2*c^2*d^
3 - 2*b^2*d^2*f + 4*a*c*d^2*f + 2*a^2*d*f^2)
```

3.107.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(d-fx^2)(a+bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}-\frac{fx}{d\left(fx^2-d\right)\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}-\frac{f^{3/2}\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^{3/2}\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{2f\left(a\left(2acf+b^2(-f)+2c^2d\right)+bcx(cd-af)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{ad\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x + c*x^2]) - (2*f
*(a*(2*c^2*d - b^2*f + 2*a*c*f) + b*c*(c*d - a*f)*x))/((b^2 - 4*a*c)*d*(b^
2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ArcTanh[(2*a + b*x)/(2*Sqr
t[a]*Sqrt[a + b*x + c*x^2])]/(a^(3/2)*d) - (f^(3/2)*ArcTanh[(b*Sqrt[d] - 2
*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d]*Sqrt[f]
+ a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c*d - b*Sqrt[d]*Sqrt[f] + a*f)^(3/2)
) + (f^(3/2)*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] + b*Sqrt[f])*
x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])]])/(2*d*(c
*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(326) = 652$.

Time = 0.75 (sec) , antiderivative size = 990, normalized size of antiderivative = 2.51

method	result
default	$\frac{\frac{1}{a\sqrt{c\,x^2+bx+a}}-\frac{b(2cx+b)}{a\left(4ac-b^2\right)\sqrt{c\,x^2+bx+a}}-\frac{\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{a^{\frac{3}{2}}}}{d} - \frac{f}{(b\sqrt{df}+fa+cd)\sqrt{\left(x-\frac{\sqrt{df}}{f}\right)^2 c+\frac{(2c\sqrt{df}+bf)\left(x-\frac{\sqrt{df}}{f}\right)}{f}+\frac{b\sqrt{df}-}{}}}$

input
```
int(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)
-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))-1/2/d*(1/(b*(d*f
)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)
^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(d*f)^(1/2)+b*f)/(b*(d*f)^
(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^(1/2)+b*f)/f)/(4*c*(b*(d*
f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/((x-(d*f)^(1/2)/f)^2*c+(2
*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-1
/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*ln((2*(b*(d*f
)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+2*((b*(d*f)^(
1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(
d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(x-(d*f)^(1/2)/f)))-1/2/d*
(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b
*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2
)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/
2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/(
(x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b
*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/
2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+
b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*((x+(d*f)^(1
/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(...
```

3.107.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.107.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=$$
$$-\int \frac{1}{-adx\sqrt{a+bx+cx^2}+afx^3\sqrt{a+bx+cx^2}-bdx^2\sqrt{a+bx+cx^2}+bfx^4\sqrt{a+bx+cx^2}-cdx^3\sqrt{a+b}$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-a*d*x*sqrt(a + b*x + c*x**2) + a*f*x**3*sqrt(a + b*x + c*x**
2) - b*d*x**2*sqrt(a + b*x + c*x**2) + b*f*x**4*sqrt(a + b*x + c*x**2) - c
*d*x**3*sqrt(a + b*x + c*x**2) + c*f*x**5*sqrt(a + b*x + c*x**2)), x)
```

3.107.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\int -\frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x), x)
```

3.107.8 Giac [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\int -\frac{1}{\left(cx^2+bx+a\right)^{\frac{3}{2}}\left(fx^2-d\right)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
sage2
```

3.107.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d-fx^2\right)}\,dx=\int \frac{1}{x\,\left(d-f\,x^2\right)\,{\left(c\,x^2+b\,x+a\right)}^{3/2}}\,dx$$

input
```
int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/(x*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.108 $\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx$

3.108.1 Optimal result

Integrand size = 28, antiderivative size = 454

$$\begin{aligned}\int \frac{1}{x^2(a+bx+cx^2)^{3/2}(d-fx^2)}\,dx &= \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,dx\sqrt{a+bx+cx^2}} \\ &- \frac{2f(b(b^2f-c(cd+3af))-c(2c^2d-b^2f+2acf)\,x)}{(b^2-4ac)\,d\,(b^2df-(cd+af)^2)\sqrt{a+bx+cx^2}} \\ &- \frac{(3b^2-8ac)\sqrt{a+bx+cx^2}}{a^2(b^2-4ac)\,dx} + \frac{3b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d} \\ &+ \frac{f^2\text{arctanh}\left(\frac{b\sqrt{d}-2a\sqrt{f}+\left(2c\sqrt{d}-b\sqrt{f}\right)x}{2\sqrt{cd-b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd-b\sqrt{d}\sqrt{f}+af\right)^{3/2}} + \frac{f^2\text{arctanh}\left(\frac{b\sqrt{d}+2a\sqrt{f}+\left(2c\sqrt{d}+b\sqrt{f}\right)x}{2\sqrt{cd+b\sqrt{d}\sqrt{f}+af}\sqrt{a+bx+cx^2}}\right)}{2d^{3/2}\left(cd+b\sqrt{d}\sqrt{f}+af\right)^{3/2}}\end{aligned}$$

output
```
3/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(5/2)/d+1/2*f^2
*arctanh(1/2*(b*d^(1/2)-2*a*f^(1/2)+x*(2*c*d^(1/2)-b*f^(1/2)))/(c*x^2+b*x+
a)^(1/2)/(c*d+a*f-b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*d+a*f-b*d^(1/2)*f^(
1/2))^(3/2)+1/2*f^2*arctanh(1/2*(b*d^(1/2)+2*a*f^(1/2)+x*(2*c*d^(1/2)+b*f^
(1/2)))/(c*x^2+b*x+a)^(1/2)/(c*d+a*f+b*d^(1/2)*f^(1/2))^(1/2))/d^(3/2)/(c*
d+a*f+b*d^(1/2)*f^(1/2))^(3/2)+2*(b*c*x-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x/(c*x
^2+b*x+a)^(1/2)-2*f*(b*(b^2*f-c*(3*a*f+c*d))-c*(2*a*c*f-b^2*f+2*c^2*d)*x)/
(-4*a*c+b^2)/d/(b^2*d*f-(a*f+c*d)^2)/(c*x^2+b*x+a)^(1/2)-(-8*a*c+3*b^2)*(c
*x^2+b*x+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x
```

3.108.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 2.10 (sec) , antiderivative size = 620, normalized size of antiderivative = 1.37

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} dx = \frac{-2\sqrt{a}(4a^4cf^2+3b^2d(-c^2d+b^2f)\,x(b+cx)+a^3f(-b^2f+4bcfx+}{}$$

input
```
Integrate[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(-2*Sqrt[a]*(4*a^4*c*f^2 + 3*b^2*d*(-(c^2*d) + b^2*f)*x*(b + c*x) + a^3*f*
(-(b^2*f) + 4*b*c*f*x + 4*c^2*(2*d + f*x^2)) + a^2*(18*b*c^2*d*f*x - b^3*f
^2*x - b^2*c*f*(6*d + f*x^2) + 4*c^3*d*(d + 3*f*x^2)) + a*d*(b^4*f + 10*b*
c^3*d*x - 16*b^3*c*f*x + 8*c^4*d*x^2 - b^2*c^2*(d + 14*f*x^2))) - 6*b*(b^2
 - 4*a*c)*(-(c^2*d^2) - 2*a*c*d*f + f*(b^2*d - a^2*f))*x*Sqrt[a + x*(b + c
*x)]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] + a^(5/2)*(b^2 -
 4*a*c)*f^2*x*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a^2*f - 4*b*Sqrt[c]*d*
#1 + 4*c*d*#1^2 + 2*a*f*#1^2 - f*#1^4 & , (b*c*d*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1] + 2*a*b*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1] - 2*c^(3/2)*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*a
*Sqrt[c]*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - b*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(b*Sqrt[c]*d - 2*c*d*#1 - a*
f*#1 + f*#1^3) & ])/(2*a^(5/2)*(-b^2 + 4*a*c)*d*(c^2*d^2 + 2*a*c*d*f + f*(
-(b^2*d) + a^2*f))*x*Sqrt[a + x*(b + c*x)])
```

3.108.3 Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2 (d-fx^2) (a+bx+cx^2)^{3/2}} dx$$

$$\Big\downarrow \; 7276$$

$$\int \left(\frac{f}{d\left(d-fx^2\right)\left(a+bx+cx^2\right)^{3/2}}+\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3b\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{5/2}d}-\frac{\left(3b^2-8ac\right)\sqrt{a+bx+cx^2}}{a^2dx\left(b^2-4ac\right)}+\frac{f^2\text{arctanh}\left(\frac{-2a\sqrt{f}+x\left(2c\sqrt{d}-b\sqrt{f}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\left(-\sqrt{d}\right)\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\left(-\sqrt{d}\right)\sqrt{f}+cd\right)^{3/2}}+\frac{f^2\text{arctanh}\left(\frac{2a\sqrt{f}+x\left(b\sqrt{f}+2c\sqrt{d}\right)+b\sqrt{d}}{2\sqrt{a+bx+cx^2}\sqrt{af+b\sqrt{d}\sqrt{f}+cd}}\right)}{2d^{3/2}\left(af+b\sqrt{d}\sqrt{f}+cd\right)^{3/2}}-\frac{2f\left(b\left(b^2f-c(3af+cd)\right)-cx\left(2acf+b^2(-f)+2c^2d\right)\right)}{d\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left(b^2df-(af+cd)^2\right)}+\frac{2\left(-2ac+b^2+bcx\right)}{adx\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[1/(x^2*(a + b*x + c*x^2)^(3/2)*(d - f*x^2)),x]
```

output
```
(2*(b^2 - 2*a*c + b*c*x))/(a*(b^2 - 4*a*c)*d*x*Sqrt[a + b*x + c*x^2]) - (2
*f*(b*(b^2*f - c*(c*d + 3*a*f)) - c*(2*c^2*d - b^2*f + 2*a*c*f)*x))/((b^2
- 4*a*c)*d*(b^2*d*f - (c*d + a*f)^2)*Sqrt[a + b*x + c*x^2]) - ((3*b^2 - 8*
a*c)*Sqrt[a + b*x + c*x^2])/(a^2*(b^2 - 4*a*c)*d*x) + (3*b*ArcTanh[(2*a +
b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(5/2)*d) + (f^2*ArcTanh[(b*S
qrt[d] - 2*a*Sqrt[f] + (2*c*Sqrt[d] - b*Sqrt[f])*x)/(2*Sqrt[c*d - b*Sqrt[d
]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*Sqrt[d]*Sqrt
[f] + a*f)^(3/2)) + (f^2*ArcTanh[(b*Sqrt[d] + 2*a*Sqrt[f] + (2*c*Sqrt[d] +
 b*Sqrt[f])*x)/(2*Sqrt[c*d + b*Sqrt[d]*Sqrt[f] + a*f]*Sqrt[a + b*x + c*x^2
])])/(2*d^(3/2)*(c*d + b*Sqrt[d]*Sqrt[f] + a*f)^(3/2))
```

3.108.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(382) = 764.

Time = 0.95 (sec) , antiderivative size = 1065, normalized size of antiderivative = 2.35

method	result	size
default	Expression too large to display	1065
risch	Expression too large to display	3015

input
```
int(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/a/x/(c*x^2+b*x+a)^(1/2)-3/2*b/a*(1/a/(c*x^2+b*x+a)^(1/2)-b/a*(2*c*
x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^
2+b*x+a)^(1/2))/x))-4*c/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2))-1/2*f
/d/(d*f)^(1/2)*(1/(b*(d*f)^(1/2)+f*a+c*d)*f/((x-(d*f)^(1/2)/f)^2*c+(2*c*(d
*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)-(2*c*(
d*f)^(1/2)+b*f)/(b*(d*f)^(1/2)+f*a+c*d)*(2*c*(x-(d*f)^(1/2)/f)+(2*c*(d*f)^
(1/2)+b*f)/f)/(4*c*(b*(d*f)^(1/2)+f*a+c*d)/f-(2*c*(d*f)^(1/2)+b*f)^2/f^2)/
((x-(d*f)^(1/2)/f)^2*c+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^
(1/2)+f*a+c*d)/f)^(1/2)-1/(b*(d*f)^(1/2)+f*a+c*d)*f/((b*(d*f)^(1/2)+f*a+c*
d)/f)^(1/2)*ln((2*(b*(d*f)^(1/2)+f*a+c*d)/f+(2*c*(d*f)^(1/2)+b*f)/f*(x-(d*
f)^(1/2)/f)+2*((b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2)*((x-(d*f)^(1/2)/f)^2*c+(2*
c*(d*f)^(1/2)+b*f)/f*(x-(d*f)^(1/2)/f)+(b*(d*f)^(1/2)+f*a+c*d)/f)^(1/2))/(
x-(d*f)^(1/2)/f)))+1/2*f/d/(d*f)^(1/2)*(f/(-b*(d*f)^(1/2)+f*a+c*d)/((x+(d*
f)^(1/2)/f)^2*c+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)
^(1/2)+f*a+c*d))^(1/2)-(-2*c*(d*f)^(1/2)+b*f)/(-b*(d*f)^(1/2)+f*a+c*d)*(2*
c*(x+(d*f)^(1/2)/f)+1/f*(-2*c*(d*f)^(1/2)+b*f))/(4*c/f*(-b*(d*f)^(1/2)+f*a
+c*d)-1/f^2*(-2*c*(d*f)^(1/2)+b*f)^2)/((x+(d*f)^(1/2)/f)^2*c+1/f*(-2*c*(d*
f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)-f/(-b*
(d*f)^(1/2)+f*a+c*d)/(1/f*(-b*(d*f)^(1/2)+f*a+c*d))^(1/2)*ln((2/f*(-b*(d*f
)^(1/2)+f*a+c*d)+1/f*(-2*c*(d*f)^(1/2)+b*f)*(x+(d*f)^(1/2)/f)+2*(1/f*(-...
```

3.108.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.108.6 Sympy [F]

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = -\int \frac{1}{-adx^2\sqrt{a+bx+cx^2}+afx^4\sqrt{a+bx+cx^2}-bdx^3\sqrt{a+bx+cx^2}+bfx^5\sqrt{a+bx+cx^2}-cdx^4\sqrt{a+}$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**(3/2)/(-f*x**2+d),x)
```

output
```
-Integral(1/(-a*d*x**2*sqrt(a + b*x + c*x**2) + a*f*x**4*sqrt(a + b*x + c*
x**2) - b*d*x**3*sqrt(a + b*x + c*x**2) + b*f*x**5*sqrt(a + b*x + c*x**2)
- c*d*x**4*sqrt(a + b*x + c*x**2) + c*f*x**6*sqrt(a + b*x + c*x**2)), x)
```

3.108.7 Maxima [F]

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \int -\frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2-d)x^2} \, dx$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="maxima")
```

output
```
-integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 - d)*x^2), x)
```

3.108.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(3/2)/(-f*x^2+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument ValueDone
```

3.108.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 (a+bx+cx^2)^{3/2} (d-fx^2)} \, dx = \int \frac{1}{x^2 \left(d-f\,x^2\right) \left(c\,x^2+b\,x+a\right)^{3/2}} \, dx$$

input
```
int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)),x)
```

output
```
int(1/(x^2*(d - f*x^2)*(a + b*x + c*x^2)^(3/2)), x)
```

3.109 $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.109.1 Optimal result

Integrand size = 30, antiderivative size = 761

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = -\frac{(4ce-bf-2cfx)\sqrt{a+bx+cx^2}}{4cf^2}$$

$$-\frac{(b^2f^2+4cf(be-af)-8c^2(e^2-df))\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}f^3}$$

$$-\frac{\left(c\left(e^4-4de^2f+2d^2f^2-e^3\sqrt{e^2-4df}+2def\sqrt{e^2-4df}\right)+f\left(af\left(e^2-2df-e\sqrt{e^2-4df}\right)-b(e^3-3c\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4d\right.}}}$$

$$+\frac{\left(c\left(e^4-4de^2f+2d^2f^2+e^3\sqrt{e^2-4df}-2def\sqrt{e^2-4df}\right)+f\left(af\left(e^2-2df+e\sqrt{e^2-4df}\right)-b(e^3-3c\right.\right.}{\sqrt{2}f^3\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4d\right.}}}$$

output
```
-1/8*(b^2*f^2+4*c*f*(-a*f+b*e)-8*c^2*(-d*f+e^2))*arctanh(1/2*(2*c*x+b)/c^(
1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f^3-1/4*(-2*c*f*x-b*f+4*c*e)*(c*x^2+b*x+
a)^(1/2)/c/f^2-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b
*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+
2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^4-4*d*e^2*f+2*d^2*f^2-
e^3*(-4*d*f+e^2)^(1/2)+2*d*e*f*(-4*d*f+e^2)^(1/2))+f*(a*f*(e^2-2*d*f-e*(-4
*d*f+e^2)^(1/2))-b*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1/2)+d*f*(-4*d*f+e^2)^(1
/2))))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+
c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*
d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(
c*(e^4-4*d*e^2*f+2*d^2*f^2+e^3*(-4*d*f+e^2)^(1/2)-2*d*e*f*(-4*d*f+e^2)^(1/
2))+f*(a*f*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))-b*(e^3-3*d*e*f+e^2*(-4*d*f+e^2
)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))))/f^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2
*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

3.109.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.14 (sec) , antiderivative size = 864, normalized size of antiderivative = 1.14

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{c}f(-4ce+bf+2cfx)\sqrt{a+x(b+cx)}+(-b^2f^2+4cf(-be+af)+8c^2(e^2-df))\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b}}\right.}{}$$

input
```
Integrate[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*f*(-4*c*e + b*f + 2*c*f*x)*Sqrt[a + x*(b + c*x)] + (-(b^2*f^2)
+ 4*c*f*(-(b*e) + a*f) + 8*c^2*(e^2 - d*f))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]
*Sqrt[a + x*(b + c*x)])] - 8*c^(3/2)*RootSum[b^2*d - a*b*e + a^2*f - 4*b*S
qrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sq
rt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]) + a*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b*
c*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*e*f*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*c*d*e*f*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] - a*b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] + a^2*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2
*c^(3/2)*d*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*c^(3/
2)*d^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d
*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f^2
*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^3*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + 2*c*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1^2 + b*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1^2 - b*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1
^2 - a*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqr
t[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2
*f*#1^3) & ])/(8*c^(3/2)*f^3)
```

3.109.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{ 2138}$$

$$-\frac{\int -\frac{-((-8(e^2-df)c^2+4f(be-af)c+b^2f^2)x^2)+(-efb^2+8c^2de+c(4be^2-4afe-8bdf))x+d(-fb^2+4ceb-4acf)}{4\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{-((-8(e^2-df)c^2+4f(be-af)c+b^2f^2)x^2)+(-efb^2+4c(e^2-2df)b+8c^2de-4acef)x+d(-fb^2+4ceb-4acf)}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{2143}$$

$$\frac{\frac{\int -\frac{8c\left(d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

3.109. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

3.109. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

3.109. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

3.109. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\Big\downarrow 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow 25$$

$$\frac{\frac{8c\int -\frac{d\left(ce^2-bfe+af^2-cdf\right)-\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

$$\Big\downarrow 25$$

$$\frac{-\frac{\left(4cf(be-af)+b^2f^2-8c^2\left(e^2-df\right)\right)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{8c\int -\frac{d\left(f(be-af)-c\left(e^2-df\right)\right)+\left(f\left(be^2-afe-bdf\right)-c\left(e^3-2def\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{8cf^2}-\frac{\sqrt{a+bx+cx^2}(-bf+4ce-2cfx)}{4cf^2}$$

input
```
Int[(x^2*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
$Aborted
```

3.109. $\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.109.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2138
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(B*c*f*(2*p + 2*q + 3) + C*(b*f*p - c*e*(2*p +
 q + 2)) + 2*c*C*f*(p + q + 1)*x)*(a + b*x + c*x^2)^p*((d + e*x + f*x^2)^(q
 + 1)/(2*c*f^2*(p + q + 1)*(2*p + 2*q + 3))), x] - Simp[1/(2*c*f^2*(p + q +
 1)*(2*p + 2*q + 3))   Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Si
mp[p*(b*d - a*e)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) +
(p + q + 1)*(b^2*C*d*f*p + a*c*(C*(2*d*f - e^2*(2*p + q + 2)) + f*(B*e - 2*
A*f)*(2*p + 2*q + 3))) + (2*p*(c*d - a*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e -
 B*f)*(2*p + 2*q + 3)) + (p + q + 1)*(C*e*f*p*(b^2 - 4*a*c) - b*c*(C*(e^2 -
 4*d*f)*(2*p + q + 2) + f*(2*C*d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x + (p*(
c*e - b*f)*(C*(c*e - b*f)*(q + 1) - c*(C*e - B*f)*(2*p + 2*q + 3)) + (p + q
 + 1)*(C*f^2*p*(b^2 - 4*a*c) - c^2*(C*(e^2 - 4*d*f)*(2*p + q + 2) + f*(2*C*
d - B*e + 2*A*f)*(2*p + 2*q + 3))))*x^2, x], x], x]] /; FreeQ[{a, b, c, d,
e, f, q}, x] && PolyQ[Px, x, 2] && GtQ[p, 0] && NeQ[p + q + 1, 0] && NeQ[2*
p + 2*q + 3, 0] &&  !IGtQ[p, 0] &&  !IGtQ[q, 0]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.109.4 Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 1148, normalized size of antiderivative = 1.51

method	result	size
risch	Expression too large to display	1148
default	Expression too large to display	1666

input
```
int(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*f*x+b*f-4*c*e)*(c*x^2+b*x+a)^(1/2)/c/f^2+1/8/c/f^2*(1/f*(4*a*c*f^
2-b^2*f^2-4*b*c*e*f-8*c^2*d*f+8*c^2*e^2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x
+a)^(1/2))/c^(1/2)+4/f^2*c*(a*e*f^2*(-4*d*f+e^2)^(1/2)+b*d*f^2*(-4*d*f+e^2
)^(1/2)-b*e^2*f*(-4*d*f+e^2)^(1/2)-2*c*d*e*f*(-4*d*f+e^2)^(1/2)+c*e^3*(-4*
d*f+e^2)^(1/2)+2*a*d*f^3-a*e^2*f^2-3*b*d*e*f^2+b*e^3*f-2*c*d^2*f^2+4*c*d*e
^2*f-c*e^4)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)
^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+
e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*
(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b
*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(
-4*d*f+e^2)^(1/2))))+4/f^2*c*(a*e*f^2*(-4*d*f+e^2)^(1/2)+b*d*f^2*(-4*d*f+e
^2)^(1/2)-b*e^2*f*(-4*d*f+e^2)^(1/2)-2*c*d*e*f*(-4*d*f+e^2)^(1/2)+c*e^3*(-
4*d*f+e^2)^(1/2)-2*a*d*f^3+a*e^2*f^2+3*b*d*e*f^2-b*e^3*f+2*c*d^2*f^2-4*c*d
*e^2*f+c*e^4)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f
+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+
e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/...
```

3.109.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.109.6 Sympy [F]

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx=\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate(x**2*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**2*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.109.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.109.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x^2*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.109.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x^2\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

output
```
int((x^2*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.110 $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.110.1 Optimal result

Integrand size = 28, antiderivative size = 549

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx=\frac{\sqrt{a+bx+cx^2}}{f}-\frac{(2ce-bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}f^2}$$
$$-\frac{\left(2df(ce-bf)+\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-\cdots}}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$
$$+\frac{\left(2df(ce-bf)+\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-\cdots}}}\right.}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-1/2*(-b*f+2*c*e)*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c
^(1/2)+(c*x^2+b*x+a)^(1/2)/f-1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+
e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-
2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c
*e)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*
f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(
1/2)+1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f
+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-
b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f*(-b*f+c*e)+(f*(-a*f+b*e)-c*(-d*
f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*
d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.110.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.17

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{2f\sqrt{a+x(b+cx)}+\frac{(2ce-bf)\log\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)}{\sqrt{c}}+2\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{ce}\right.}{}$$

input
```
Integrate[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
(2*f*Sqrt[a + x*(b + c*x)] + ((2*c*e - b*f)*Log[b + 2*c*x - 2*Sqrt[c]*Sqrt
[a + x*(b + c*x)]])/Sqrt[c] + 2*RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c
]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]
*e*#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
 #1]) + a*c*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*d*f*L
og[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*f*Log[-(Sqrt[c]*x) +
 Sqrt[a + b*x + c*x^2] - #1] - a*b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] + a^2*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c
^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]
*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 - c*e^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1^2 + b*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1^2 - a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/
(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*
#1^2 - 2*f*#1^3) & ])/(2*f^2)
```

3.110.3 Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 554, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1352, 27, 2143, 27, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

$$\Big\downarrow \text{ 1352}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{(2ce-bf)x^2+(2cd+be-2af)x+bd}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\int \frac{(2ce-bf)x^2+(2cd+be-2af)x+bd}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{2f}$$

$$\Big\downarrow \text{ 2143}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{\int -\frac{2\left(d(ce-bf)-\left(f(be-af)-c\left(e^2-df\right)\right)x\right)}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}+\frac{(2ce-bf)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}}{2f}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{(2ce-bf)\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{2\int \frac{d(ce-bf)-\left(f(be-af)-c\left(e^2-df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{2f}$$

$$\Big\downarrow \text{ 1092}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{2(2ce-bf)\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}-\frac{2\int \frac{d(ce-bf)-\left(f(be-af)-c\left(e^2-df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{2f}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{(2ce-bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{2\int \frac{d(ce-bf)-\left(f(be-af)-c\left(e^2-df\right)\right)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{f}}{2f}$$

$$\Big\downarrow \text{ 1365}$$

$$\frac{\sqrt{a+bx+cx^2}}{f}-$$

$$\frac{\frac{(2ce-bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{2\Bigg(\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(\left(\sqrt{e^2-4df}\right.\right.\cdots}{\cdots}}{f}}{2f}$$

$$\Big\downarrow \text{ 1154}$$

3.110. $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

$$\frac{\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{(2ce-bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{2\left(\frac{2\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\int\frac{}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-}}{\sqrt{e^2-}}\right.}{}}{}}{}$$

$\Big\downarrow$ 219

$$\frac{\frac{\sqrt{a+bx+cx^2}}{f}-\frac{\frac{(2ce-bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{2\left(\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(f(be-af)-c\left(e^2-df\right)\right)+2df(ce-bf)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{}\right.}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-b}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right.}{}}{2f}$$

input
```
Int[(x*Sqrt[a + b*x + c*x^2])/(d + e*x + f*x^2),x]
```

output
```
Sqrt[a + b*x + c*x^2]/f - (((2*c*e - b*f)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/(Sqrt[c]*f) - (2*(-(((2*d*f*(c*e - b*f) + (e - Sqr
t[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - S
qrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt
[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
 + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((2*d*f*(c*e - b*f) + (e +
 Sqrt[e^2 - 4*d*f])*(f*(b*e - a*f) - c*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e
 + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*
Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sq
rt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/f)/(2*f)
```

3.110. $\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1352
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[h*(a + b*x + c*x^2)^p*((d
+ e*x + f*x^2)^(q + 1)/(2*f*(p + q + 1))), x] - Simp[1/(2*f*(p + q + 1))
Int[(a + b*x + c*x^2)^(p - 1)*(d + e*x + f*x^2)^q*Simp[h*p*(b*d - a*e) + a*
(h*e - 2*g*f)*(p + q + 1) + (2*h*p*(c*d - a*f) + b*(h*e - 2*g*f)*(p + q + 1
))*x + (h*p*(c*e - b*f) + c*(h*e - 2*g*f)*(p + q + 1))*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*
d*f, 0] && GtQ[p, 0] && NeQ[p + q + 1, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1021 vs. $2(490) = 980$.

Time = 0.91 (sec) , antiderivative size = 1022, normalized size of antiderivative = 1.86

method	result	size
risch	Expression too large to display	1022
default	Expression too large to display	1580

input
```
int(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1/2)/f+1/2/f*(1/f*(b*f-2*c*e)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2
+b*x+a)^(1/2))/c^(1/2)-1/2*(2*a*f^2*(-4*d*f+e^2)^(1/2)-2*b*e*f*(-4*d*f+e^2
)^(1/2)-2*c*d*f*(-4*d*f+e^2)^(1/2)+2*c*e^2*(-4*d*f+e^2)^(1/2)-2*a*e*f^2-4*
b*d*f^2+2*b*e^2*f+6*c*d*e*f-2*c*e^3)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*
(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^
2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2
*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))-1/2*(2*a*f^2*(-4*d*f+e^2
)^(1/2)-2*b*e*f*(-4*d*f+e^2)^(1/2)-2*c*d*f*(-4*d*f+e^2)^(1/2)+2*c*e^2*(-4*
d*f+e^2)^(1/2)+2*a*e*f^2+4*b*d*f^2-2*b*e^2*f-6*c*d*e*f+2*c*e^3)/f^2/(-4*d*
f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^
(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*...
```

3.110.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.110.6 Sympy [F]

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate(x*(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x*sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.110.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.110.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.110.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{x\,\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d}\,dx$$

input
```
int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2),x)
```

output
```
int((x*(a + b*x + c*x^2)^(1/2))/(d + e*x + f*x^2), x)
```

3.111 $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} \, dx$

3.111.1 Optimal result

Integrand size = 27, antiderivative size = 431

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} \, dx = \frac{\sqrt{c}\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}$$
$$- \frac{\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{\right.\right.}}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$
$$+ \frac{\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{\right.\right.}}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-}}}\right.}{\sqrt{2}f\sqrt{e^2-4df}}$$

output
```
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/f-1/2*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d
*f+e^2)^(1/2)))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)+1/2*arctanh(1/4*(4*a*f-
b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^
2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))
^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/
2))))^(1/2)/f*2^(1/2)/(-4*d*f+e^2)^(1/2)
```

3.111.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$= \frac{2\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{-\sqrt{a}+\sqrt{a+x(b+cx)}}\right) + \operatorname{RootSum}\left[c^2d - bce + b^2f + 2\sqrt{a}ce\#1 - 4\sqrt{a}bf\#1 - 2cd\#1^2 + be\#1^2\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

output
```
(2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/(-Sqrt[a] + Sqrt[a + x*(b + c*x)])] + RootS
um[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^
2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*d*Log[x])
 + b*c*e*Log[x] - b^2*f*Log[x] + a*c*f*Log[x] + c^2*d*Log[-Sqrt[a] + Sqrt[
a + b*x + c*x^2] - x*#1] - b*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*
#1] + b^2*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - a*c*f*Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*Log[x]*#1 + 2*Sqrt[a]*
b*f*Log[x]*#1 + 2*Sqrt[a]*c*e*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]
*#1 - 2*Sqrt[a]*b*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1 + c*d*
Log[x]*#1^2 - a*f*Log[x]*#1^2 - c*d*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] -
 x*#1]*#1^2 + a*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(-(Sq
rt[a]*c*e) + 2*Sqrt[a]*b*f + 2*c*d*#1 - b*e*#1 - 4*a*f*#1 + 3*Sqrt[a]*e*#1
^2 - 2*d*#1^3) & ])/f
```

3.111.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1320, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$
$$\Big\downarrow \text{ 1320}$$

$$\frac{c\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$\downarrow$ 1092

$$\frac{2c\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$\downarrow$ 219

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\int \frac{cd-af+(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$\downarrow$ 1365

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2 ...}}}{\sqrt{e^2-4df}}}{f}$$

$\downarrow$ 1154

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{2\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af- ...}{}}{\sqrt{e^2-4df}}$$

$\downarrow$ 219

$$\frac{\sqrt{c}\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{f}-\frac{\frac{\left(2f(cd-af)-\left(\sqrt{e^2-4df}+e\right)(ce-bf)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(2f(cd-af)-\left(e-\sqrt{e^2-4df}\right)(ce- ...}{\sqrt{2}\sqrt{e^2 ...}}}{f}$$

input

```
Int[Sqrt[a + b*x + c*x^2]/(d + e*x + f*x^2),x]
```

3.111. $\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$

output
```
(Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/f - (-(((
2*f*(c*d - a*f) - (c*e - b*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + ((2*f*(c*d - a*f) -
(c*e - b*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 +
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/f
```

3.111.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1320
```
Int[Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2]/((d_) + (e_.)*(x_) + (f_.)*(x_)^
2), x_Symbol] :> Simp[c/f   Int[1/Sqrt[a + b*x + c*x^2], x], x] - Simp[1/f
  Int[(c*d - a*f + (c*e - b*f)*x)/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2))
, x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2
- 4*d*f, 0]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1546 vs. $2(376) = 752$.

Time = 0.94 (sec) , antiderivative size = 1547, normalized size of antiderivative = 3.59

method	result	size
default	Expression too large to display	1547

input
```
int((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c
*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*
d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1
/2)+1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)+c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+
e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+
c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2
))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2
-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+1/(-4*d*f+e^2)^(1/2)*(1/2*(4
*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*
(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2*(c*(-4*d*f+e^2)^(1...
```

3.111.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.111.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(d + e*x + f*x**2), x)
```

3.111.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.111.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.111.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{d+ex+fx^2} \, dx = \int \frac{\sqrt{c\,x^2+b\,x+a}}{f\,x^2+e\,x+d} \,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(d + e*x + f*x^2), x)
```

3.112 $\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)}\,dx$

3.112.1 Optimal result

Integrand size = 30, antiderivative size = 523

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx = -\frac{\sqrt{a}\,\mathrm{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

$$+\frac{\left(cd\left(e-\sqrt{e^2-4df}\right)-f\left(2bd-a\left(e+\sqrt{e^2-4df}\right)\right)\right)\mathrm{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}$$

$$-\frac{\left(cd\left(e+\sqrt{e^2-4df}\right)-f\left(2bd-a\left(e-\sqrt{e^2-4df}\right)\right)\right)\mathrm{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx\cdots}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d+1/2*arctanh(
1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^
(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+
e^2)^(1/2))^(1/2))*(c*d*(e-(-4*d*f+e^2)^(1/2))-f*(2*b*d-a*(e+(-4*d*f+e^2)^
(1/2))))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+
c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*
d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(
-f*(2*b*d-a*(e-(-4*d*f+e^2)^(1/2)))+c*d*(e+(-4*d*f+e^2)^(1/2)))/d*2^(1/2)/
(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d
*f+e^2)^(1/2))))^(1/2)
```

3.112.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 467, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)}\,dx$$
$$= \frac{2\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right) - \text{RootSum}\left[b^2d - abe + a^2f - 4b\sqrt{c}d\#1 + 2a\sqrt{c}e\#1 + 4cd\#1^2 + be\#1^2\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

output
```
(2*Sqrt[a]*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]] - RootSum[
b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 +
 b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1] - a*b*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a
^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + c*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1]*#1^2 - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#
1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[
c]*e*#1^2 - 2*f*#1^3) & ])/d
```

3.112.3 Rubi [A] (verified)

Time = 2.78 (sec) , antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{(-e-fx)\sqrt{a+bx+cx^2}}{d(d+ex+fx^2)} + \frac{\sqrt{a+bx+cx^2}}{dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\left(-af\left(\sqrt{e^2-4df}+e\right)+2bdf-cd\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$\frac{\left(-af\left(e-\sqrt{e^2-4df}\right)+2bdf-cd\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$
$$\frac{\sqrt{a}\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x*(d + e*x + f*x^2)),x]
```

output
```
-((Sqrt[a]*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/d) - ((
2*b*d*f - c*d*(e - Sqrt[e^2 - 4*d*f]) - a*f*(e + Sqrt[e^2 - 4*d*f]))*ArcTa
nh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f])
)*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[
e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c
*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]
))]) + ((2*b*d*f - a*f*(e - Sqrt[e^2 - 4*d*f]) - c*d*(e + Sqrt[e^2 - 4*d*f
]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2
- 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b
*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*
f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt[e^2
 - 4*d*f]))])
```

3.112.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.112.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. $2(460) = 920$.

Time = 1.05 (sec) , antiderivative size = 1691, normalized size of antiderivative = 3.23

method	result	size
default	Expression too large to display	1691

input
```
int((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*((c*x^2+b*x+a)^(1/2)+1
/2*b*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-a^(1/2)*ln((2*a+b
*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e
^2)^(1/2)*(1/2*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)+1/2/f*(-
c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*ln((1/2/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)+c*
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))/c^(1/2)+((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)
+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)/f^2)^(1/2))/c^(1/2)-1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^
(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^
2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))+2*f/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e...
```

3.112.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.112.6 Sympy [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x/(f*x**2+e*x+d),x)
```

output
```
Integral(sqrt(a + b*x + c*x**2)/(x*(d + e*x + f*x**2)), x)
```

3.112.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)x}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x), x)
```

3.112.8 Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x*(d + e*x + f*x^2)), x)
```

3.113 $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx$

3.113.1 Optimal result

Integrand size = 30, antiderivative size = 736

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx = -\frac{\sqrt{a+bx+cx^2}}{dx} - \frac{b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}$$
$$+ \frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2} + \frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}$$
$$- \frac{be\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2} - \frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}$$
$$- \frac{f\left(2cd^2-bd\left(e+\sqrt{e^2-4df}\right)+a\left(e^2-2df+e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2\cdots}\right)\right)}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4\cdots}}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}$$
$$+ \frac{f\left(2cd^2-bd\left(e-\sqrt{e^2-4df}\right)+a\left(e^2-2df-e\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2\cdots}\right)\right)}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4\cdots}}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)}}$$

output
```
-1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+e*arct
anh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))*a^(1/2)/d^2-1/2*b*e*arctanh
(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)-1/2*(-b*e+2*c*d)*a
rctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/c^(1/2)+arctanh(1/2*
(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))*c^(1/2)/d-(c*x^2+b*x+a)^(1/2)/d/x-1
/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^
2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+
c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*c*d^2-b*d*(e+(-4*d*f+e^2)^(1/2))+a*(e^2
-2*d*f+e*(-4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(
e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arc
tanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))
))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2))*(2*c*d^2-b*d*(e-(-4*d*f+e^2)^(1/2))+a*(e^2-2*d*f-e
*(-4*d*f+e^2)^(1/2)))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d
*f+e^2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

3.113.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 582, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx$$

$$= \frac{-\frac{d\sqrt{a+x(b+cx)}}{x} + \frac{(-bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \text{RootSum}\left[b^2d - abe + a^2f - 4b\sqrt{c}d\#1 + 2a\sqrt{c}e\#1\right.}{}$$

input
```
Integrate[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

output
```
(-((d*Sqrt[a + x*(b + c*x)])/x) + ((-(b*d) + 2*a*e)*ArcTanh[(-(Sqrt[c]*x)
+ Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] + RootSum[b^2*d - a*b*e + a^2*f
 - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^
2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1]) + b^2*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1
] - a*b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*e*f*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d^2*Log[-(Sqrt[c]*x)
 + Sqrt[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*e^2*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1 - 2*a*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^
2 - a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c
]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*
#1^3) & ])/d^2
```

3.113.3 Rubi [A] (verified)

Time = 2.86 (sec) , antiderivative size = 736, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2 (d+ex+fx^2)} \, dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{\sqrt{a+bx+cx^2}\left(-df+e^2+efx\right)}{d^2 (d+ex+fx^2)}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{\sqrt{a+bx+cx^2}}{dx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{f\left(a\left(e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(\sqrt{e^2-4df}+e\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4\cdots}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{f\left(a\left(-e\sqrt{e^2-4df}-2df+e^2\right)-bd\left(e-\sqrt{e^2-4df}\right)+2cd^2\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}\cdots\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-\cdots}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(\sqrt{e^2-4df}+e\right)\right)+c\left(e\sqrt{e^2-4df}-2df+e^2\right)}}$$

$$\frac{\sqrt{a}e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{d^2}-\frac{be\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-\frac{(2cd-be)\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}d^2}-$$

$$\frac{b\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{a}d}+\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{d}-\frac{\sqrt{a+bx+cx^2}}{dx}$$

input
```
Int[Sqrt[a + b*x + c*x^2]/(x^2*(d + e*x + f*x^2)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(d*x)) - (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a
+ b*x + c*x^2])])/(2*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x)/(2*Sqrt[a
]*Sqrt[a + b*x + c*x^2])])/d^2 + (Sqrt[c]*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*S
qrt[a + b*x + c*x^2])])/d - (b*e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(2*Sqrt[c]*d^2) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x)/(2*Sqr
t[c]*Sqrt[a + b*x + c*x^2])])/(2*Sqrt[c]*d^2) - (f*(2*c*d^2 - b*d*(e + Sqr
t[e^2 - 4*d*f]) + a*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt
[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]
]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*(e^2 - 2*
d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2 - 4*d*f]))]) + (f*
(2*c*d^2 - b*d*(e - Sqrt[e^2 - 4*d*f]) + a*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d
*f]))*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^
2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e -
 b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 -
4*d*f]*Sqrt[c*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e + Sqrt
[e^2 - 4*d*f]))])
```

3.113. $\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx$

3.113.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.113.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 976, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{\sqrt{c\,x^{2}+bx+a}}{dx}-\frac{\frac{4f(2ae-bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^{2}+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^{2}}\right)\left(e+\sqrt{-4df+e^{2}}\right)\sqrt{a}}+\frac{2\left(fa\sqrt{-4df+e^{2}}-cd\sqrt{-4df+e^{2}}-aef+2bdf-cde\right)\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^{2}}+\sqrt{\cdots}}{\cdots}\right.}{\cdots}}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/d/x-1/2/d*(4*f*(2*a*e-b*d)/(-e+(-4*d*f+e^2)^(1/2))/(e
+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2))/x)
+2*(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2)^(1/2)-a*e*f+2*b*d*f-c*d*e)/(-4
*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-
4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1
/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^
(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d
*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^
2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*(f*a*(-4*d*f+e^2)^(1/2)-c*d*(-4*d*f+e^2
)^(1/2)+a*e*f-2*b*d*f+c*d*e)/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2))*2^
(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-
e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^
2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e...
```

3.113.5 Fricas [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x+a)**(1/2)/x**2/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.113.7 Maxima [F]

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx = \int \frac{\sqrt{cx^2+bx+a}}{(fx^2+ex+d)x^2}\,dx$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x + a)/((f*x^2 + e*x + d)*x^2), x)
```

3.113.8 Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x+a)^(1/2)/x^2/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+bx+cx^2}}{x^2\left(d+ex+fx^2\right)}\,dx=\int \frac{\sqrt{c\,x^2+b\,x+a}}{x^2\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)),x)
```

output
```
int((a + b*x + c*x^2)^(1/2)/(x^2*(d + e*x + f*x^2)), x)
```

3.114 $\int \frac{x^3}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.114.1 Optimal result

Integrand size = 30, antiderivative size = 545

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx &= \frac{\sqrt{a+bx+cx^2}}{cf}-\frac{e\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}-\frac{b\,\mathrm{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}\\
&-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}\\
&+\frac{\left(2def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\arctan\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}\end{aligned}$$

output
```
-1/2*b*arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/c^(3/2)/f-e*arct
anh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f^2/c^(1/2)+(c*x^2+b*x+a)^(
1/2)/c/f-1/2*arctan(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-
4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^
2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(-d*f+e^2)*(e-(-4*d*f+e^2
)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*
f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*arctan(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^
(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*
e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*e*f-(
-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/f^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*
c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.114.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.78

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx =$$
$$-\frac{-\frac{2f\sqrt{a+x(b+cx)}}{c}+\frac{(2ce+bf)\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+x(b+cx)}}\right)}{c^{3/2}}+2\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1\right.}{}$$

input
```
Integrate[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-1/2*((-2*f*Sqrt[a + x*(b + c*x)])/c + ((2*c*e + b*f)*ArcTanh[(b + 2*c*x)/
(2*Sqrt[c]*Sqrt[a + x*(b + c*x)])])/c^(3/2) + 2*RootSum[b^2*d - a*b*e + a^
2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*
#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1] - a*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] +
a*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*d*e*Log[-
(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + e^2*Log[-(Sqrt[c]*x) + Sqrt
[a + b*x + c*x^2] - #1]*#1^2 - d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1
+ 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/f^2
```

3.114.3 Rubi [A] (verified)

Time = 2.52 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{x(e^2-df)+de}{f^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}-\frac{e}{f^2\sqrt{a+bx+cx^2}}+\frac{x}{f\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{3/2}f}-\frac{\left(2def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(2def-(e^2-df)\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}f^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{e\,\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f^2}+\frac{\sqrt{a+bx+cx^2}}{cf}$$

input
```
Int[x^3/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
Sqrt[a + b*x + c*x^2]/(c*f) - (e*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b
*x + c*x^2])])/(Sqrt[c]*f^2) - (b*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a +
b*x + c*x^2])])/(2*c^(3/2)*f) - ((2*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*
d*f]))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e
^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e
- b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 -
 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]) + ((2*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f -
 b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqr
t[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f
]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*f^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*
c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

3.114.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.114.4 Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 914, normalized size of antiderivative = 1.68

method	result
risch	$\frac{\sqrt{c\,x^2+bx+a}}{cf}-\frac{\frac{(bf+2ce)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}}-\frac{c\left(df\sqrt{-4df+e^2}-e^2\sqrt{-4df+e^2}+3def-e^3\right)\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a}{f^2}\right.}{}}{}$
default	$\frac{\frac{\sqrt{c\,x^2+bx+a}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{2c^{\frac{3}{2}}}}{f}-\frac{e\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f^2\sqrt{c}}-\frac{\left(e^3-3def+e^2\sqrt{-4df+e^2}-df\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{-b}{}\right.}{}$

input
```
int(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
(c*x^2+b*x+a)^(1/2)/c/f-1/2/f/c*(1/f*(b*f+2*c*e)*ln((1/2*b+c*x)/c^(1/2)+(c
*x^2+b*x+a)^(1/2))/c^(1/2)-1/f^2*c*(d*f*(-4*d*f+e^2)^(1/2)-e^2*(-4*d*f+e^2
)^(1/2)+3*d*e*f-e^3)/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+
1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e
^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/
(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/f^2*c*(e^3-3*d*e*f-e^2*(-4*d*f+e^2)^(1
/2)+d*f*(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^
(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((
b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)
/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/
2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
)/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))))
```

3.114.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.114.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**3/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.114.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.114.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.114.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\int \frac{x^3}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^3/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.115 $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.115.1 Optimal result

Integrand size = 30, antiderivative size = 463

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\operatorname{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f} - \frac{\left(e^2-2df-e\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(2df-e\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}f\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
arctanh(1/2*(2*c*x+b)/c^(1/2)/(c*x^2+b*x+a)^(1/2))/f/c^(1/2)-1/2*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/f*2^(1/2)/(-4*d*f+e^2)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/
2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(
1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e
)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*f-e*(e+(-4*d*f+e^2)^(1/2)))/f*2^(1/2)/(-
4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2
))^(1/2)
```

3.115.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.44 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.69

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{-\frac{\log\left(f\left(b+2cx-2\sqrt{c}\sqrt{a+x(b+cx)}\right)\right)}{\sqrt{c}} + \text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-\right.}{}$$

input
```
Integrate[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
(-(Log[f*(b + 2*c*x - 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/Sqrt[c]) + RootSum
[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
+ b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*d*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1] - a*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
c*x^2] - #1] - 2*Sqrt[c]*d*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*
#1 + e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d
 - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^
3) & ])/f
```

3.115.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2143, 25, 1092, 219, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\big\downarrow \text{ 2143}$$
$$\frac{\int -\frac{d+ex}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}+\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}$$
$$\big\downarrow \text{ 25}$$

$$\frac{\int \frac{1}{\sqrt{cx^2+bx+a}}dx}{f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \ 1092$$

$$\frac{2\int \frac{1}{4c-\frac{(b+2cx)^2}{cx^2+bx+a}}d\frac{b+2cx}{\sqrt{cx^2+bx+a}}}{f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{\int \frac{d+ex}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{f}$$

$$\Big\downarrow \ 1365$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{-\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}}{f}$$

$$\Big\downarrow \ 1154$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{\frac{2\left(-e\sqrt{e^2-4df}-2df+e^2\right)\int \frac{1}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e-\sqrt{e^2-4df}\right)\ldots}{\sqrt{c\ldots}}}{\sqrt{e^2-4df}}\ldots}{\ldots}$$

$$\Big\downarrow \ 219$$

$$\frac{\text{arctanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{c}f}-\frac{\frac{\left(-e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{\left(2df-e\left(\sqrt{e^2-4df}+e\right)\right)\text{arctanh}\left(\frac{4af+2x\ldots}{2\sqrt{2}\sqrt{a+bx-\ldots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4\ldots}}}}{f}$$

input
```
Int[x^2/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

3.115. $\int \frac{x^2}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

output
```
ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])]/(Sqrt[c]*f) - (((e^
2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]
) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f
 - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])
])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*
e - b*f)*Sqrt[e^2 - 4*d*f]]) + ((2*d*f - e*(e + Sqrt[e^2 - 4*d*f]))*ArcTan
h[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))
*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e
^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^
2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/f
```

3.115.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 843 vs. $2(407) = 814$.

Time = 0.88 (sec) , antiderivative size = 844, normalized size of antiderivative = 1.82

method	result
default	$\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx+a}\right)}{f\sqrt{c}} - \frac{\left(-e\sqrt{-4df+e^2}+2df-e^2\right)\sqrt{2}\,\ln\left(\frac{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}\right.}{}}{}\right.}{}$

input
```
int(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/f*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))/c^(1/2)-1/2*(-e*(-4*d*f+e^
2)^(1/2)+2*d*f-e^2)/f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*
f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f
^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2))/
f^2/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)
*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/
2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+
e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*
(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/
f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)
^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e
^2)^(1/2))))
```

3.115.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.115.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x**2/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.115.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.115.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

3.115.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x^2}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^2/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.116 $\int \frac{x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.116.1 Optimal result

Integrand size = 28, antiderivative size = 402

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \frac{\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2
)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c
*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)
^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1
/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^
(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*
e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(-4*d*f+e^2)^
(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.116.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.36 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.51

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$=\text{RootSum}\Bigg[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}e\#1^3$$
$$+f\#1^4\&,\frac{-a\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)+\log\left(-\sqrt{c}x+\sqrt{a+bx+cx^2}-\#1\right)\#1^2}{-2b\sqrt{c}d+a\sqrt{c}e+4cd\#1+be\#1-2af\#1-3\sqrt{c}e\#1^2+2f\#1^3}\&\Bigg]$$

input
```
Integrate[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*
d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (-(a*Log[-(
Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + Log[-(Sqrt[c]*x) + Sqrt[a + b*
x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 -
 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ]
```

3.116.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 382, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{1365}$$
$$\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx+$$
$$\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx$$

$$\Big\downarrow \text{1154}$$

$$-2\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\int \frac{1}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e\right.\right.\right.}{cx^2+bx+a}}$$
$$2\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\right.\right.\right.}{cx^2+bx+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(1-\frac{e}{\sqrt{e^2-4df}}\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\left(\frac{e}{\sqrt{e^2-4df}}+1\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

input
```
Int[x/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-(((1 - e/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]])) - ((1 + e/Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*
d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c
*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x
^2])])/(Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[
e^2 - 4*d*f]])
```

3.116. $\int \frac{x}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.116.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(355) = 710$.

Time = 0.90 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.98

method	result
default	$-\frac{\left(e+\sqrt{-4df+e^2}\right)\sqrt{2}\,\ln\left(\frac{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{-b \cdots}}{\cdots}}{\cdots}\right.}{2\sqrt{-4df+\cdots}}$

input
```
int(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*2^(1/2)/((-b*f*(-4*d*f+e^
2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln
(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*
e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))
/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f
*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*
(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2
)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-1/2*(-e+(-4*d*f+e^2)^(1/2))/(-4
*d*f+e^2)^(1/2)/f*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*
f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^
(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x
-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-
1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(
1/2))))
```

3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11311 vs. $2(353) = 706$.

Time = 3.57 (sec) , antiderivative size = 11311, normalized size of antiderivative = 28.14

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.116.6 Sympy [F]

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(x/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.116.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.116.8 Giac [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.116.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{x}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.117 $\int \frac{1}{\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.117.1 Optimal result

Integrand size = 27, antiderivative size = 374

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{\sqrt{2}f\,\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)
^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*
e)*(-4*d*f+e^2)^(1/2))^(1/2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*
e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+f*arctanh(1/4*(4*a*f-b*(e
+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*
x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/
2))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2)
```

3.117.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = -\text{RootSum}\Big[c^2d-bce+b^2f+2\sqrt{a}ce\#1-4\sqrt{a}bf\#1 -2cd\#1^2+be\#1^2+4af\#1^2-2\sqrt{a}e\#1^3 +d\#1^4\&, \frac{c\log(x)-c\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-x\#1\right)-\log(x)\#1^2+\log\left(-\sqrt{a}+\sqrt{a+bx+cx^2}-\right.}{\sqrt{a}ce-2\sqrt{a}bf-2cd\#1+be\#1+4af\#1-3\sqrt{a}e\#1^2+2d\#1^3}$$

input
```
Integrate[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-RootSum[c^2*d - b*c*e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c
*d*#1^2 + b*e*#1^2 + 4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (c*Log[x]
- c*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - Log[x]*#1^2 + Log[-Sqrt
[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2
*c*d*#1 + b*e*#1 + 4*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ]
```

3.117.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {1314, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1314}$$

$$\frac{2f\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{2f\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \text{1154}$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$$\frac{4f\int \frac{1}{4\left(4af^2-2b\left(e-\sqrt{e^2-4df}\right)f+c\left(e-\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\right.\right.}{\sqrt{cx^2+bx+a}}}{\sqrt{e^2-4df}}$$

$$\Big\downarrow \; 219$$

$$\frac{\sqrt{2}f\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\sqrt{2}f\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

input
```
Int[1/(Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-((Sqrt[2]*f*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e -
Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 -
 (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d*f
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
) + (Sqrt[2]*f*ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e
+ Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2
 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[e^2 - 4*d
*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]
])
```

3.117.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1314
```
Int[1/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*(
x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int[1/(
(b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[2*(c/q)   Int[1/((b +
 q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e - b*f, 0] && PosQ
[b^2 - 4*a*c]
```

3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 760 vs. $2(330) = 660$.

Time = 0.83 (sec) , antiderivative size = 761, normalized size of antiderivative = 2.03

method	result
default	$\sqrt{2}\,\ln\left(\frac{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(x+\frac{e+\sqrt{-4df+e^2}}{2f}\right)}{f}+\frac{\sqrt{2}\sqrt{\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df-}\cdots}{\cdots}}}{\cdots}}{\cdots}\right) \Big/ \left(\sqrt{-4df+e^2}\sqrt{\frac{-bf\sqrt{\cdots}}{\cdots}}\cdots\right)$

input
```
int(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(
-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2
)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d
*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/
2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-
c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f))-1/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d
*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*(
(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2
)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)
-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*
(-e+(-4*d*f+e^2)^(1/2))))
```

3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11287 vs. $2(328) = 656$.

Time = 4.27 (sec) , antiderivative size = 11287, normalized size of antiderivative = 30.18

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Too large to include
```

3.117.6 Sympy [F]

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.117.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.117.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.117.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.118 $\int \frac{1}{x\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)} \, dx$

3.118.1 Optimal result

Integrand size = 30, antiderivative size = 451

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)} \, dx = -\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d} + \frac{f\left(e+\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} - \frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d/a^(1/2)+1/2*f*arctan
h(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*
2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*
f+e^2)^(1/2))^(1/2))*(e+(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2)/(
c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arc
tanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))
))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4
*d*f+e^2)^(1/2))^(1/2))*(e-(-4*d*f+e^2)^(1/2))/d*2^(1/2)/(-4*d*f+e^2)^(1/2
)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.118.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.45 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.71

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{\sqrt{a}} - \text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2a\right.}{}$$

input
```
Integrate[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
((2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/Sqrt[a] - RootSu
m[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2
 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1] - 2*Sqrt[c]*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]
*#1 + f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]
*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#
1^3) & ])/d
```

3.118.3 Rubi [A] (verified)

Time = 1.83 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{ 7279}$$
$$\int \left(\frac{-e-fx}{d\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}+\frac{1}{dx\sqrt{a+bx+cx^2}}\right)dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{f\left(\sqrt{e^2-4df}+e\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(e-\sqrt{e^2-4df}\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d}$$

input
```
Int[1/(x*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-(ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])]/(Sqrt[a]*d)) + (f
*(e + Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b
*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f
 + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt
[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*
f)*Sqrt[e^2 - 4*d*f]]) - (f*(e - Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*(e
+ Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*S
qrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqr
t[a + b*x + c*x^2])])/(Sqrt[2]*d*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

3.118.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 858 vs. $2(396) = 792$.

Time = 0.89 (sec) , antiderivative size = 859, normalized size of antiderivative = 1.90

method	result
default	$\frac{4f\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}} - \frac{2f\sqrt{2}\ln\left(\frac{-bf\sqrt{-4df+e^2}+\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf+c\,e^2}{f^2}+\frac{\left(-c\sqrt{-4df+e^2}+bf-ce\right)\left(\cdots\right.}{f}\cdots\right.}{\cdots}$

input
```
int(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x)-2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/
2)*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-
2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*
c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-
4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*
f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f))-2*f
/(-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/
2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f
*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2
^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^
2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(
x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))
```

3.118.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.118.6 Sympy [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.118.7 Maxima [F]

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x), x)
```

3.118.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.118.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\int \frac{1}{x\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.119 $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.119.1 Optimal result

Integrand size = 30, antiderivative size = 543

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= -\frac{\sqrt{a+bx+cx^2}}{adx}+\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}+\frac{e\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}$$
$$-\frac{f\left(e^2-2df+e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)}}$$
$$+\frac{f\left(e^2-2df-e\sqrt{e^2-4df}\right)\text{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
1/2*b*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+e*arcta
nh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^2/a^(1/2)-(c*x^2+b*x+a)^(1
/2)/a/d/x+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+
(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)
)/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(
-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^
2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*
c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(e^2-2*d*f+e*(-4
*d*f+e^2)^(1/2))/d^2*2^(1/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2
)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))^(1/2)
```

3.119.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.78

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$= \frac{-\frac{d\sqrt{a+x(b+cx)}}{ax}+\frac{(bd+2ae)\text{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}}+\text{RootSum}\left[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+\right.}{}$$

input
```
Integrate[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
(-((d*Sqrt[a + x*(b + c*x)])/(a*x)) + ((b*d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x)
 + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(3/2) + RootSum[b^2*d - a*b*e + a^2*
f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1
^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
+ c*x^2] - #1] - b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a*
e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^2*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*Sqrt[c]*d*f*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1]*#1 + e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x +
 c*x^2] - #1]*#1^2)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*
a*f*#1 - 3*Sqrt[c]*e*#1^2 + 2*f*#1^3) & ])/d^2
```

3.119.3 Rubi [A] (verified)

Time = 2.92 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{-df+e^2+efx}{d^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}-\frac{e}{d^2x\sqrt{a+bx+cx^2}}+\frac{1}{dx^2\sqrt{a+bx+cx^2}}\right)dx$$

$$\downarrow 2009$$

$$\frac{b\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d}-\frac{f\left(e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)+c\left(-e\sqrt{e^2-4df}-2df+e^2\right)}}+\frac{f\left(-e\sqrt{e^2-4df}-2df+e^2\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^2\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\,\text{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^2}-\frac{\sqrt{a+bx+cx^2}}{adx}$$

input
```
Int[1/(x^2*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-(Sqrt[a + b*x + c*x^2]/(a*d*x)) + (b*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[
a + b*x + c*x^2])])/(2*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt
[a + b*x + c*x^2])])/(Sqrt[a]*d^2) - (f*(e^2 - 2*d*f + e*Sqrt[e^2 - 4*d*f]
)*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 -
4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f
)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*
f]*Sqrt[c*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f]) + f*(2*a*f - b*(e - Sqrt[e^2
 - 4*d*f]))]) + (f*(e^2 - 2*d*f - e*Sqrt[e^2 - 4*d*f])*ArcTanh[(4*a*f - b*
(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^2*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d
*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

3.119.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.119. $\int \frac{1}{x^2\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.119.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 927, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}}{adx}-\frac{\frac{4f(2ae+bd)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}-\frac{2f\left(e+\sqrt{-4df+e^2}\right)a\sqrt{2}\,\ln\left(\frac{bf\sqrt{-4df+e^2}-\sqrt{-4df+e^2}\,ce+2a\,f^2-bef-2cdf\cdots}{f^2}\cdots\right.}{\cdots}}{\cdots}$
default	$-\frac{4f\left(-\frac{\sqrt{c\,x^2+bx+a}}{ax}+\frac{b\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{2a^{\frac{3}{2}}}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)}+\frac{16f^2e\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)^2\left(e+\sqrt{-4df+e^2}\right)^2\sqrt{a}}+\frac{4f^2\sqrt{2}\,\ln\left(\frac{-bf\sqrt{-4df+e\cdots}}{\cdots}\right.}{\cdots}$

input
```
int(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(c*x^2+b*x+a)^(1/2)/a/d/x-1/2/a/d*(4*f*(2*a*e+b*d)/(-e+(-4*d*f+e^2)^(1/2)
)/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)^(1/2)
)/x)-2*f*(e+(-4*d*f+e^2)^(1/2))*a/(-4*d*f+e^2)^(1/2)/(-e+(-4*d*f+e^2)^(1/2
))*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*
e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2
/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e
^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x-1/2/f*(-e+(-4*
d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d
*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2))))+2*f*a*(
-e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/(e+(-4*d*f+e^2)^(1/2))*2^(1/2)/(
(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^
2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4
*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(
1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1
/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*
c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)))
```

3.119.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.119.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x**2/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x**2*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.119.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x^2}\,dx$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^2), x)
```

3.119.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^2/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.119.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^2\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x^2*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.120 $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.120.1 Optimal result

Integrand size = 30, antiderivative size = 679

$$\begin{aligned}
\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx &= -\frac{\sqrt{a+bx+cx^2}}{2adx^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx} \\
&+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d} \\
&-\frac{b e\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}-\frac{(e^2-df)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3} \\
&+\frac{f\left(2e^3-4def-(e^2-df)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} \\
&-\frac{f\left(2e^3-4def-(e^2-df)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}
\end{aligned}$$

output
```
-1/8*(-4*a*c+3*b^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(
5/2)/d-1/2*b*e*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/
d^2-(-d*f+e^2)*arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/d^3/a^(1
/2)-1/2*(c*x^2+b*x+a)^(1/2)/a/d/x^2+3/4*b*(c*x^2+b*x+a)^(1/2)/a^2/d/x+e*(c
*x^2+b*x+a)^(1/2)/a/d^2/x+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e
^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2
*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*e^3-4*d*e*f-
(-d*f+e^2)*(e-(-4*d*f+e^2)^(1/2)))/d^3*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2
*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/
4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1
/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2))*(2*e^3-4*d*e*f-(-d*f+e^2)*(e+(-4*d*f+e^2)^(1/2)))/d^3*2^(
1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2)
```

3.120.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.22 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.81

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

$$= \frac{\frac{d(-2ad+3bdx+4aex)\sqrt{a+x(b+cx)}}{a^2x^2}+\frac{(-3b^2d^2-4abde+4a(cd^2-2ae^2+2adf))\operatorname{arctanh}\left(\frac{-\sqrt{c}x+\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{5/2}}-4\text{RootSum}\Big[b^2d-c}{}$$

input
```
Integrate[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
((d*(-2*a*d + 3*b*d*x + 4*a*e*x)*Sqrt[a + x*(b + c*x)])/(a^2*x^2) + ((-3*b
^2*d^2 - 4*a*b*d*e + 4*a*(c*d^2 - 2*a*e^2 + 2*a*d*f))*ArcTanh[(-(Sqrt[c]*x
) + Sqrt[a + x*(b + c*x)])/Sqrt[a]])/a^(5/2) - 4*RootSum[b^2*d - a*b*e + a
^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f
*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b
*x + c*x^2] - #1] - 2*b*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1] - a*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*d*f^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*Sqrt[c]*e^3*Log[-(Sqrt[c]*x
) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 4*Sqrt[c]*d*e*f*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1]*#1^2 - d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2
)/(-2*b*Sqrt[c]*d + a*Sqrt[c]*e + 4*c*d*#1 + b*e*#1 - 2*a*f*#1 - 3*Sqrt[c]
*e*#1^2 + 2*f*#1^3) & ])/(4*d^3)
```

3.120.3 Rubi [A] (verified)

Time = 7.01 (sec) , antiderivative size = 679, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{e^2-df}{d^3x\sqrt{a+bx+cx^2}}+\frac{-fx(e^2-df)-e(e^2-2df)}{d^3\sqrt{a+bx+cx^2}\left(d+ex+fx^2\right)}-\frac{e}{d^2x^2\sqrt{a+bx+cx^2}}+\frac{1}{dx^3\sqrt{a+bx+cx^2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(3b^2-4ac)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{8a^{5/2}d}-\frac{be\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{2a^{3/2}d^2}+\frac{3b\sqrt{a+bx+cx^2}}{4a^2dx}-\frac{(e^2-df)\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{\sqrt{a}d^3}+\frac{f\left(-(e^2-df)\left(e-\sqrt{e^2-4df}\right)-4def+2e^3\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(-(e^2-df)\left(\sqrt{e^2-4df}+e\right)-4def+2e^3\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}d^3\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}+\frac{e\sqrt{a+bx+cx^2}}{ad^2x}-\frac{\sqrt{a+bx+cx^2}}{2adx^2}$$

input
```
Int[1/(x^3*Sqrt[a + b*x + c*x^2]*(d + e*x + f*x^2)),x]
```

output
```
-1/2*Sqrt[a + b*x + c*x^2]/(a*d*x^2) + (3*b*Sqrt[a + b*x + c*x^2])/(4*a^2*
d*x) + (e*Sqrt[a + b*x + c*x^2])/(a*d^2*x) - ((3*b^2 - 4*a*c)*ArcTanh[(2*a
 + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(8*a^(5/2)*d) - (b*e*ArcTanh[(
2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(2*a^(3/2)*d^2) - ((e^2 - d
*f)*ArcTanh[(2*a + b*x)/(2*Sqrt[a]*Sqrt[a + b*x + c*x^2])])/(Sqrt[a]*d^3)
+ (f*(2*e^3 - 4*d*e*f - (e^2 - d*f)*(e - Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*
f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*
Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*
d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 -
 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - (f*(2*e^3 -
 4*d*e*f - (e^2 - d*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*(e + Sq
rt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[
c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a
+ b*x + c*x^2])])/(Sqrt[2]*d^3*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f - b*
e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

3.120. $\int \frac{1}{x^3\sqrt{a+bx+cx^2}(d+ex+fx^2)}\,dx$

3.120.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.120.4 Maple [A] (verified)

Time = 1.15 (sec) , antiderivative size = 991, normalized size of antiderivative = 1.46

method	result
risch	$-\frac{\sqrt{c\,x^2+bx+a}\,(-4aex-3bdx+2ad)}{4a^2d^2x^2}+\frac{-\frac{4f\left(8a^2df-8e^2a^2-4abde+4c\,d^2a-3b^2d^2\right)\ln\left(\frac{2a+bx+2\sqrt{a}\,\sqrt{c\,x^2+bx+a}}{x}\right)}{\left(-e+\sqrt{-4df+e^2}\right)\left(e+\sqrt{-4df+e^2}\right)\sqrt{a}}+\frac{8f\,a^2\left(e\sqrt{-4df+e^2}+2\right.}{}}{}$
default	Expression too large to display

input
```
int(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x^2+b*x+a)^(1/2)*(-4*a*e*x-3*b*d*x+2*a*d)/a^2/d^2/x^2+1/8/d^2/a^2*
(-4*f*(8*a^2*d*f-8*a^2*e^2-4*a*b*d*e+4*a*c*d^2-3*b^2*d^2)/(-e+(-4*d*f+e^2)
^(1/2))/(e+(-4*d*f+e^2)^(1/2))/a^(1/2)*ln((2*a+b*x+2*a^(1/2)*(c*x^2+b*x+a)
^(1/2))/x)+8*f*a^2*(e*(-4*d*f+e^2)^(1/2)+2*d*f-e^2)/(-4*d*f+e^2)^(1/2)/(e+
(-4*d*f+e^2)^(1/2))*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c
*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)
^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*
f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2
)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c
*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f))-8*f*a^2*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/
(-e+(-4*d*f+e^2)^(1/2))*2^(1/2)/((b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+e^2)^(1/2
)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((b*f*(-4*d*f+e^2)^(1/2)-
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+(c*(-4*d*f+e^2)^(1
/2)+b*f-c*e)/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+1/2*2^(1/2)*((b*f*(-4*d*f
+e^2)^(1/2)-(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)
*(4*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))^2*c+4*(c*(-4*d*f+e^2)^(1/2)+b*f-c*e)
/f*(x-1/2/f*(-e+(-4*d*f+e^2)^(1/2)))+2*(b*f*(-4*d*f+e^2)^(1/2)-(-4*d*f+...
```

3.120.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.120.6 Sympy [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx$$

input
```
integrate(1/x**3/(c*x**2+b*x+a)**(1/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x**3*sqrt(a + b*x + c*x**2)*(d + e*x + f*x**2)), x)
```

3.120.7 Maxima [F]

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\int \frac{1}{\sqrt{cx^2+bx+a}(fx^2+ex+d)x^3}\,dx$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x + a)*(f*x^2 + e*x + d)*x^3), x)
```

3.120.8 Giac [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx=\text{Timed out}$$

input
```
integrate(1/x^3/(c*x^2+b*x+a)^(1/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.120.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{a+bx+cx^2}\,(d+ex+fx^2)}\,dx = \int \frac{1}{x^3\,\sqrt{c\,x^2+b\,x+a}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x^3*(a + b*x + c*x^2)^(1/2)*(d + e*x + f*x^2)), x)
```

3.121 $\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

3.121.1 Optimal result

Integrand size = 30, antiderivative size = 779

$$\int \frac{x^3}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \frac{2(2a+bx)}{\left(b^2-4ac\right)f\sqrt{a+bx+cx^2}}$$
$$+\frac{2e(b+2cx)}{\left(b^2-4ac\right)f^2\sqrt{a+bx+cx^2}}$$
$$+\frac{2(cde(bcd-2ace+abf)-\left(bde-ae^2+adf\right)\left(2c^2d+b^2f-c(be+2af)\right)+c((bcd-2ace+abf)\left(e^2-df\right)\cdots}{\left(b^2-4ac\right)f^2\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{\left(2d(bd-ae)f+\left(e-\sqrt{e^2-4df}\right)\left(cd^2-bde+a\left(e^2-df\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4\cdots}}}$$
$$-\frac{\left(2d(bd-ae)f+\left(e+\sqrt{e^2-4df}\right)\left(cd^2-bde+a\left(e^2-df\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-\cdots}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}\cdots}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4\cdots}}}$$

output
```
2*(b*x+2*a)/(-4*a*c+b^2)/f/(c*x^2+b*x+a)^(1/2)+2*e*(2*c*x+b)/(-4*a*c+b^2)/
f^2/(c*x^2+b*x+a)^(1/2)+2*(c*d*e*(a*b*f-2*a*c*e+b*c*d)-(a*d*f-a*e^2+b*d*e)
*(2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*((a*b*f-2*a*c*e+b*c*d)*(-d*f+e^2)-d*e*(2*
c^2*d+b^2*f-c*(2*a*f+b*e)))*x)/(-4*a*c+b^2)/f^2/((-a*f+c*d)^2-(-a*e+b*d)*(
-b*f+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*
f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^
2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d*(-a*e+b
*d)*f+(c*d^2-b*d*e+a*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-a
*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^
2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*arctanh(1/4*(4*a*f-b*(e+(-4*d*f
+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/
2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(2*d
*(-a*e+b*d)*f+(c*d^2-b*d*e+a*(-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*
d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*
f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.121.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.13 (sec) , antiderivative size = 690, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(2a^3f+b^3dx+ab(bd-3cdx-bex)+a^2(-2cd-be+2cex+b\ldots}{}$$

input
```
Integrate[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(2*(2*a^3*f + b^3*d*x + a*b*(b*d - 3*c*d*x - b*e*x) + a^2*(-2*c*d - b*e +
2*c*e*x + b*f*x)) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*
b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2
- 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b^2*d^2*Log[-(Sqrt[c]*x) + S
qrt[a + b*x + c*x^2] - #1] + a*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x
^2] - #1] - 2*a*b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2
*e^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*d*f*Log[-(Sqrt[c
]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*b*Sqrt[c]*d^2*Log[-(Sqrt[c]*x) + Sq
rt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*e*Log[-(Sqrt[c]*x) + Sqrt[a +
 b*x + c*x^2] - #1]*#1 - c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1^2 + b*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*e
^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + a*d*f*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4
*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/((b^2 - 4*
a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*S
qrt[a + x*(b + c*x)])
```

3.121.3 Rubi [A] (verified)

Time = 7.39 (sec) , antiderivative size = 779, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{x(e^2-df)+de}{f^2(a+bx+cx^2)^{3/2}(d+ex+fx^2)}-\frac{e}{f^2(a+bx+cx^2)^{3/2}}+\frac{x}{f(a+bx+cx^2)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\left(e-\sqrt{e^2-4df}\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)-b\left(e-\sqrt{e^2-4df}\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2c}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2-\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$\frac{\left(\left(\sqrt{e^2-4df}+e\right)\left(a\left(e^2-df\right)-bde+cd^2\right)+2df(bd-ae)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2c}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}$$

$$\frac{2\left(cx\left(\left(e^2-df\right)(abf-2ace+bcd)-de\left(-c(2af+be)+b^2f+2c^2d\right)\right)-\left(adf-ae^2+bde\right)\left(-c(2af+be)+b^2f\right.\right.}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\frac{2e(b+2cx)}{f^2\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}+\frac{2(2a+bx)}{f\left(b^2-4ac\right)\sqrt{a+bx+cx^2}}$$

input
```
Int[x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(2*(2*a + b*x))/((b^2 - 4*a*c)*f*Sqrt[a + b*x + c*x^2]) + (2*e*(b + 2*c*x)
)/((b^2 - 4*a*c)*f^2*Sqrt[a + b*x + c*x^2]) + (2*(c*d*e*(b*c*d - 2*a*c*e +
 a*b*f) - (b*d*e - a*e^2 + a*d*f)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) + c*
((b*c*d - 2*a*c*e + a*b*f)*(e^2 - d*f) - d*e*(2*c^2*d + b^2*f - c*(b*e + 2
*a*f)))*x))/((b^2 - 4*a*c)*f^2*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*S
qrt[a + b*x + c*x^2]) + ((2*d*(b*d - a*e)*f + (e - Sqrt[e^2 - 4*d*f])*(c*d
^2 - b*d*e + a*(e^2 - d*f)))*ArcTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) +
2*(b*f - c*(e - Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b
*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(
Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c
*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]]) - ((2*d
*(b*d - a*e)*f + (e + Sqrt[e^2 - 4*d*f])*(c*d^2 - b*d*e + a*(e^2 - d*f)))*
ArcTanh[(4*a*f - b*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*
d*f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*
Sqrt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*((c
*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*
f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]])
```

3.121. $\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.121.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.121.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2082 vs. 2(726) = 1452.

Time = 1.00 (sec) , antiderivative size = 2083, normalized size of antiderivative = 2.67

method	result	size
default	Expression too large to display	2083

input
```
int(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/f*(-1/c/(c*x^2+b*x+a)^(1/2)-b/c*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2
))-2*e/f^2*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*(e^3-3*d*e*f+e^2*
(-4*d*f+e^2)^(1/2)-d*f*(-4*d*f+e^2)^(1/2))/f^3/(-4*d*f+e^2)^(1/2)*(2/(-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^
2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e
)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^
2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^
(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e
*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^
2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*
a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(
1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2
)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2
*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+
2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2)...
```

3.121.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.121.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.121.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.121.8 Giac [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x^3/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Timed out
```

3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^3}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^3/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.122 $\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.122.1 Optimal result

Integrand size = 30, antiderivative size = 609

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = -\frac{2(a(bcd-2ace+abf)+c(b^2d-abe-2a(cd-af))x)}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))\sqrt{a+bx+cx^2}} - \frac{f\left(2d(cd-af)-(bd-ae)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}} + \frac{f\left(2d(cd-af)-(bd-ae)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\,((cd-af)^2-(bd-ae)(ce-bf))\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
-2*(a*(a*b*f-2*a*c*e+b*c*d)+c*(b^2*d-a*b*e-2*a*(-a*f+c*d))*x)/(-4*a*c+b^2)
/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)-1/2*f*arctanh(1/
4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(1
/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^
2)^(1/2))^(1/2))*(2*d*(-a*f+c*d)-(-a*e+b*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*f
+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b
*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)+1/2*f*arctanh(1/4*(4*a*f
-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x
^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2)
)^(1/2))*(2*d*(-a*f+c*d)-(-a*e+b*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-
(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a
*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.122.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.90 (sec) , antiderivative size = 534, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-2(b^2cdx+ac(bd-2cdx-bex)+a^2(-2ce+bf+2cfx))-(b^2}{}$$

input
```
Integrate[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(b^2*c*d*x + a*c*(b*d - 2*c*d*x - b*e*x) + a^2*(-2*c*e + b*f + 2*c*f*x
)) - (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*e + a^2*f - 4
*b*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 -
2*Sqrt[c]*e*#1^3 + f*#1^4 & , (b*c*d^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1] - 2*a*b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a
^2*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*c^(3/2)*d^2*Log[
-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + 2*a*Sqrt[c]*d*f*Log[-(Sqrt
[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1 + b*d*f*Log[-(Sqrt[c]*x) + Sqrt[a
+ b*x + c*x^2] - #1]*#1^2 - a*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2]
 - #1]*#1^2)/(2*b*Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 +
 3*Sqrt[c]*e*#1^2 - 2*f*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + f*(
b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)])
```

3.122.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 580, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2135, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}dx$$

$$\Big\downarrow \text{ 2135}$$

$$-\frac{2\int -\frac{(b^2-4ac)(d(cd-af)+(bd-ae)fx)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(b^2-4ac)((cd-af)^2-(bd-ae)(ce-bf))}-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{d(cd-af)+(bd-ae)fx}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}-\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 1365}$$

$$\frac{\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(bd-ae)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{c}}}{\sqrt{e^2-4df}}}{(cd-af)^2-(bd-ae)(ce-bf)}$$
$$\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{\frac{2f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af}{}}{\sqrt{e^2-4df}}}{}$$

$$\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}((cd-af)^2-(bd-ae)(ce-bf))}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{f\left(2d(cd-af)-\left(\sqrt{e^2-4df}+e\right)(bd-ae)\right)\operatorname{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(2d(cd-af)-\left(e-\sqrt{e^2-4df}\right)(\cdots}{\sqrt{2}\sqrt{e\cdots}}}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\frac{2\left(cx\left(-abe-2a(cd-af)+b^2d\right)+a(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

input
```
Int[x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(a*(b*c*d - 2*a*c*e + a*b*f) + c*(b^2*d - a*b*e - 2*a*(c*d - a*f))*x))
/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x + c
*x^2]) + (-((f*(2*d*(c*d - a*f) - (b*d - a*e)*(e - Sqrt[e^2 - 4*d*f]))*Arc
Tanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*f
]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqr
t[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c
*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (f*(
2*d*(c*d - a*f) - (b*d - a*e)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b*
(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[2
]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]*
Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f -
 b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/((c*d - a*f)^2 - (b*d
- a*e)*(c*e - b*f))
```

3.122.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

rule 2135
```
Int[(Px_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e_.)*(x_) + (f_.
)*(x_)^2)^(q_), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1]
, C = Coeff[Px, x, 2]}, Simp[(a + b*x + c*x^2)^(p + 1)*((d + e*x + f*x^2)^(
q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1)))*(
(A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b
*e + 2*a*f)) + c*(A*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - B*(b*c*d - 2*a*c*
e + a*b*f) + C*(b^2*d - a*b*e - 2*a*(c*d - a*f)))*x), x] + Simp[1/((b^2 - 4
*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c
*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Simp[(b*B - 2*A*c - 2*a*C)*((c*d - a*f)^2
 - (b*d - a*e)*(c*e - b*f))*(p + 1) + (b^2*(C*d + A*f) - b*(B*c*d + A*c*e +
 a*C*e + a*B*f) + 2*(A*c*(c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(a*f*(p
+ 1) - c*d*(p + 2)) - e*((A*c - a*C)*(2*a*c*e - b*(c*d + a*f)) + (A*b - a*B
)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((A*c - a*C)*(2*a
*c*e - b*(c*d + a*f)) + (A*b - a*B)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p
 + q + 2) - (b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(
c*d - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))
*x - c*f*(b^2*(C*d + A*f) - b*(B*c*d + A*c*e + a*C*e + a*B*f) + 2*(A*c*(c*d
 - a*f) - a*(c*C*d - B*c*e - a*C*f)))*(2*p + 2*q + 5)*x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e, f, q}, x] && PolyQ[Px, x, 2] && LtQ[p, -1] && NeQ[(c*d
 - a*f)^2 - (b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
 &&  !IGtQ[q, 0]
```

3.122.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(560) = 1120$.

Time = 0.86 (sec) , antiderivative size = 1992, normalized size of antiderivative = 3.27

method	result	size
default	Expression too large to display	1992

input
```
int(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
2/f*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)+1/2*(-e*(-4*d*f+e^2)^(1/2)+2
*d*f-e^2)/f^2/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/
f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f
)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)
^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+
(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/
f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2
*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/
2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*
e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-
b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1
/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/
2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f
+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)
^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+
b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-...
```

3.122.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.122.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.122.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x^2/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \int \frac{x^2}{(c\,x^2+b\,x+a)^{3/2}\,(f\,x^2+e\,x+d)}\,dx$$

input
```
int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x^2/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.123 $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.123.1 Optimal result

Integrand size = 28, antiderivative size = 609

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(a(2c^2d-bce+b^2f-2acf)+c(bcd-2ace+abf)x)}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$+\frac{f\left(2d(ce-bf)-(cd-af)\left(e-\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4df}}}$$
$$-\frac{f\left(2d(ce-bf)-(cd-af)\left(e+\sqrt{e^2-4df}\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}\sqrt{a+bx+cx^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}$$

output
```
2*(a*(-2*a*c*f+b^2*f-b*c*e+2*c^2*d)+c*(a*b*f-2*a*c*e+b*c*d)*x)/(-4*a*c+b^2
)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2)+1/2*f*arctanh(1
/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f+e^2)^(1/2)))*2^(
1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e
^2)^(1/2))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e-(-4*d*f+e^2)^(1/2)))/((-a*
f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-
b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a*
f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*
x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2
))^(1/2))*(2*d*(-b*f+c*e)-(-a*f+c*d)*(e+(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2
-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)
```

3.123.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.85 (sec) , antiderivative size = 570, normalized size of antiderivative = 0.94

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-4a^2cf+2bc^2dx+2a(b^2f+2c^2(d-ex)+bc(-e+fx))-(b^2-}{}$$

input
```
Integrate[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-4*a^2*c*f + 2*b*c^2*d*x + 2*a*(b^2*f + 2*c^2*(d - e*x) + b*c*(-e + f*x))
 - (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[b^2*d - a*b*e + a^2*f - 4*b
*Sqrt[c]*d*#1 + 2*a*Sqrt[c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*
Sqrt[c]*e*#1^3 + f*#1^4 & , (-(b*c*d*e*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]) + b^2*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a*
c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - a^2*f^2*Log[-(Sqrt[
c]*x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*d*e*Log[-(Sqrt[c]*x) + Sqr
t[a + b*x + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*d*f*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1]*#1 - c*d*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #
1]*#1^2 + a*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2)/(2*b*
Sqrt[c]*d - a*Sqrt[c]*e - 4*c*d*#1 - b*e*#1 + 2*a*f*#1 + 3*Sqrt[c]*e*#1^2
- 2*f*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^
2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)])
```

3.123.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 581, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1349, 27, 1365, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \text{1349}$$

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{2\int \frac{(b^2-4ac)(d(ce-bf)+f(cd-af)x)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$\downarrow$ 27

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int \frac{d(ce-bf)+f(cd-af)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$\downarrow$ 1365

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-$$
$$\frac{\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(cd-af)\right)\int \frac{1}{\left(e+2fx-\sqrt{e^2-4df}\right)\sqrt{cx^2+bx+a}}dx}{\sqrt{e^2-4df}}-\frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\int \frac{1}{\left(e+2fx+\sqrt{e^2-4df}\right)\sqrt{c}}}{\sqrt{e^2-4df}}}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$\downarrow$ 1154

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-$$
$$\frac{2f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\int \frac{1}{4\left(4af^2-2b\left(e+\sqrt{e^2-4df}\right)f+c\left(e+\sqrt{e^2-4df}\right)^2\right)-\frac{\left(4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-4df}\right)\right)x\right)^2}{cx^2+bx+a}}d\frac{4af}{}}{\sqrt{e^2-4df}}$$

$\downarrow$ 219

$$\frac{2\left(a\left(-2acf+b^2f-bce+2c^2d\right)+cx(abf-2ace+bcd)\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-$$
$$\frac{\frac{f\left(2d(ce-bf)-\left(\sqrt{e^2-4df}+e\right)(cd-af)\right)\text{arctanh}\left(\frac{4af+2x\left(bf-c\left(\sqrt{e^2-4df}+e\right)\right)-b\left(\sqrt{e^2-4df}+e\right)}{2\sqrt{2}\sqrt{a+bx+cx^2}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}\right)}{\sqrt{2}\sqrt{e^2-4df}\sqrt{2af^2+\sqrt{e^2-4df}(ce-bf)-bef-2cdf+ce^2}}-\frac{f\left(2d(ce-bf)-\left(e-\sqrt{e^2-4df}\right)(c\right)}{\sqrt{2}\sqrt{e}}}{(cd-af)^2-(bd-ae)(ce-bf)}$$

input
```
Int[x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

3.123. $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

output
```
(2*(a*(2*c^2*d - b*c*e + b^2*f - 2*a*c*f) + c*(b*c*d - 2*a*c*e + a*b*f)*x)
)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*Sqrt[a + b*x +
c*x^2]) - (-((f*(2*d*(c*e - b*f) - (c*d - a*f)*(e - Sqrt[e^2 - 4*d*f]))*Ar
cTanh[(4*a*f - b*(e - Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e - Sqrt[e^2 - 4*d*
f]))*x)/(2*Sqrt[2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sq
rt[e^2 - 4*d*f]]*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[
c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 - (c*e - b*f)*Sqrt[e^2 - 4*d*f]])) + (f*
(2*d*(c*e - b*f) - (c*d - a*f)*(e + Sqrt[e^2 - 4*d*f]))*ArcTanh[(4*a*f - b
*(e + Sqrt[e^2 - 4*d*f]) + 2*(b*f - c*(e + Sqrt[e^2 - 4*d*f]))*x)/(2*Sqrt[
2]*Sqrt[c*e^2 - 2*c*d*f - b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]
*Sqrt[a + b*x + c*x^2])])/(Sqrt[2]*Sqrt[e^2 - 4*d*f]*Sqrt[c*e^2 - 2*c*d*f
- b*e*f + 2*a*f^2 + (c*e - b*f)*Sqrt[e^2 - 4*d*f]]))/((c*d - a*f)^2 - (b*d
 - a*e)*(c*e - b*f))
```

3.123.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

3.123. $\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

rule 1349
```
Int[((g_.) + (h_.)*(x_))*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_) + (e
_.)*(x_) + (f_.)*(x_)^2)^(q_), x_Symbol] :> Simp[(a + b*x + c*x^2)^(p + 1)*
((d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e
 - b*f))*(p + 1)))*(g*c*(2*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d +
b^2*f - c*(b*e + 2*a*f)) + c*(g*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)) - h*(b*
c*d - 2*a*c*e + a*b*f))*x), x] + Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*
d - a*e)*(c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f
*x^2)^q*Simp[(b*h - 2*g*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1
) + (b^2*(g*f) - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a*((-h)*c
*e)))*(a*f*(p + 1) - c*d*(p + 2)) - e*((g*c)*(2*a*c*e - b*(c*d + a*f)) + (g
*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*(p + q + 2) - (2*f*((g*c)*(2
*a*c*e - b*(c*d + a*f)) + (g*b - a*h)*(2*c^2*d + b^2*f - c*(b*e + 2*a*f)))*
(p + q + 2) - (b^2*g*f - b*(h*c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) - a
*((-h)*c*e)))*(b*f*(p + 1) - c*e*(2*p + q + 4)))*x - c*f*(b^2*(g*f) - b*(h*
c*d + g*c*e + a*h*f) + 2*(g*c*(c*d - a*f) + a*h*c*e))*(2*p + 2*q + 5)*x^2,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, h, q}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 - (b*d - a*e)*(c*
e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1])
```

rule 1365
```
Int[((g_.) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (
e_.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Sim
p[(2*c*g - h*(b - q))/q   Int[1/((b - q + 2*c*x)*Sqrt[d + e*x + f*x^2]), x]
, x] - Simp[(2*c*g - h*(b + q))/q   Int[1/((b + q + 2*c*x)*Sqrt[d + e*x + f
*x^2]), x], x]] /; FreeQ[{a, b, c, d, e, f, g, h}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[e^2 - 4*d*f, 0] && PosQ[b^2 - 4*a*c]
```

3.123.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1938 vs. $2(560) = 1120$.

Time = 0.86 (sec) , antiderivative size = 1939, normalized size of antiderivative = 3.18

method	result	size
default	Expression too large to display	1939

input
```
int(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/2*(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1/2)/f*(2/(-b*f*(-4*d*f+e^2)^(1/2
)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d
*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*
f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f
^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b
*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*
(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))
/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*
f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e
^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^
2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b
*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+
(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(
-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+
1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2
^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*
d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*
d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e
^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2...
```

3.123.5 Fricas [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.123.6 Sympy [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.123.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.123.8 Giac [F(-2)]

Exception generated.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx=\int \frac{x}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(x/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.124 $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.124.1 Optimal result

Integrand size = 27, antiderivative size = 666

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(b^2ce-2ac^2e-b^3f-bc(cd-3af)-c(2c^2d-bce+b^2f-2acf}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$
$$-\frac{f\left(c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e-\sqrt{e^2-4df}\right)+2\left(bf-c\left(e-\sqrt{e^2-}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2-(ce-bf)\sqrt{e^2-4d}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2}\right.\right.}}$$
$$+\frac{f\left(c\left(e^2-2df-e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e-\sqrt{e^2-4df}\right)\right)\right)\operatorname{arctanh}\left(\frac{4af-b\left(e+\sqrt{e^2-4df}\right)+2\left(bf-c\left(e+\sqrt{e^2-}\right.\right.}{2\sqrt{2}\sqrt{ce^2-2cdf-bef+2af^2+(ce-bf)\sqrt{e^2-4df}}}\right.}{\sqrt{2}\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{c\left(e^2-2df+e\sqrt{e^2-4df}\right)+f\left(2af-b\left(e+\sqrt{e^2-}\right.\right.}}$$

output
```
2*(b^2*c*e-2*a*c^2*e-b^3*f-b*c*(-3*a*f+c*d)-c*(-2*a*c*f+b^2*f-b*c*e+2*c^2*
d)*x)/(-4*a*c+b^2)/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*x+a)^(1/2
)-1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(e-(-4*d*f
+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2-(-b
*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(c*(e^2-2*d*f+e*(-4*d*f+e^2)^(1/2))+f*(
2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1
/2)/(-4*d*f+e^2)^(1/2)/(f*(2*a*f-b*(e-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*
(-4*d*f+e^2)^(1/2)))^(1/2)+1/2*f*arctanh(1/4*(4*a*f-b*(e+(-4*d*f+e^2)^(1/2
))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-
2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(f*(2*a*f-b*(e
-(-4*d*f+e^2)^(1/2)))+c*(e^2-2*d*f-e*(-4*d*f+e^2)^(1/2)))/((-a*f+c*d)^2-(-
a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*(e^2-2*d*f+e*(-4*d*f+e^
2)^(1/2))+f*(2*a*f-b*(e+(-4*d*f+e^2)^(1/2))))^(1/2)
```

3.124.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.21

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{-2(b^3f+b^2c(-e+fx)+bc(-3af+c(d-ex))+2c^2(cdx+a(e$$

input
```
Integrate[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(b^3*f + b^2*c*(-e + f*x) + b*c*(-3*a*f + c*(d - e*x)) + 2*c^2*(c*d*x
+ a*(e - f*x))) + (b^2 - 4*a*c)*Sqrt[a + x*(b + c*x)]*RootSum[c^2*d - b*c*
e + b^2*f + 2*Sqrt[a]*c*e*#1 - 4*Sqrt[a]*b*f*#1 - 2*c*d*#1^2 + b*e*#1^2 +
4*a*f*#1^2 - 2*Sqrt[a]*e*#1^3 + d*#1^4 & , (-(c^2*e^2*Log[x]) + c^2*d*f*Lo
g[x] + 2*b*c*e*f*Log[x] - b^2*f^2*Log[x] - a*c*f^2*Log[x] + c^2*e^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - c^2*d*f*Log[-Sqrt[a] + Sqrt[a + b
*x + c*x^2] - x*#1] - 2*b*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#
1] + b^2*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] + a*c*f^2*Log[-S
qrt[a] + Sqrt[a + b*x + c*x^2] - x*#1] - 2*Sqrt[a]*c*e*f*Log[x]*#1 + 2*Sqr
t[a]*b*f^2*Log[x]*#1 + 2*Sqrt[a]*c*e*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2
] - x*#1]*#1 - 2*Sqrt[a]*b*f^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1
]*#1 + c*e^2*Log[x]*#1^2 - c*d*f*Log[x]*#1^2 - b*e*f*Log[x]*#1^2 + a*f^2*L
og[x]*#1^2 - c*e^2*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + c*d
*f*Log[-Sqrt[a] + Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 + b*e*f*Log[-Sqrt[a]
+ Sqrt[a + b*x + c*x^2] - x*#1]*#1^2 - a*f^2*Log[-Sqrt[a] + Sqrt[a + b*x +
 c*x^2] - x*#1]*#1^2)/(Sqrt[a]*c*e - 2*Sqrt[a]*b*f - 2*c*d*#1 + b*e*#1 + 4
*a*f*#1 - 3*Sqrt[a]*e*#1^2 + 2*d*#1^3) & ])/((b^2 - 4*a*c)*(c^2*d^2 - b*c*
d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a + x*(b + c*x)]
)
```

3.124.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$$

$$\Big\downarrow \ 1305$$

$$\frac{2\int -\frac{(b^2-4ac)\left(f(be-af)-c(e^2-df)-f(ce-bf)x\right)}{2\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(b^2-4ac)\left((cd-af)^2-(bd-ae)(ce-bf)\right)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

3.124. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

3.124. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c\left(e^2-df\right)-f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c\left(e^2-df\right)-f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int -\frac{f(be-af)-c\left(e^2-df\right)-f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{\left(b^2-4ac\right)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}\left(fx^2+ex+d\right)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

3.124. $\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \ 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \ 25$$

3.124. $\int \frac{1}{(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

$$\Big\downarrow \; 25$$

$$\frac{\int -\frac{f(be-af)-c(e^2-df)-f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}+\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}$$

$$\Big\downarrow \; 25$$

$$\frac{2\left(-cx\left(-2acf+b^2f-bce+2c^2d\right)-bc(cd-3af)-2ac^2e+b^3(-f)+b^2ce\right)}{(b^2-4ac)\sqrt{a+bx+cx^2}\left((cd-af)^2-(bd-ae)(ce-bf)\right)}-\frac{\int -\frac{ce^2-bfe+af^2-cdf+f(ce-bf)x}{\sqrt{cx^2+bx+a}(fx^2+ex+d)}dx}{(cd-af)^2-(bd-ae)(ce-bf)}$$

input
```
Int[1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
$Aborted
```

3.124.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1305
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.) + (e_.)*(x_) + (f_.)*(x
_)^2)^(q_), x_Symbol] :> Simp[(2*a*c^2*e - b^2*c*e + b^3*f + b*c*(c*d - 3*a
*f) + c*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*x)*(a + b*x + c*x^2)^(p + 1)*((
d + e*x + f*x^2)^(q + 1)/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(c*e -
 b*f))*(p + 1))), x] - Simp[1/((b^2 - 4*a*c)*((c*d - a*f)^2 - (b*d - a*e)*(
c*e - b*f))*(p + 1))   Int[(a + b*x + c*x^2)^(p + 1)*(d + e*x + f*x^2)^q*Si
mp[2*c*((c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f))*(p + 1) - (2*c^2*d + b^2*f
 - c*(b*e + 2*a*f))*(a*f*(p + 1) - c*d*(p + 2)) - e*(b^2*c*e - 2*a*c^2*e -
b^3*f - b*c*(c*d - 3*a*f))*(p + q + 2) + (2*f*(2*a*c^2*e - b^2*c*e + b^3*f
+ b*c*(c*d - 3*a*f))*(p + q + 2) - (2*c^2*d + b^2*f - c*(b*e + 2*a*f))*(b*f
*(p + 1) - c*e*(2*p + q + 4)))*x + c*f*(2*c^2*d + b^2*f - c*(b*e + 2*a*f))*
(2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b
^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && LtQ[p, -1] && NeQ[(c*d - a*f)^2 -
(b*d - a*e)*(c*e - b*f), 0] &&  !( !IntegerQ[p] && ILtQ[q, -1]) &&  !IGtQ[q
, 0]
```

3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1905 vs. $2(609) = 1218$.

Time = 1.09 (sec) , antiderivative size = 1906, normalized size of antiderivative = 2.86

method	result	size
default	Expression too large to display	1906

input
```
int(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/(-4*d*f+e^2)^(1/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2
*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*
(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f
*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^
2)^(1/2)-2*f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*
d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2
)^(1/2))/f)+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(
1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4
*d*f+e^2)^(1/2)+b*f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*
(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4
*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(
1/2)-2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)*f^2*2^(1/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+
e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)
^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(
x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+4/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x
+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+2*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/
2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2))/(x+1/2*(e+(-4*d*f+e^2)^...
```

3.124.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Timed out
```

3.124.7 Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*d*f-e^2>0)', see `assume?` for
 more deta
```

3.124.8 Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(1/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

3.124.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/((a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.125 $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.125.1 Optimal result

Integrand size = 30, antiderivative size = 816

$$\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx = \frac{2(b^2-2ac+bcx)}{a(b^2-4ac)\,d\sqrt{a+bx+cx^2}}$$

$$+\frac{2(ce(2ace-b(cd+af))+(be-af)\left(2c^2d+b^2f-c(be+2af)\right)+c(2c^2de+bf(be-af)-bc(e^2+df))\,x}{(b^2-4ac)\,d\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{a+bx+cx^2}}$$

$$-\frac{\operatorname{arctanh}\left(\frac{2a+bx}{2\sqrt{a}\sqrt{a+bx+cx^2}}\right)}{a^{3/2}d}$$

$$+\frac{f\left(\left(e-\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)-2(f(be^2-bdf-aef)-c(e^3-2def))\right)\operatorname{arctanh}\left(\frac{4af-}{2\sqrt{2}\sqrt{ce^2-}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2-(ce}}$$

$$-\frac{f\left(\left(e+\sqrt{e^2-4df}\right)\left(f(be-af)-c(e^2-df)\right)-2(f(be^2-bdf-aef)-c(e^3-2def))\right)\operatorname{arctanh}\left(\frac{4af-}{2\sqrt{2}\sqrt{ce^2-}}\right.}{\sqrt{2}d\sqrt{e^2-4df}\left((cd-af)^2-(bd-ae)(ce-bf)\right)\sqrt{ce^2-2cdf-bef+2af^2+(ce}}$$

output
```
-arctanh(1/2*(b*x+2*a)/a^(1/2)/(c*x^2+b*x+a)^(1/2))/a^(3/2)/d+2*(b*c*x-2*a
*c+b^2)/a/(-4*a*c+b^2)/d/(c*x^2+b*x+a)^(1/2)+2*(c*e*(2*a*c*e-b*(a*f+c*d))+
(-a*f+b*e)*(2*c^2*d+b^2*f-c*(2*a*f+b*e))+c*(2*c^2*d*e+b*f*(-a*f+b*e)-b*c*(
d*f+e^2))*x)/(-4*a*c+b^2)/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))/(c*x^2+b*
x+a)^(1/2)+1/2*f*arctanh(1/4*(4*a*f+2*x*(b*f-c*(e-(-4*d*f+e^2)^(1/2)))-b*(
e-(-4*d*f+e^2)^(1/2)))*2^(1/2)/(c*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*
a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2))*(-2*f*(-a*e*f-b*d*f+b*e^2)+2*c
*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(-d*f+e^2))*(e-(-4*d*f+e^2)^(1/2)))/d/((-a
*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f
-b*e*f+2*a*f^2-(-b*f+c*e)*(-4*d*f+e^2)^(1/2))^(1/2)-1/2*f*arctanh(1/4*(4*a
*f-b*(e+(-4*d*f+e^2)^(1/2))+2*x*(b*f-c*(e+(-4*d*f+e^2)^(1/2))))*2^(1/2)/(c
*x^2+b*x+a)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f+e^2)^(1/
2))^(1/2))*(-2*f*(-a*e*f-b*d*f+b*e^2)+2*c*(-2*d*e*f+e^3)+(f*(-a*f+b*e)-c*(
-d*f+e^2))*(e+(-4*d*f+e^2)^(1/2)))/d/((-a*f+c*d)^2-(-a*e+b*d)*(-b*f+c*e))*
2^(1/2)/(-4*d*f+e^2)^(1/2)/(c*e^2-2*c*d*f-b*e*f+2*a*f^2+(-b*f+c*e)*(-4*d*f
+e^2)^(1/2))^(1/2)
```

3.125.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 6.73 (sec) , antiderivative size = 1025, normalized size of antiderivative = 1.26

$$\int \frac{1}{x\,(a+bx+cx^2)^{3/2}\,(d+ex+fx^2)}\,dx =$$

$$-\frac{2(b^4f+2ac^2(-cd+af+cex)+b^3c(-e+fx)+b^2c(-4af+c(d-ex))+bc^2(cdx+3a(e-fx)))}{a\,(-b^2+4ac)\,(c^2d^2-bcde+f\,(b^2d-abe+a^2f)+ac\,(e^2-2df))\,\sqrt{a+x(b+cx)}}$$

$$+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x-\sqrt{a+x(b+cx)}}{\sqrt{a}}\right)}{a^{3/2}d}$$

$$-\frac{\text{RootSum}\Big[b^2d-abe+a^2f-4b\sqrt{c}d\#1+2a\sqrt{c}e\#1+4cd\#1^2+be\#1^2-2af\#1^2-2\sqrt{c}e\#1^3+f\#1^4\&}{}$$

input
```
Integrate[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
(-2*(b^4*f + 2*a*c^2*(-(c*d) + a*f + c*e*x) + b^3*c*(-e + f*x) + b^2*c*(-4
*a*f + c*(d - e*x)) + b*c^2*(c*d*x + 3*a*(e - f*x))))/(a*(-b^2 + 4*a*c)*(c
^2*d^2 - b*c*d*e + f*(b^2*d - a*b*e + a^2*f) + a*c*(e^2 - 2*d*f))*Sqrt[a +
 x*(b + c*x)]) + (2*ArcTanh[(Sqrt[c]*x - Sqrt[a + x*(b + c*x)])/Sqrt[a]])/
(a^(3/2)*d) - RootSum[b^2*d - a*b*e + a^2*f - 4*b*Sqrt[c]*d*#1 + 2*a*Sqrt[
c]*e*#1 + 4*c*d*#1^2 + b*e*#1^2 - 2*a*f*#1^2 - 2*Sqrt[c]*e*#1^3 + f*#1^4 &
 , (-(b*c*e^3*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]) + 2*b*c*d*e*
f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + b^2*e^2*f*Log[-(Sqrt[c]
*x) + Sqrt[a + b*x + c*x^2] - #1] + a*c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a +
b*x + c*x^2] - #1] - b^2*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1] - a*c*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] - 2*a*b*e*f
^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1] + a^2*f^3*Log[-(Sqrt[c]*
x) + Sqrt[a + b*x + c*x^2] - #1] + 2*c^(3/2)*e^3*Log[-(Sqrt[c]*x) + Sqrt[a
 + b*x + c*x^2] - #1]*#1 - 4*c^(3/2)*d*e*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x
 + c*x^2] - #1]*#1 - 2*b*Sqrt[c]*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c
*x^2] - #1]*#1 + 2*b*Sqrt[c]*d*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2
] - #1]*#1 + 2*a*Sqrt[c]*e*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] -
#1]*#1 - c*e^2*f*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + c*d
*f^2*Log[-(Sqrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 + b*e*f^2*Log[-(S
qrt[c]*x) + Sqrt[a + b*x + c*x^2] - #1]*#1^2 - a*f^3*Log[-(Sqrt[c]*x) +...
```

3.125.3 Rubi [F]

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x (a+bx+cx^2)^{3/2} (d+ex+fx^2)} \, dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d (a+bx+cx^2)^{3/2} (d+ex+fx^2)}+\frac{1}{dx (a+bx+cx^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x (a+bx+cx^2)^{3/2} (d+ex+fx^2)} dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d (a+bx+cx^2)^{3/2} (d+ex+fx^2)}+\frac{1}{dx (a+bx+cx^2)^{3/2}}\right) dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

3.125. $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

3.125. $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{-e-fx}{d\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}+\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}}\right)dx$$

$$\Big\downarrow \text{7239}$$

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}dx$$

input
```
Int[1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x]
```

output
```
$Aborted
```

3.125. $\int \frac{1}{x(a+bx+cx^2)^{3/2}(d+ex+fx^2)}\,dx$

3.125.3.1 Defintions of rubi rules used

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.125.4 Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. $2(753) = 1506$.

Time = 1.16 (sec) , antiderivative size = 2059, normalized size of antiderivative = 2.52

method	result	size
default	Expression too large to display	2059

input
```
int(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-4*f/(-e+(-4*d*f+e^2)^(1/2))/(e+(-4*d*f+e^2)^(1/2))*(1/a/(c*x^2+b*x+a)^(1/
2)-b/a*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^(1/2)-1/a^(3/2)*ln((2*a+b*x+2*a
^(1/2)*(c*x^2+b*x+a)^(1/2))/x))+2*f/(e+(-4*d*f+e^2)^(1/2))/(-4*d*f+e^2)^(1
/2)*(2/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d
*f+c*e^2)*f^2/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(
1/2)+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2
)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2*f*(-c*(
-4*d*f+e^2)^(1/2)+b*f-c*e)/(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e
+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*(2*c*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/f*(-
c*(-4*d*f+e^2)^(1/2)+b*f-c*e))/(2*c*(-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^
(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2-1/f^2*(-c*(-4*d*f+e^2)^(1/2)+b*
f-c*e)^2)/((x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)^2*c+1/f*(-c*(-4*d*f+e^2)^(1/2)
+b*f-c*e)*(x+1/2*(e+(-4*d*f+e^2)^(1/2))/f)+1/2*(-b*f*(-4*d*f+e^2)^(1/2)+(-
4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)-2/(-b*f*(-4*d
*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)*f^2*2^(1
/2)/((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f
+c*e^2)/f^2)^(1/2)*ln(((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2)^(1/2)*c*e+2*a
*f^2-b*e*f-2*c*d*f+c*e^2)/f^2+1/f*(-c*(-4*d*f+e^2)^(1/2)+b*f-c*e)*(x+1/2*(
e+(-4*d*f+e^2)^(1/2))/f)+1/2*2^(1/2)*((-b*f*(-4*d*f+e^2)^(1/2)+(-4*d*f+e^2
)^(1/2)*c*e+2*a*f^2-b*e*f-2*c*d*f+c*e^2)/f^2)^(1/2)*(4*(x+1/2*(e+(-4*d*...
```

3.125.5 Fricas [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
Timed out
```

3.125.6 Sympy [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\left(a+bx+cx^2\right)^{\frac{3}{2}}\left(d+ex+fx^2\right)}\,dx$$

input
```
integrate(1/x/(c*x**2+b*x+a)**(3/2)/(f*x**2+e*x+d),x)
```

output
```
Integral(1/(x*(a + b*x + c*x**2)**(3/2)*(d + e*x + f*x**2)), x)
```

3.125.7 Maxima [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x + a)^(3/2)*(f*x^2 + e*x + d)*x), x)
```

3.125.8 Giac [F]

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{(cx^2+bx+a)^{\frac{3}{2}}(fx^2+ex+d)x}\,dx$$

input
```
integrate(1/x/(c*x^2+b*x+a)^(3/2)/(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
sage0*x
```

3.125.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(a+bx+cx^2\right)^{3/2}\left(d+ex+fx^2\right)}\,dx=\int \frac{1}{x\left(c\,x^2+b\,x+a\right)^{3/2}\,\left(f\,x^2+e\,x+d\right)}\,dx$$

input
```
int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)),x)
```

output
```
int(1/(x*(a + b*x + c*x^2)^(3/2)*(d + e*x + f*x^2)), x)
```

3.126 $\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.126.1 Optimal result

Integrand size = 30, antiderivative size = 140

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{5}{2}\sqrt{-3-4x-x^2}-\frac{1}{4}x\sqrt{-3-4x-x^2}+\frac{11}{2}\arcsin(2+x)+\frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
11/2*arcsin(2+x)-5/4*arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x
)/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3
)^(1/2))*2^(1/2))*2^(1/2)+5/2*(-x^2-4*x-3)^(1/2)-1/4*x*(-x^2-4*x-3)^(1/2)
```

3.126.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.71

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{4}\left(-\left((-10+x)\sqrt{-3-4x-x^2}\right) - \sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) - 44\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right) - 5\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

input
```
Integrate[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(-((-10 + x)*Sqrt[-3 - 4*x - x^2]) - Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqr
t[-3 - 4*x - x^2])] - 44*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] - 5*ArcTanh[
x/Sqrt[-3 - 4*x - x^2]])/4
```

3.126.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{x^2}{2\sqrt{-x^2-4x-3}} - \frac{x}{\sqrt{-x^2-4x-3}} + \frac{5}{4\sqrt{-x^2-4x-3}} - \frac{8x+15}{4\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{11}{2}\arcsin(x+2)+\frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{5}{4}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{4}\sqrt{-x^2-4x-3}x+\frac{5}{2}\sqrt{-x^2-4x-3}$$

input
```
Int[x^4/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(5*Sqrt[-3 - 4*x - x^2])/2 - (x*Sqrt[-3 - 4*x - x^2])/4 + (11*ArcSin[2 + x
])/2 + ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - Ar
cTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - (5*ArcTanh[
x/Sqrt[-3 - 4*x - x^2]])/4
```

3.126.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.126.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.11

method	result
risch	$\frac{(x-10)(x^2+4x+3)}{4\sqrt{-x^2-4x-3}}+\frac{11\arcsin(2+x)}{2}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)+5\,\mathrm{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\ldots}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
default	$\frac{5\sqrt{-x^2-4x-3}}{2}+\frac{11\arcsin(2+x)}{2}-\frac{x\sqrt{-x^2-4x-3}}{4}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)+5\,\mathrm{arctanh}\left(\ldots\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\left(-\frac{x}{4}+\frac{5}{2}\right)\sqrt{-x^2-4x-3}-\frac{5\ln\left(\frac{12\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)^2x+28\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)x+12\,\mathrm{RootOf}\left(12\_Z^2+\ldots\right.}{2\,\mathrm{RootOf}\left(12\_Z^2+20\_Z+9\right)x+x-1}\right.}{4}$

input
```
int(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(x-10)*(x^2+4*x+3)/(-x^2-4*x-3)^(1/2)+11/2*arcsin(2+x)+1/24*3^(1/2)*4^
(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12
)^(1/2)*2^(1/2))+5*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^
2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

3.126. $\int \frac{x^4}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.126.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.27

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{4}\sqrt{-x^2-4\,x-3}(x-10) + \frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) - \frac{11}{2}\arctan\left(\frac{\sqrt{-x^2-4\,x-3}(x+2)}{x^2+4\,x+3}\right) + \frac{5}{16}\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) - \frac{5}{16}\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

input
```
integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3
*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(
2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x + 3)) - 11/2*arctan(sqrt(-x^2
- 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) + 5/16*log(-(2*sqrt(-x^2 - 4*x - 3)*x
+ 4*x + 3)/x^2) - 5/16*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.126.6 Sympy [F]

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^4}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(x**4/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(x**4/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.126.7 Maxima [F]

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^4}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.126.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.34

$$\begin{aligned}
&\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= -\frac{1}{4}\sqrt{-x^2-4\,x-3}(x-10) + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) + \frac{11}{2}\arcsin\left(x+2\right) \\
&\quad - \frac{5}{8}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) \\
&\quad + \frac{5}{8}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)
\end{aligned}$$

input
```
integrate(x^4/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(-x^2 - 4*x - 3)*(x - 10) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sq
rt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)) + 11/2*arcsin(x + 2) - 5/8*log(2*(s
qrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^
2 + 1) + 5/8*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x -
 3) - 1)^2/(x + 2)^2 + 3)
```

3.126.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^4}{\sqrt{-x^2-4\,x-3}\,(2\,x^2+4\,x+3)}\,dx$$

input
```
int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(x^4/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.127 $\int \frac{x^3}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.127.1 Optimal result

Integrand size = 30, antiderivative size = 115

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{2}\sqrt{-3-4x-x^2}-2\arcsin(2+x)+\frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{2\sqrt{2}}+\operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
-2*arcsin(2+x)+arctanh(x/(-x^2-4*x-3)^(1/2))+1/4*arctan(1/2*(1+(-3-x)/(-x^
2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)-1/4*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2
))*2^(1/2))*2^(1/2)-1/2*(-x^2-4*x-3)^(1/2)
```

3.127.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{2}\sqrt{-3-4x-x^2}-\frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{2\sqrt{2}}+4\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right)+\operatorname{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-1/2*Sqrt[-3 - 4*x - x^2] - ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2]
)]/(2*Sqrt[2]) + 4*ArcTan[Sqrt[-3 - 4*x - x^2]/(3 + x)] + ArcTanh[x/Sqrt[-
3 - 4*x - x^2]]
```

3.127.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$
$$\Big\downarrow \; 7279$$
$$\int \left(\frac{x}{2\sqrt{-x^2-4x-3}}-\frac{1}{\sqrt{-x^2-4x-3}}+\frac{5x+6}{2\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-2\arcsin(x+2)+\frac{\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)}{2\sqrt{2}}-\frac{\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)}{2\sqrt{2}}+\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{1}{2}\sqrt{-x^2-4x-3}$$

input
```
Int[x^3/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-1/2*Sqrt[-3 - 4*x - x^2] - 2*ArcSin[2 + x] + ArcTan[(1 - (3 + x)/Sqrt[-3
- 4*x - x^2])/Sqrt[2]]/(2*Sqrt[2]) - ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x
^2])/Sqrt[2]]/(2*Sqrt[2]) + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]
```

3.127.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.127.4 Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.25

method	result
default	$-2\arcsin(2+x)-\frac{\sqrt{-x^2-4x-3}}{2}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)-4\,\mathrm{arctanh}\left(\frac{3\ldots}{\left(-\frac{3}{2}-x\right)\sqrt{\ldots}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
risch	$\frac{x^2+4x+3}{2\sqrt{-x^2-4x-3}}-2\arcsin(2+x)+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)-4\,\mathrm{arctanh}\left(\frac{3x\ldots}{\left(-\frac{3}{2}-x\right)\sqrt{\ldots}}\right.\right.}{24\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$-\frac{\sqrt{-x^2-4x-3}}{2}+\frac{3\,\mathrm{RootOf}\left(24\_\_Z^2-16\_\_Z+3\right)\ln\left(\frac{72\,\mathrm{RootOf}\left(24\_\_Z^2-16\_\_Z+3\right)^2x-72\,\mathrm{RootOf}\left(24\_\_Z^2-16\_\_Z+3\right)x-36\,\mathrm{RootOf}\ldots}{12\,\mathrm{RootOf}\left(24\_\_Z^2-16\_\_Z+3\right)x-\ldots}\right.}{2}$

input
```
int(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arcsin(2+x)-1/2*(-x^2-4*x-3)^(1/2)+1/24*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)
^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-4*ar
ctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/
(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

3.127.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.52

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{8}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) + \frac{1}{8}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2(2x+3)}\right) - \frac{1}{2}\sqrt{-x^2-4x-3} + 2\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right) - \frac{1}{4}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right) + \frac{1}{4}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)$$

input
```
integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x +
 3)) + 1/8*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3)
)/(2*x + 3)) - 1/2*sqrt(-x^2 - 4*x - 3) + 2*arctan(sqrt(-x^2 - 4*x - 3)*(x
 + 2)/(x^2 + 4*x + 3)) - 1/4*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2
) + 1/4*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.127.6 Sympy [F]

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(x**3/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(x**3/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.127.7 Maxima [F]

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^3/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.127.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.61

$$\begin{aligned}
&\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) - \frac{1}{2}\sqrt{-x^2-4\,x-3} \\
&\quad - 2\arcsin\left(x+2\right) + \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+1\right) \\
&\quad - \frac{1}{2}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2} + \frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{\left(x+2\right)^2}+3\right)
\end{aligned}$$

input
```
integrate(x^3/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/4*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 1/2*sqrt(-x^2 - 4*x - 3) - 2*arcsin(x + 2) + 1/2*log(2*(sqrt(-x^2 - 4*
x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) - 1/2*
log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x
 + 2)^2 + 3)
```

3.127.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^3}{\sqrt{-x^2-4\,x-3}\,(2\,x^2+4\,x+3)}\,dx$$

input
```
int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(x^3/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.128 $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.128.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{2}\arcsin(2+x)-\frac{\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}+\frac{\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right)}{\sqrt{2}}-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
1/2*arcsin(2+x)-1/2*arctanh(x/(-x^2-4*x-3)^(1/2))-1/2*arctan(1/2*(1+(-3-x)
/(-x^2-4*x-3)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)
^(1/2))*2^(1/2))*2^(1/2)
```

3.128.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}}-\arctan\left(\frac{\sqrt{-3-4x-x^2}}{3+x}\right)-\frac{1}{2}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

input
```
Integrate[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2] - ArcTan[Sqrt[-3
- 4*x - x^2]/(3 + x)] - ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/2
```

3.128.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.01, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2143, 25, 1090, 223, 1361, 27, 1317, 27, 1359, 27, 1360, 219, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{2143}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-x^2-4x-3}}dx+\frac{1}{2}\int -\frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{-x^2-4x-3}}dx-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{4}\int \frac{1}{\sqrt{1-\frac{1}{4}(-2x-4)^2}}d(-2x-4)$$

$$\Big\downarrow \text{223}$$

$$-\frac{1}{2}\int \frac{4x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{1361}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx\right)-\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(3\int \frac{1}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx-2\int \frac{2x+3}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx\right)-\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1317

$$\frac{1}{2}\left(3\left(\frac{1}{6}\int -\frac{4x}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx-\frac{1}{6}\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx\right)-2\int \frac{2x}{\sqrt{-x^2-4x-}}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx-\frac{2}{3}\int \frac{x}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx\right)-2\int \frac{2x+}{\sqrt{-x^2-4x-3}\,(2}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1359

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx-\frac{16}{3}\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 27

$$\frac{1}{2}\left(3\left(\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\left(2x^2+4x+3\right)}dx+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-2\int\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1360

$$\frac{1}{2}\left(3\left(\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}\right)-6\int \frac{}{3-}\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 219

3.128. $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

$$\frac{1}{2}\left(3\left(\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{}{\sqrt{-}}\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1475

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{}{\frac{(x+}{9(-x^2-}}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 1083

$$\frac{1}{2}\left(3\left(\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-\frac{4}{3}\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}}\right.\right.\right.$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

$\big\downarrow$ 217

$$\frac{1}{2}\left(3\left(\frac{2}{3}\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-2\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)\right)-$$
$$\frac{1}{2}\arcsin\left(\frac{1}{2}(-2x-4)\right)$$

input
```
Int[x^2/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-1/2*ArcSin[(-4 - 2*x)/2] + (3*((2*Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[
-3 - 4*x - x^2])])/3 + ArcTanh[x/Sqrt[-3 - 4*x - x^2]]/3) - 2*ArcTanh[x/Sq
rt[-3 - 4*x - x^2]])/2
```

3.128. $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.128.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

3.128. $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

rule 1361
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[-(2*h*d - g*e)/e   Int[1/((a +
b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] + Simp[h/e   Int[(2*d + e*x)/((
a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e,
0] && NeQ[2*h*d - g*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 2143
```
Int[(Px_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_
.)*(x_)^2]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 1], C
 = Coeff[Px, x, 2]}, Simp[C/c   Int[1/Sqrt[d + e*x + f*x^2], x], x] + Simp[
1/c   Int[(A*c - a*C + (B*c - b*C)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x
^2]), x], x]] /; FreeQ[{a, b, c, d, e, f}, x] && PolyQ[Px, x, 2]
```

3.128. $\int \frac{x^2}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.128.4 Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.33

method	result
default	$\frac{\arcsin(2+x)}{2}-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)-\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\ln\left(-\frac{16\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)^2 x+8\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)x+24\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)+6\sqrt{-x^2-4x-3}-3x-6}{4\,\mathrm{RootOf}\left(16\_Z^2-8\_Z+3\right)x-3x-3}\right)}{2}-\ln\Big($

input
```
int(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(2+x)-1/12*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*
arctan(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-arctanh(3*x/(-3/2-x)/(3*x^
2/(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/
(-3/2-x))
```

3.128.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.64

$$\begin{aligned}\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx &= -\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\sqrt{2}\sqrt{-x^2-4x-3}}{2\,(2x+3)}\right)\\ &\quad-\frac{1}{4}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\sqrt{2}\sqrt{-x^2-4x-3}}{2\,(2x+3)}\right)\\ &\quad-\frac{1}{2}\arctan\left(\frac{\sqrt{-x^2-4x-3}(x+2)}{x^2+4x+3}\right)\\ &\quad+\frac{1}{8}\log\left(-\frac{2\sqrt{-x^2-4x-3}x+4x+3}{x^2}\right)\\ &\quad-\frac{1}{8}\log\left(\frac{2\sqrt{-x^2-4x-3}x-4x-3}{x^2}\right)\end{aligned}$$

input
```
integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x
+ 3)) - 1/4*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3
))/(2*x + 3)) - 1/2*arctan(sqrt(-x^2 - 4*x - 3)*(x + 2)/(x^2 + 4*x + 3)) +
 1/8*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) - 1/8*log((2*sqrt(-x^2
 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.128.6 Sympy [F]

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(x**2/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.128.7 Maxima [F]

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(82) = 164.

Time = 0.29 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.74

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) - \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) + \frac{1}{2}\,\arcsin\left(x+2\right) - \frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) + \frac{1}{4}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)$$

input
```
integrate(x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 1/2*arcsin(x + 2) - 1/4*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*
(sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 1/4*log(2*(sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.128.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x^2}{\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

input
```
int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(x^2/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.129 $\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)} dx$

3.129.1 Optimal result

Integrand size = 28, antiderivative size = 68

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{1-\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}} + \frac{\arctan\left(\frac{1+\frac{3\sqrt{-1-x}}{\sqrt{3+x}}}{\sqrt{2}}\right)}{\sqrt{2}}$$

output
```
-1/2*arctan(1/2*(1-3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)+1/2*arctan
(1/2*(1+3*(-1-x)^(1/2)/(3+x)^(1/2))*2^(1/2))*2^(1/2)
```

3.129.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)}{\sqrt{2}}$$

input
```
Integrate[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-(ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]/Sqrt[2])
```

3.129.3 Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.50, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1359, 27, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

$$\Big\downarrow \ 1359$$

$$8\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \ 27$$

$$-2\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \ 1475$$

$$2\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}+\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x}\right.$$

$$\Big\downarrow \ 1083$$

$$2\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}+\frac{2}{3}\right)\right)$$

$$\Big\downarrow \ 217$$

$$-\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)$$

input
```
Int[x/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-(Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[-3 - 4*x - x^2])])
```

3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.129. $\int \frac{x}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.129.4 Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.69 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result	size
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+2\right)\ln\left(\frac{6\operatorname{RootOf}\left(\_Z^2+2\right)x^2+20\operatorname{RootOf}\left(\_Z^2+2\right)x+8x\sqrt{-x^2-4x-3}+15\operatorname{RootOf}\left(\_Z^2+2\right)+12\sqrt{-x^2-4x-3}}{2x^2+4x+3}\right)}{4}$	81
default	$\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)}{12\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$	92

input
```
int(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*RootOf(_Z^2+2)*ln((6*RootOf(_Z^2+2)*x^2+20*RootOf(_Z^2+2)*x+8*x*(-x^2-
4*x-3)^(1/2)+15*RootOf(_Z^2+2)+12*(-x^2-4*x-3)^(1/2))/(2*x^2+4*x+3))
```

3.129.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}(6\,x^2+20\,x+15)\sqrt{-x^2-4\,x-3}}{4\,(2\,x^3+11\,x^2+18\,x+9)}\right)$$

input
```
integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*sqrt(2)*(6*x^2 + 20*x + 15)*sqrt(-x^2 - 4*x - 3)/(2
*x^3 + 11*x^2 + 18*x + 9))
```

3.129.6 Sympy [F]

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.129.7 Maxima [F]

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.129.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00

$$\begin{aligned}\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx &= \frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\,(\sqrt{-x^2-4\,x-3}-1)}{x+2}+1\right)\right)\\&\quad+\frac{1}{2}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right)\end{aligned}$$

input
```
integrate(x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 1/2*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
)
```

3.129.9 Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{x}{\sqrt{-x^2-4\,x-3}\,(2\,x^2+4\,x+3)}\,dx$$

input
```
int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(x/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.130 $\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.130.1 Optimal result

Integrand size = 27, antiderivative size = 95

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
1/3*arctanh(x/(-x^2-4*x-3)^(1/2))-1/3*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(1
/2))*2^(1/2))*2^(1/2)+1/3*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2))
*2^(1/2)
```

3.130.2 Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{3}\left(\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right) + \text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

input
```
Integrate[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])] + ArcTanh[x/Sqrt
[-3 - 4*x - x^2]])/3
```

3.130.3 Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.61, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1317, 27, 1359, 27, 1360, 219, 1475, 1083, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1317}$$

$$\frac{1}{6}\int -\frac{4x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{1}{6}\int -\frac{2(2x+3)}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{2}{3}\int \frac{x}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx$$

$$\Big\downarrow \text{1359}$$

$$\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx-\frac{16}{3}\int -\frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{4\left(\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1\right)}d\frac{x+3}{3\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{2x+3}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}dx+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \text{1360}$$

$$\int \frac{1}{3-\frac{3x^2}{-x^2-4x-3}}d\frac{x}{\sqrt{-x^2-4x-3}}+\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{4}{3}\int \frac{\frac{(x+3)^2}{3(-x^2-4x-3)}+1}{\frac{(x+3)^4}{9(-x^2-4x-3)^2}+\frac{2(x+3)^2}{9(-x^2-4x-3)}+1}d\frac{x+3}{3\sqrt{-x^2-4x-3}}+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

$$\Big\downarrow \text{ 1475}$$

$$\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$\frac{4}{3}\left(-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x-3}}-\frac{1}{6}\int \frac{1}{\frac{(x+3)^2}{9(-x^2-4x-3)}+\frac{2(x+3)}{9\sqrt{-x^2-4x-3}}+\frac{1}{3}}d\frac{x+3}{3\sqrt{-x^2-4x}}\right.$$

$$\Big\downarrow \text{ 1083}$$

$$\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)-$$
$$\frac{4}{3}\left(\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}-\frac{2}{3}\right)+\frac{1}{3}\int \frac{1}{-\frac{(x+3)^2}{9(-x^2-4x-3)}-\frac{8}{9}}d\left(\frac{2(x+3)}{3\sqrt{-x^2-4x-3}}+\frac{2}{3}\right)\right)$$

$$\Big\downarrow \text{ 217}$$

$$\frac{2}{3}\sqrt{2}\arctan\left(\frac{x+3}{2\sqrt{2}\sqrt{-x^2-4x-3}}\right)+\frac{1}{3}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

input
```
Int[1/(Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(2*Sqrt[2]*ArcTan[(3 + x)/(2*Sqrt[2]*Sqrt[-3 - 4*x - x^2])])/3 + ArcTanh[x
/Sqrt[-3 - 4*x - x^2]]/3
```

3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1317
```
Int[1/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_.) + (e_.)*(x_) + (f_.)*
(x_)^2]), x_Symbol] :> With[{q = Rt[(c*d - a*f)^2 - (b*d - a*e)*(c*e - b*f)
, 2]}, Simp[1/(2*q)   Int[(c*d - a*f + q + (c*e - b*f)*x)/((a + b*x + c*x^2
)*Sqrt[d + e*x + f*x^2]), x], x] - Simp[1/(2*q)   Int[(c*d - a*f - q + (c*e
 - b*f)*x)/((a + b*x + c*x^2)*Sqrt[d + e*x + f*x^2]), x], x]] /; FreeQ[{a,
b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && NeQ[c*e
 - b*f, 0] && NegQ[b^2 - 4*a*c]
```

rule 1359
```
Int[(x_)/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_.)*(x_) + (f_.)
*(x_)^2]), x_Symbol] :> Simp[-2*e   Subst[Int[(1 - d*x^2)/(c*e - b*f - e*(2
*c*d - b*e + 2*a*f)*x^2 + d^2*(c*e - b*f)*x^4), x], x, (1 + (e + Sqrt[e^2 -
 4*d*f])*(x/(2*d)))/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0]
```

rule 1360
```
Int[((g_) + (h_.)*(x_))/(((a_) + (b_.)*(x_) + (c_.)*(x_)^2)*Sqrt[(d_) + (e_
.)*(x_) + (f_.)*(x_)^2]), x_Symbol] :> Simp[g   Subst[Int[1/(a + (c*d - a*f
)*x^2), x], x, x/Sqrt[d + e*x + f*x^2]], x] /; FreeQ[{a, b, c, d, e, f, g,
h}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[e^2 - 4*d*f, 0] && EqQ[b*d - a*e, 0] &
& EqQ[2*h*d - g*e, 0]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

3.130. $\int \frac{1}{\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.130.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.27

method	result
default	$-\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+\operatorname{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\right)\right)}{18\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\ln\left(\frac{36\operatorname{RootOf}\left(12\_Z^2-4\_Z+1\right)^2x-36\operatorname{RootOf}\left(12\_Z^2-4\_Z+1\right)x-36\operatorname{RootOf}\left(12\_Z^2-4\_Z+1\right)-6\sqrt{-x^2-4x-3}+5x+6}{6\operatorname{RootOf}\left(12\_Z^2-4\_Z+1\right)x+x+3}\right)}{3}-\ln\Big($

input
```
int(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/18*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x
^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))+arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12
)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

3.130.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.39

$$\begin{aligned}\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = &-\frac{1}{6}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)\\ &-\frac{1}{6}\,\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)\\ &-\frac{1}{12}\,\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right)\\ &+\frac{1}{12}\,\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)\end{aligned}$$

input
```
integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/6*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3))/(2*x
+ 3)) - 1/6*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 - 4*x - 3
))/(2*x + 3)) - 1/12*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2) + 1/12
*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.130.6 Sympy [F]

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(1/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(1/(sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.130.7 Maxima [F]

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}}\,dx$$

input
```
integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)), x)
```

3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(76) = 152$.

Time = 0.29 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

$$\begin{aligned}
&\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad -\frac{1}{3}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4x-3}-1}{x+2}+1\right)\right) \\
&\quad +\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+1\right) \\
&\quad -\frac{1}{6}\log\left(\frac{2\left(\sqrt{-x^2-4x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4x-3}-1\right)^2}{(x+2)^2}+3\right)
\end{aligned}$$

input
```
integrate(1/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
-1/3*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 1/3*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1
)) + 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3
) - 1)^2/(x + 2)^2 + 1) - 1/6*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (
sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.130.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

input
```
int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(1/((- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.131 $\int \frac{1}{x\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.131.1 Optimal result

Integrand size = 30, antiderivative size = 130

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{\arctan\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{1}{9}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{4}{9}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
-4/9*arctanh(x/(-x^2-4*x-3)^(1/2))+1/9*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)^(
1/2))*2^(1/2))*2^(1/2)-1/9*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1/2)
)*2^(1/2)-1/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)
```

3.131.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{9}\left(-\sqrt{2}\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)+2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3-4x-x^2}}{3+x}\right)-4\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)\right)$$

input
```
Integrate[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(-(Sqrt[2]*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])]) + 2*Sqrt[3]*A
rcTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)] - 4*ArcTanh[x/Sqrt[-3 - 4*x
- x^2]])/9
```

3.131.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$

$$\Big\downarrow \; 7279$$

$$\int \left(\frac{1}{3x\sqrt{-x^2-4x-3}}-\frac{2(x+2)}{3\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\arctan\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{1}{9}\sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{1}{9}\sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)-\frac{4}{9}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)$$

input
```
Int[1/(x*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
-1/3*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x - x^2])]/Sqrt[3] + (Sqrt[2]*A
rcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (Sqrt[2]*ArcTan[(1
+ (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[2]])/9 - (4*ArcTanh[x/Sqrt[-3 - 4*x -
 x^2]])/9
```

3.131.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.131.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.17

method	result
default	$\frac{\sqrt{3}\,\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\,\sqrt{4}\,\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\,\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\,\sqrt{2}}{6}\right)+4\,\text{arctanh}\left(\frac{3x}{\left(-\frac{3}{2}-x\right)\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}}\cdots\right)\cdots\right)}{54\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\text{RootOf}\left(18\_Z^2+8\_Z+1\right)\ln\left(\frac{40500\,\text{RootOf}\left(18\_Z^2+8\_Z+1\right)^2 x-40500\,\text{RootOf}\left(18\_Z^2+8\_Z+1\right)^2+4680\sqrt{-\cdots}}{18\,\text{Roo}\cdots}\cdots\right)$

input
```
int(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/54*3^(1/2)*4
^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arctan(1/6*(3*x^2/(-3/2-x)^2-1
2)^(1/2)*2^(1/2))+4*arctanh(3*x/(-3/2-x)/(3*x^2/(-3/2-x)^2-12)^(1/2)))/((x
^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-3/2-x))
```

3.131.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.31

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{1}{9}\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4\,x-3}(2\,x+3)}{3\,(x^2+4\,x+3)}\right) + \frac{1}{18}\sqrt{2}\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{18}\sqrt{2}\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right) + \frac{1}{9}\log\left(-\frac{2\,\sqrt{-x^2-4\,x-3}x+4\,x+3}{x^2}\right) - \frac{1}{9}\log\left(\frac{2\,\sqrt{-x^2-4\,x-3}x-4\,x-3}{x^2}\right)$$

input
```
integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
1/9*sqrt(3)*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2 + 4*x +
 3)) + 1/18*sqrt(2)*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4*x - 3)
)/(2*x + 3)) + 1/18*sqrt(2)*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(-x^2 -
 4*x - 3))/(2*x + 3)) + 1/9*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3)/x^2)
 - 1/9*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2)
```

3.131.6 Sympy [F]

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x\sqrt{-(x+1)(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(1/x/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(1/(x*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.131.7 Maxima [F]

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}x}\,dx$$

input
```
integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x), x)
```

3.131.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.53

$$\begin{aligned}
&\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad + \frac{1}{9}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) \\
&\quad - \frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) \\
&\quad + \frac{2}{9}\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)
\end{aligned}$$

input
```
integrate(1/x/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
1/9*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1))
 + 2/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
1)) + 1/9*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
 1)) - 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x -
 3) - 1)^2/(x + 2)^2 + 1) + 2/9*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
 (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx=\int \frac{1}{x\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

input
```
int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(1/(x*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.132 $\int \frac{1}{x^2\sqrt{-3-4x-x^2}(3+4x+2x^2)}\,dx$

3.132.1 Optimal result

Integrand size = 30, antiderivative size = 151

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \frac{\sqrt{-3-4x-x^2}}{9x} + \frac{2\arctan\left(\frac{3+2x}{\sqrt{3}\sqrt{-3-4x-x^2}}\right)}{3\sqrt{3}} + \frac{2}{27}\sqrt{2}\arctan\left(\frac{1-\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) - \frac{2}{27}\sqrt{2}\arctan\left(\frac{1+\frac{3+x}{\sqrt{-3-4x-x^2}}}{\sqrt{2}}\right) + \frac{10}{27}\text{arctanh}\left(\frac{x}{\sqrt{-3-4x-x^2}}\right)$$

output
```
10/27*arctanh(x/(-x^2-4*x-3)^(1/2))+2/27*arctan(1/2*(1+(-3-x)/(-x^2-4*x-3)
^(1/2))*2^(1/2))*2^(1/2)-2/27*arctan(1/2*(1+(3+x)/(-x^2-4*x-3)^(1/2))*2^(1
/2))*2^(1/2)+2/9*arctan(1/3*(3+2*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))*3^(1/2)+1/
9*(-x^2-4*x-3)^(1/2)/x
```

3.132.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx$$
$$= \frac{-2\sqrt{2}x\arctan\left(\frac{3+2x}{\sqrt{2}\sqrt{-3-4x-x^2}}\right)+3\left(\sqrt{-3-4x-x^2}-4\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-3-4x-x^2}}{3+x}\right)\right)+10x\text{arctanh}\left(\frac{}{\sqrt{-3-}}\right.}{27x}$$

input
```
Integrate[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
(-2*Sqrt[2]*x*ArcTan[(3 + 2*x)/(Sqrt[2]*Sqrt[-3 - 4*x - x^2])] + 3*(Sqrt[-
3 - 4*x - x^2] - 4*Sqrt[3]*x*ArcTan[(Sqrt[3]*Sqrt[-3 - 4*x - x^2])/(3 + x)
]) + 10*x*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/(27*x)
```

3.132.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{2(4x+5)}{9\sqrt{-x^2-4x-3}\,(2x^2+4x+3)}-\frac{4}{9x\sqrt{-x^2-4x-3}}+\frac{1}{3x^2\sqrt{-x^2-4x-3}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2\arctan\left(\frac{2x+3}{\sqrt{3}\sqrt{-x^2-4x-3}}\right)}{3\sqrt{3}}+\frac{2}{27}\sqrt{2}\arctan\left(\frac{1-\frac{x+3}{\sqrt{-x^2-4x-3}}}{\sqrt{2}}\right)-\frac{2}{27}\sqrt{2}\arctan\left(\frac{\frac{x+3}{\sqrt{-x^2-4x-3}}+1}{\sqrt{2}}\right)+\frac{10}{27}\text{arctanh}\left(\frac{x}{\sqrt{-x^2-4x-3}}\right)+\frac{\sqrt{-x^2-4x-3}}{9x}$$

input
```
Int[1/(x^2*Sqrt[-3 - 4*x - x^2]*(3 + 4*x + 2*x^2)),x]
```

output
```
Sqrt[-3 - 4*x - x^2]/(9*x) + (2*ArcTan[(3 + 2*x)/(Sqrt[3]*Sqrt[-3 - 4*x -
x^2])])/(3*Sqrt[3]) + (2*Sqrt[2]*ArcTan[(1 - (3 + x)/Sqrt[-3 - 4*x - x^2])
/Sqrt[2]])/27 - (2*Sqrt[2]*ArcTan[(1 + (3 + x)/Sqrt[-3 - 4*x - x^2])/Sqrt[
2]])/27 + (10*ArcTanh[x/Sqrt[-3 - 4*x - x^2]])/27
```

3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

3.132.4 Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.12

method	result
default	$-\frac{2\sqrt{3}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{-x^2-4x-3}}{9x}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)-5\operatorname{arctanh}\left(\cdots\right)\cdots\right)}{81\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
risch	$-\frac{x^2+4x+3}{9x\sqrt{-x^2-4x-3}}-\frac{2\sqrt{3}\arctan\left(\frac{(-6-4x)\sqrt{3}}{6\sqrt{-x^2-4x-3}}\right)}{9}+\frac{\sqrt{3}\sqrt{4}\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\left(\sqrt{2}\arctan\left(\frac{\sqrt{\frac{3x^2}{\left(-\frac{3}{2}-x\right)^2}-12}\sqrt{2}}{6}\right)-5\operatorname{arctanh}\left(\cdots\right)\cdots\right)}{81\sqrt{\frac{\frac{x^2}{\left(-\frac{3}{2}-x\right)^2}-4}{\left(1+\frac{x}{-\frac{3}{2}-x}\right)^2}}\left(1+\frac{x}{-\frac{3}{2}-x}\right)}$
trager	$\frac{\sqrt{-x^2-4x-3}}{9x}-\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(\frac{2\operatorname{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-x^2-4x-3}+3\operatorname{RootOf}\left(\_Z^2+3\right)}{x}\right)}{9}+\frac{10\ln\left(-\frac{-96000\operatorname{RootOf}\left(7\cdots\right.}{\cdots}\right)\cdots}{\cdots}$

input
```
int(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*3^(1/2)*arctan(1/6*(-6-4*x)*3^(1/2)/(-x^2-4*x-3)^(1/2))+1/9*(-x^2-4*x
-3)^(1/2)/x+1/81*3^(1/2)*4^(1/2)*(3*x^2/(-3/2-x)^2-12)^(1/2)*(2^(1/2)*arct
an(1/6*(3*x^2/(-3/2-x)^2-12)^(1/2)*2^(1/2))-5*arctanh(3*x/(-3/2-x)/(3*x^2/
(-3/2-x)^2-12)^(1/2)))/((x^2/(-3/2-x)^2-4)/(1+x/(-3/2-x))^2)^(1/2)/(1+x/(-
3/2-x))
```

3.132.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.28

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = -\frac{12\,\sqrt{3}x\arctan\left(\frac{\sqrt{3}\sqrt{-x^2-4\,x-3}(2\,x+3)}{3\,(x^2+4\,x+3)}\right)-2\,\sqrt{2}x\arctan\left(\frac{\sqrt{2}x+3\,\sqrt{2}\sqrt{-x^2-4\,x-3}}{2\,(2\,x+3)}\right)-2\,\sqrt{2}x\arctan\left(-\frac{\sqrt{2}x-3\,\sqrt{2}\cdots}{2\,(2\cdots}\right.\cdots}{54\,x}$$

input
```
integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="fricas")
```

output
```
-1/54*(12*sqrt(3)*x*arctan(1/3*sqrt(3)*sqrt(-x^2 - 4*x - 3)*(2*x + 3)/(x^2
 + 4*x + 3)) - 2*sqrt(2)*x*arctan(1/2*(sqrt(2)*x + 3*sqrt(2)*sqrt(-x^2 - 4
*x - 3))/(2*x + 3)) - 2*sqrt(2)*x*arctan(-1/2*(sqrt(2)*x - 3*sqrt(2)*sqrt(
-x^2 - 4*x - 3))/(2*x + 3)) + 5*x*log(-(2*sqrt(-x^2 - 4*x - 3)*x + 4*x + 3
)/x^2) - 5*x*log((2*sqrt(-x^2 - 4*x - 3)*x - 4*x - 3)/x^2) - 6*sqrt(-x^2 -
 4*x - 3))/x
```

3.132.6 Sympy [F]

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x^2\sqrt{-(x+1)\,(x+3)}\,(2x^2+4x+3)}\,dx$$

input
```
integrate(1/x**2/(2*x**2+4*x+3)/(-x**2-4*x-3)**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(-(x + 1)*(x + 3))*(2*x**2 + 4*x + 3)), x)
```

3.132.7 Maxima [F]

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{(2\,x^2+4\,x+3)\sqrt{-x^2-4\,x-3}x^2}\,dx$$

input
```
integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((2*x^2 + 4*x + 3)*sqrt(-x^2 - 4*x - 3)*x^2), x)
```

3.132.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(121) = 242.

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx \\
&= \frac{2}{27}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad -\frac{4}{9}\,\sqrt{3}\arctan\left(\frac{1}{3}\,\sqrt{3}\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+1\right)\right) \\
&\quad +\frac{2}{27}\,\sqrt{2}\arctan\left(\frac{1}{2}\,\sqrt{2}\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+1\right)\right) \\
&\quad -\frac{\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+2}{18\left(\frac{\sqrt{-x^2-4\,x-3}-1}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right)} \\
&\quad +\frac{5}{27}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{3\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+1\right) \\
&\quad -\frac{5}{27}\,\log\left(\frac{2\left(\sqrt{-x^2-4\,x-3}-1\right)}{x+2}+\frac{\left(\sqrt{-x^2-4\,x-3}-1\right)^2}{(x+2)^2}+3\right)
\end{aligned}$$

input
```
integrate(1/x^2/(2*x^2+4*x+3)/(-x^2-4*x-3)^(1/2),x, algorithm="giac")
```

output
```
2/27*sqrt(2)*arctan(1/2*sqrt(2)*(3*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 1)
) - 4/9*sqrt(3)*arctan(1/3*sqrt(3)*(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) +
 1)) + 2/27*sqrt(2)*arctan(1/2*sqrt(2)*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2)
 + 1)) - 1/18*((sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 2)/((sqrt(-x^2 - 4*x -
 3) - 1)/(x + 2) + (sqrt(-x^2 - 4*x - 3) - 1)^2/(x + 2)^2 + 1) + 5/27*log(
2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + 3*(sqrt(-x^2 - 4*x - 3) - 1)^2/(x +
 2)^2 + 1) - 5/27*log(2*(sqrt(-x^2 - 4*x - 3) - 1)/(x + 2) + (sqrt(-x^2 -
4*x - 3) - 1)^2/(x + 2)^2 + 3)
```

3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{-3-4x-x^2}\,(3+4x+2x^2)}\,dx = \int \frac{1}{x^2\,\sqrt{-x^2-4\,x-3}\,\left(2\,x^2+4\,x+3\right)}\,dx$$

input
```
int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)),x)
```

output
```
int(1/(x^2*(- 4*x - x^2 - 3)^(1/2)*(4*x + 2*x^2 + 3)), x)
```

3.133 $\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$

3.133.1 Optimal result

Integrand size = 34, antiderivative size = 149

$$\begin{aligned}
&\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx \\
&= \frac{125455(17+24x)\sqrt{6+17x+12x^2}}{150994944} - \frac{125455(17+24x)\left(6+17x+12x^2\right)^{3/2}}{4718592} \\
&\quad + \frac{25091(17+24x)\left(6+17x+12x^2\right)^{5/2}}{24576} - \frac{873\left(6+17x+12x^2\right)^{7/2}}{1792} \\
&\quad - \frac{1}{32}(10-3x)\left(6+17x+12x^2\right)^{7/2} - \frac{125455\,\text{arctanh}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{603979776\sqrt{3}}
\end{aligned}$$

output
```
-125455/4718592*(17+24*x)*(12*x^2+17*x+6)^(3/2)+25091/24576*(17+24*x)*(12*
x^2+17*x+6)^(5/2)-873/1792*(12*x^2+17*x+6)^(7/2)-1/32*(10-3*x)*(12*x^2+17*
x+6)^(7/2)-125455/1811939328*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6
)^(1/2))*3^(1/2)+125455/150994944*(17+24*x)*(12*x^2+17*x+6)^(1/2)
```

3.133.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.60

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$

$$= \frac{6\sqrt{6+17x+12x^2}(474999091769+3132157281976x+7899203409792x^2+8974844476416x^3+3438453}{6341}$$

input
```
Integrate[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

output
```
(6*Sqrt[6 + 17*x + 12*x^2]*(474999091769 + 3132157281976*x + 7899203409792
*x^2 + 8974844476416*x^3 + 3438453030912*x^4 - 1190083166208*x^5 - 7328162
11968*x^6 + 171228266496*x^7) - 878185*Sqrt[3]*ArcTanh[(2*Sqrt[2 + (17*x)/
3 + 4*x^2])/(3 + 4*x)])/6341787648
```

3.133.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.265$, Rules used = {1335, 1166, 27, 1160, 1087, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2)^2 \left(-12x^2+31x+30\right)^2 \sqrt{12x^2+17x+6}\,dx$$

$$\Big\downarrow \; 1335$$

$$\int (10-3x)^2 \left(12x^2+17x+6\right)^{5/2}dx$$

$$\Big\downarrow \; 1166$$

$$\frac{1}{96}\int \frac{9}{2}(2518-873x)\left(12x^2+17x+6\right)^{5/2}dx-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$$\Big\downarrow \; 27$$

$$\frac{3}{64}\int (2518-873x)\left(12x^2+17x+6\right)^{5/2}dx-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$$\Big\downarrow \; 1160$$

$$\frac{3}{64}\left(\frac{25091}{8}\int \left(12x^2+17x+6\right)^{5/2}dx-\frac{291}{28}\left(12x^2+17x+6\right)^{7/2}\right)-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$\big\downarrow$ 1087

$$\frac{3}{64}\left(\frac{25091}{8}\left(\frac{1}{144}(24x+17)\left(12x^2+17x+6\right)^{5/2}-\frac{5}{288}\int \left(12x^2+17x+6\right)^{3/2}dx\right)-\frac{291}{28}\left(12x^2+17x+6\right)^{7/2}\right)-\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$\big\downarrow$ 1087

$$\frac{3}{64}\left(\frac{25091}{8}\left(\frac{1}{144}(24x+17)\left(12x^2+17x+6\right)^{5/2}-\frac{5}{288}\left(\frac{1}{96}(24x+17)\left(12x^2+17x+6\right)^{3/2}-\frac{1}{64}\int \sqrt{12x^2+17x}\right.\right.\right.$$
$$\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$\big\downarrow$ 1087

$$\frac{3}{64}\left(\frac{25091}{8}\left(\frac{1}{144}(24x+17)\left(12x^2+17x+6\right)^{5/2}-\frac{5}{288}\left(\frac{1}{64}\left(\frac{1}{96}\int \frac{1}{\sqrt{12x^2+17x+6}}dx-\frac{1}{48}(24x+17)\sqrt{12x^2+}\right.\right.\right.\right.$$
$$\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$\big\downarrow$ 1092

$$\frac{3}{64}\left(\frac{25091}{8}\left(\frac{1}{144}(24x+17)\left(12x^2+17x+6\right)^{5/2}-\frac{5}{288}\left(\frac{1}{64}\left(\frac{1}{48}\int \frac{1}{48-\frac{(24x+17)^2}{12x^2+17x+6}}d\frac{24x+17}{\sqrt{12x^2+17x+6}}-\frac{1}{48}(24x\right.\right.\right.\right.$$
$$\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

$\big\downarrow$ 219

$$\frac{3}{64}\left(\frac{25091}{8}\left(\frac{1}{144}(24x+17)\left(12x^2+17x+6\right)^{5/2}-\frac{5}{288}\left(\frac{1}{64}\left(\frac{\operatorname{arctanh}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{192\sqrt{3}}-\frac{1}{48}(24x+17)\sqrt{12x^2}\right.\right.\right.\right.$$
$$\frac{1}{32}(10-3x)\left(12x^2+17x+6\right)^{7/2}$$

input
```
Int[(2 + 3*x)^2*(30 + 31*x - 12*x^2)^2*Sqrt[6 + 17*x + 12*x^2],x]
```

3.133. $\int (2+3x)^2\left(30+31x-12x^2\right)^2\sqrt{6+17x+12x^2}\,dx$

output
```
-1/32*((10 - 3*x)*(6 + 17*x + 12*x^2)^(7/2)) + (3*((-291*(6 + 17*x + 12*x^
2)^(7/2))/28 + (25091*(((17 + 24*x)*(6 + 17*x + 12*x^2)^(5/2))/144 - (5*((
(17 + 24*x)*(6 + 17*x + 12*x^2)^(3/2))/96 + (-1/48*((17 + 24*x)*Sqrt[6 + 1
7*x + 12*x^2]) + ArcTanh[(17 + 24*x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])]/
(192*Sqrt[3]))/64))/288))/8))/64
```

3.133.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1335
```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

3.133.4 Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.54

method	result
risch	$\frac{(171228266496x^7-732816211968x^6-1190083166208x^5+3438453030912x^4+8974844476416x^3+7899203409792x^2+3132157281976}{1056964608}$
trager	$\left(162x^7-\frac{19413}{28}x^6-\frac{504423}{448}x^5+\frac{11659251}{3584}x^4+\frac{139118993}{16384}x^3+\frac{20570842213}{2752512}x^2+\frac{391519660247}{132120576}x+\frac{47499909176}{1056964608}\right.$
default	$\frac{125455(17+24x)\sqrt{12x^2+17x+6}}{150994944}-\frac{125455\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{3623878656}+\frac{2473875847(12x^2+17x+6)^{\frac{3}{2}}}{33030144}+\frac{27x^5(12}{}$

input
```
int((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x,method=_RETURNVE
RBOSE)
```

output
```
1/1056964608*(171228266496*x^7-732816211968*x^6-1190083166208*x^5+34384530
30912*x^4+8974844476416*x^3+7899203409792*x^2+3132157281976*x+474999091769
)*(12*x^2+17*x+6)^(1/2)-125455/3623878656*ln(1/12*(17/2+12*x)*12^(1/2)+(12
*x^2+17*x+6)^(1/2))*12^(1/2)
```

3.133.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.59

$$\int (2+3x)^2\left(30+31x-12x^2\right)^2\sqrt{6+17x+12x^2}\,dx$$
$$=\frac{1}{1056964608}\left(171228266496\,x^7-732816211968\,x^6-1190083166208\,x^5+3438453030912\,x^4+897484447\right.$$
$$+\frac{125455}{3623878656}\sqrt{3}\log\left(-8\sqrt{3}\sqrt{12\,x^2+17\,x+6}(24\,x+17)+1152\,x^2+1632\,x+577\right)$$

input
```
integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm
="fricas")
```

output
```
1/1056964608*(171228266496*x^7 - 732816211968*x^6 - 1190083166208*x^5 + 34
38453030912*x^4 + 8974844476416*x^3 + 7899203409792*x^2 + 3132157281976*x
+ 474999091769)*sqrt(12*x^2 + 17*x + 6) + 125455/3623878656*sqrt(3)*log(-8
*sqrt(3)*sqrt(12*x^2 + 17*x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

3.133.6 Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.64

$$\begin{aligned}\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx &= \sqrt{12x^2+17x+6}\cdot\left(162x^7-\frac{19413x^6}{28}-\frac{504423x^5}{448}+\frac{11659251x^4}{3584}+\frac{139118993x^3}{16384}\right.\\ &\quad\left.+\frac{20570842213x^2}{2752512}+\frac{391519660247x}{132120576}+\frac{474999091769}{1056964608}\right)\\ &\quad-\frac{125455\sqrt{3}\log\left(24x+4\sqrt{3}\sqrt{12x^2+17x+6}+17\right)}{1811939328}\end{aligned}$$

input
```
integrate((2+3*x)**2*(-12*x**2+31*x+30)**2*(12*x**2+17*x+6)**(1/2),x)
```

output
```
sqrt(12*x**2 + 17*x + 6)*(162*x**7 - 19413*x**6/28 - 504423*x**5/448 + 116
59251*x**4/3584 + 139118993*x**3/16384 + 20570842213*x**2/2752512 + 391519
660247*x/132120576 + 474999091769/1056964608) - 125455*sqrt(3)*log(24*x +
4*sqrt(3)*sqrt(12*x**2 + 17*x + 6) + 17)/1811939328
```

3.133.7 Maxima [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.04

$$\begin{aligned}\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx &= \frac{27}{2}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^5-\frac{8613}{112}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^4+\frac{14991}{1792}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^3\\ &\quad+\frac{4267751}{14336}\left(12x^2+17x+6\right)^{\frac{3}{2}}x^2+\frac{129220757}{458752}\left(12x^2+17x+6\right)^{\frac{3}{2}}x\\ &\quad+\frac{2473875847}{33030144}\left(12x^2+17x+6\right)^{\frac{3}{2}}+\frac{125455}{6291456}\sqrt{12x^2+17x+6}x\\ &\quad-\frac{125455}{1811939328}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12x^2+17x+6}+24x+17\right)\\ &\quad+\frac{2132735}{150994944}\sqrt{12x^2+17x+6}\end{aligned}$$

input
```
integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm
="maxima")
```

output
```
27/2*(12*x^2 + 17*x + 6)^(3/2)*x^5 - 8613/112*(12*x^2 + 17*x + 6)^(3/2)*x^
4 + 14991/1792*(12*x^2 + 17*x + 6)^(3/2)*x^3 + 4267751/14336*(12*x^2 + 17*
x + 6)^(3/2)*x^2 + 129220757/458752*(12*x^2 + 17*x + 6)^(3/2)*x + 24738758
47/33030144*(12*x^2 + 17*x + 6)^(3/2) + 125455/6291456*sqrt(12*x^2 + 17*x
+ 6)*x - 125455/1811939328*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) +
 24*x + 17) + 2132735/150994944*sqrt(12*x^2 + 17*x + 6)
```

3.133.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.57

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= \frac{1}{1056964608} (8 (48 (24 (96 (24 (48 (168 x - 719)x - 56047)x + 3886417)x + 973832951)x + 20570842213)$$
$$+ \frac{125455}{1811939328} \sqrt{3} \log \left(\left| -4 \sqrt{3} \left(2 \sqrt{3}x - \sqrt{12 x^2 + 17 x + 6} \right) - 17 \right| \right)$$

input
```
integrate((2+3*x)^2*(-12*x^2+31*x+30)^2*(12*x^2+17*x+6)^(1/2),x, algorithm
="giac")
```

output
```
1/1056964608*(8*(48*(24*(96*(24*(48*(168*x - 719)*x - 56047)*x + 3886417)*
x + 973832951)*x + 20570842213)*x + 391519660247)*x + 474999091769)*sqrt(1
2*x^2 + 17*x + 6) + 125455/1811939328*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3
)*x - sqrt(12*x^2 + 17*x + 6)) - 17))
```

3.133.9 Mupad [B] (verification not implemented)

Time = 13.92 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.26

$$\int (2+3x)^2 \left(30+31x-12x^2\right)^2 \sqrt{6+17x+12x^2}\,dx$$
$$= \frac{4267751\,x^2\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{14336}+\frac{14991\,x^3\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{1792}$$
$$-\frac{8613\,x^4\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{112}+\frac{27\,x^5\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{2}$$
$$-\frac{146030443\,\sqrt{12}\,\ln\left(\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\,\left(12\,x+\frac{17}{2}\right)}{12}\right)}{88080384}$$
$$+\frac{438091329\,\left(\frac{x}{2}+\frac{17}{48}\right)\,\sqrt{12\,x^2+17\,x+6}}{229376}$$
$$+\frac{2473875847\,\sqrt{12\,x^2+17\,x+6}\,\left(1152\,x^2+408\,x-291\right)}{3170893824}$$
$$+\frac{129220757\,x\,{\left(12\,x^2+17\,x+6\right)}^{3/2}}{458752}$$
$$+\frac{42055889399\,\sqrt{12}\,\ln\left(2\,\sqrt{12\,x^2+17\,x+6}+\frac{\sqrt{12}\,\left(24\,x+17\right)}{12}\right)}{25367150592}$$

input
```
int((3*x + 2)^2*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30)^2,x)
```

output
```
(4267751*x^2*(17*x + 12*x^2 + 6)^(3/2))/14336 + (14991*x^3*(17*x + 12*x^2
+ 6)^(3/2))/1792 - (8613*x^4*(17*x + 12*x^2 + 6)^(3/2))/112 + (27*x^5*(17*
x + 12*x^2 + 6)^(3/2))/2 - (146030443*12^(1/2)*log((17*x + 12*x^2 + 6)^(1/
2) + (12^(1/2)*(12*x + 17/2))/12))/88080384 + (438091329*(x/2 + 17/48)*(17
*x + 12*x^2 + 6)^(1/2))/229376 + (2473875847*(17*x + 12*x^2 + 6)^(1/2)*(40
8*x + 1152*x^2 - 291))/3170893824 + (129220757*x*(17*x + 12*x^2 + 6)^(3/2)
)/458752 + (42055889399*12^(1/2)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/
2)*(24*x + 17))/12))/25367150592
```

3.134 $\int (2+3x)(30+31x-12x^2)\sqrt{6+17x+12x^2}\,dx$

3.134.1 Optimal result

Integrand size = 30, antiderivative size = 103

$$\begin{aligned}&\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx\\&=-\frac{97(17+24x)\sqrt{6+17x+12x^2}}{24576}+\frac{97}{768}(17+24x)\left(6+17x+12x^2\right)^{3/2}\\&\quad-\frac{1}{20}\left(6+17x+12x^2\right)^{5/2}+\frac{97\text{arctanh}\left(\frac{17+24x}{4\sqrt{3}\sqrt{6+17x+12x^2}}\right)}{98304\sqrt{3}}\end{aligned}$$

output
```
97/768*(17+24*x)*(12*x^2+17*x+6)^(3/2)-1/20*(12*x^2+17*x+6)^(5/2)+97/29491
2*arctanh(1/12*(17+24*x)*3^(1/2)/(12*x^2+17*x+6)^(1/2))*3^(1/2)-97/24576*(
17+24*x)*(12*x^2+17*x+6)^(1/2)
```

3.134.2 Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.72

$$\begin{aligned}&\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx\\&=\frac{6\sqrt{6+17x+12x^2}(1353611+5455144x+6837888x^2+1963008x^3-884736x^4)+485\sqrt{3}\text{arctanh}\left(\frac{2\sqrt{2+\cdots}}{3\cdots}\right.}{737280}\end{aligned}$$

input
```
Integrate[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

output
```
(6*Sqrt[6 + 17*x + 12*x^2]*(1353611 + 5455144*x + 6837888*x^2 + 1963008*x^
3 - 884736*x^4) + 485*Sqrt[3]*ArcTanh[(2*Sqrt[2 + (17*x)/3 + 4*x^2])/(3 +
4*x)])/737280
```

3.134.3 Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1335, 1160, 1087, 1087, 1092, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x+2) \left(-12x^2+31x+30\right) \sqrt{12x^2+17x+6} \, dx$$

$$\Big\downarrow \text{1335}$$

$$\int (10-3x) \left(12x^2+17x+6\right)^{3/2} dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{97}{8} \int \left(12x^2+17x+6\right)^{3/2} dx - \frac{1}{20} \left(12x^2+17x+6\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{97}{8} \left(\frac{1}{96}(24x+17) \left(12x^2+17x+6\right)^{3/2} - \frac{1}{64} \int \sqrt{12x^2+17x+6} dx\right) - \frac{1}{20} \left(12x^2+17x+6\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{97}{8} \left(\frac{1}{64} \left(\frac{1}{96} \int \frac{1}{\sqrt{12x^2+17x+6}} dx - \frac{1}{48}(24x+17)\sqrt{12x^2+17x+6}\right) + \frac{1}{96}(24x+17) \left(12x^2+17x+6\right)^{3/2}\right) - \frac{1}{20} \left(12x^2+17x+6\right)^{5/2}$$

$$\Big\downarrow \text{1092}$$

$$\frac{97}{8} \left(\frac{1}{64} \left(\frac{1}{48} \int \frac{1}{48-\frac{(24x+17)^2}{12x^2+17x+6}} d\frac{24x+17}{\sqrt{12x^2+17x+6}} - \frac{1}{48}(24x+17)\sqrt{12x^2+17x+6}\right) + \frac{1}{96}(24x+17) \left(12x^2+1\right.\right.$$
$$\frac{1}{20} \left(12x^2+17x+6\right)^{5/2}$$

$$\Big\downarrow \text{219}$$

$$\frac{97}{8}\left(\frac{1}{64}\left(\frac{\text{arctanh}\left(\frac{24x+17}{4\sqrt{3}\sqrt{12x^2+17x+6}}\right)}{192\sqrt{3}}-\frac{1}{48}(24x+17)\sqrt{12x^2+17x+6}\right)+\frac{1}{96}(24x+17)\left(12x^2+17x+6\right)^{3/2}\right)-\frac{1}{20}\left(12x^2+17x+6\right)^{5/2}$$

input
```
Int[(2 + 3*x)*(30 + 31*x - 12*x^2)*Sqrt[6 + 17*x + 12*x^2],x]
```

output
```
-1/20*(6 + 17*x + 12*x^2)^(5/2) + (97*(((17 + 24*x)*(6 + 17*x + 12*x^2)^(3
/2))/96 + (-1/48*((17 + 24*x)*Sqrt[6 + 17*x + 12*x^2]) + ArcTanh[(17 + 24*
x)/(4*Sqrt[3]*Sqrt[6 + 17*x + 12*x^2])]/(192*Sqrt[3]))/64))/8
```

3.134.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1092
```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[I
nt[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a
, b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1335
```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

3.134.4 Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.63

method	result
risch	$-\frac{(884736x^4-1963008x^3-6837888x^2-5455144x-1353611)\sqrt{12x^2+17x+6}}{122880}+\frac{97\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{589824}$
trager	$\left(-\frac{36}{5}x^4+\frac{639}{40}x^3+\frac{17807}{320}x^2+\frac{681893}{15360}x+\frac{1353611}{122880}\right)\sqrt{12x^2+17x+6}-\frac{97\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-24\,\mathrm{RootOf}\left(\_\right.\right.}{}$
default	$-\frac{97(17+24x)\sqrt{12x^2+17x+6}}{24576}+\frac{97\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12x^2+17x+6}\right)\sqrt{12}}{589824}+\frac{7093(12x^2+17x+6)^{\frac{3}{2}}}{3840}-\frac{3x^2(12x^2+17x+6)^{\frac{3}{2}}}{5}$

input
```
int((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x,method=_RETURNVERBOS
E)
```

output
```
-1/122880*(884736*x^4-1963008*x^3-6837888*x^2-5455144*x-1353611)*(12*x^2+1
7*x+6)^(1/2)+97/589824*ln(1/12*(17/2+12*x)*12^(1/2)+(12*x^2+17*x+6)^(1/2))
*12^(1/2)
```

3.134.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.71

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx =$$
$$-\frac{1}{122880}\left(884736\,x^4-1963008\,x^3-6837888\,x^2-5455144\,x-1353611\right)\sqrt{12\,x^2+17\,x+6}$$
$$+\frac{97}{589824}\sqrt{3}\log\left(8\sqrt{3}\sqrt{12\,x^2+17\,x+6}(24\,x+17)+1152\,x^2+1632\,x+577\right)$$

input
```
integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="fr
icas")
```

output
```
-1/122880*(884736*x^4 - 1963008*x^3 - 6837888*x^2 - 5455144*x - 1353611)*s
qrt(12*x^2 + 17*x + 6) + 97/589824*sqrt(3)*log(8*sqrt(3)*sqrt(12*x^2 + 17*
x + 6)*(24*x + 17) + 1152*x^2 + 1632*x + 577)
```

3.134.6 Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$=\sqrt{12x^2+17x+6}\left(-\frac{36x^4}{5}+\frac{639x^3}{40}+\frac{17807x^2}{320}+\frac{681893x}{15360}+\frac{1353611}{122880}\right)$$
$$+\frac{97\sqrt{3}\log\left(24x+4\sqrt{3}\sqrt{12x^2+17x+6}+17\right)}{294912}$$

input
```
integrate((2+3*x)*(-12*x**2+31*x+30)*(12*x**2+17*x+6)**(1/2),x)
```

output
```
sqrt(12*x**2 + 17*x + 6)*(-36*x**4/5 + 639*x**3/40 + 17807*x**2/320 + 6818
93*x/15360 + 1353611/122880) + 97*sqrt(3)*log(24*x + 4*sqrt(3)*sqrt(12*x**
2 + 17*x + 6) + 17)/294912
```

3.134.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.01

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$=-\frac{3}{5}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x^2+\frac{349}{160}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}x$$
$$+\frac{7093}{3840}\left(12\,x^2+17\,x+6\right)^{\frac{3}{2}}-\frac{97}{1024}\sqrt{12\,x^2+17\,x+6}x$$
$$+\frac{97}{294912}\sqrt{3}\log\left(4\sqrt{3}\sqrt{12\,x^2+17\,x+6}+24\,x+17\right)-\frac{1649}{24576}\sqrt{12\,x^2+17\,x+6}$$

input
```
integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="ma
xima")
```

output
```
-3/5*(12*x^2 + 17*x + 6)^(3/2)*x^2 + 349/160*(12*x^2 + 17*x + 6)^(3/2)*x +
 7093/3840*(12*x^2 + 17*x + 6)^(3/2) - 97/1024*sqrt(12*x^2 + 17*x + 6)*x +
 97/294912*sqrt(3)*log(4*sqrt(3)*sqrt(12*x^2 + 17*x + 6) + 24*x + 17) - 16
49/24576*sqrt(12*x^2 + 17*x + 6)
```

3.134.8 Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.68

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$= -\frac{1}{122880}(8(48(72(32x-71)x-17807)x-681893)x-1353611)\sqrt{12x^2+17x+6} - \frac{97}{294912}\sqrt{3}\log\left(\left|-4\sqrt{3}\left(2\sqrt{3}x-\sqrt{12x^2+17x+6}\right)-17\right|\right)$$

input
```
integrate((2+3*x)*(-12*x^2+31*x+30)*(12*x^2+17*x+6)^(1/2),x, algorithm="gi
ac")
```

output
```
-1/122880*(8*(48*(72*(32*x - 71)*x - 17807)*x - 681893)*x - 1353611)*sqrt(
12*x^2 + 17*x + 6) - 97/294912*sqrt(3)*log(abs(-4*sqrt(3)*(2*sqrt(3)*x - s
qrt(12*x^2 + 17*x + 6)) - 17))
```

3.134.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.32

$$\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$$
$$= \frac{3753\left(\frac{x}{2}+\frac{17}{48}\right)\sqrt{12x^2+17x+6}}{80} - \frac{417\sqrt{12}\,\ln\left(\sqrt{12x^2+17x+6}+\frac{\sqrt{12}\left(12x+\frac{17}{2}\right)}{12}\right)}{10240} - \frac{3x^2\left(12x^2+17x+6\right)^{3/2}}{5} + \frac{7093\sqrt{12x^2+17x+6}\left(1152x^2+408x-291\right)}{368640} + \frac{349x\left(12x^2+17x+6\right)^{3/2}}{160} + \frac{120581\sqrt{12}\,\ln\left(2\sqrt{12x^2+17x+6}+\frac{\sqrt{12}(24x+17)}{12}\right)}{2949120}$$

input
```
int((3*x + 2)*(17*x + 12*x^2 + 6)^(1/2)*(31*x - 12*x^2 + 30),x)
```

output
```
(3753*(x/2 + 17/48)*(17*x + 12*x^2 + 6)^(1/2))/80 - (417*12^(1/2)*log((17*
x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(12*x + 17/2))/12))/10240 - (3*x^2*(17*x
 + 12*x^2 + 6)^(3/2))/5 + (7093*(17*x + 12*x^2 + 6)^(1/2)*(408*x + 1152*x^
2 - 291))/368640 + (349*x*(17*x + 12*x^2 + 6)^(3/2))/160 + (120581*12^(1/2
)*log(2*(17*x + 12*x^2 + 6)^(1/2) + (12^(1/2)*(24*x + 17))/12))/2949120
```

3.134. $\int (2+3x)\left(30+31x-12x^2\right)\sqrt{6+17x+12x^2}\,dx$

3.135 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)(30+31x-12x^2)}\,dx$

3.135.1 Optimal result

Integrand size = 34, antiderivative size = 28

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = \frac{1}{42}\text{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)$$

output
```
1/42*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))
```

3.135.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = \frac{1}{21}\text{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)$$

input
```
Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

output
```
ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*x))]/21
```

3.135.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.088$, Rules used = {1335, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12x^2+17x+6}}{(3x+2)(-12x^2+31x+30)}dx$$

$$\Big\downarrow \text{ 1335}$$

$$\int \frac{1}{(10-3x)\sqrt{12x^2+17x+6}}dx$$

$$\Big\downarrow \text{ 1154}$$

$$-2\int \frac{1}{7056-\frac{(291x+206)^2}{12x^2+17x+6}}d\left(-\frac{291x+206}{\sqrt{12x^2+17x+6}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{42}\text{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)$$

input
```
Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)*(30 + 31*x - 12*x^2)),x]
```

output
```
ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x + 12*x^2])]/42
```

3.135.3.1 Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1335
```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

3.135.4 Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.14

method	result
trager	$-\frac{\ln\left(-\frac{84\sqrt{12x^2+17x+6}-206-291x}{3x-10}\right)}{42}$
default	$\frac{\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{12}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{12}}{288}-\frac{4\sqrt{12\left(x+\frac{3}{4}\right)^2-x-\frac{3}{4}}}{49}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{3}{4}\right)^2 \ldots}\right.}{294}$

input
```
int((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x,method=_RETURNVERBOS
E)
```

output
```
-1/42*ln(-(84*(12*x^2+17*x+6)^(1/2)-206-291*x)/(3*x-10))
```

3.135.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 53 vs. 2(22) = 44.

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx=\frac{1}{84}\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-\frac{1}{84}\log\left(\frac{291\,x-84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="fr
icas")
```

output
```
1/84*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) - 1/84*log((291*x -
 84*sqrt(12*x^2 + 17*x + 6) + 206)/x)
```

3.135.6 Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = -\int \frac{\sqrt{12x^2+17x+6}}{36x^3-69x^2-152x-60}\,dx$$

input
```
integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)/(-12*x**2+31*x+30),x)
```

output
```
-Integral(sqrt(12*x**2 + 17*x + 6)/(36*x**3 - 69*x**2 - 152*x - 60), x)
```

3.135.7 Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = \int -\frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)(3\,x+2)}\,dx$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="ma
xima")
```

output
```
-integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)*(3*x + 2)), x)
```

3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(22) = 44$.

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\left(30+31x-12x^2\right)}\,dx = \frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12\,x^2+17\,x+6}+42\right|\right) - \frac{1}{42}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12\,x^2+17\,x+6}-42\right|\right)$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)/(-12*x^2+31*x+30),x, algorithm="gi
ac")
```

output
```
1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42))
- 1/42*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) - 42)
)
```

3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)\,(30+31x-12x^2)}\,dx=\int \frac{\sqrt{12\,x^2+17\,x+6}}{(3\,x+2)\,\left(-12\,x^2+31\,x+30\right)}\,dx$$

input
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)),x)
```

output
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)*(31*x - 12*x^2 + 30)), x)
```

3.136 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx$

3.136.1 Optimal result

Integrand size = 34, antiderivative size = 84

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = -\frac{275+388x}{98(10-3x)\sqrt{6+17x+12x^2}} + \frac{3137\sqrt{6+17x+12x^2}}{38416(10-3x)} + \frac{97\operatorname{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{3226944}$$

output
```
97/3226944*arctanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/98*(-275-388*
x)/(10-3*x)/(12*x^2+17*x+6)^(1/2)+3137/38416*(12*x^2+17*x+6)^(1/2)/(10-3*x
)
```

3.136.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2\left(30+31x-12x^2\right)^2}\,dx = \frac{(88978+98767x-37644x^2)\sqrt{6+17x+12x^2}}{38416(-10+3x)(2+3x)(3+4x)} + \frac{97\operatorname{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)}{1613472}$$

input
```
Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

output
```
((88978 + 98767*x - 37644*x^2)*Sqrt[6 + 17*x + 12*x^2])/(38416*(-10 + 3*x)
*(2 + 3*x)*(3 + 4*x)) + (97*ArcTanh[(6*Sqrt[6 + 17*x + 12*x^2])/(7*(2 + 3*
x))])/1613472
```

3.136.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1335, 1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12x^2+17x+6}}{(3x+2)^2\left(-12x^2+31x+30\right)^2}dx$$

$$\Big\downarrow \text{ 1335}$$

$$\int \frac{1}{(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1165}$$

$$-\frac{1}{882}\int -\frac{9(2328x+1651)}{2(10-3x)^2\sqrt{12x^2+17x+6}}dx-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{196}\int \frac{2328x+1651}{(10-3x)^2\sqrt{12x^2+17x+6}}dx-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}$$

$$\Big\downarrow \text{ 1228}$$

$$\frac{1}{196}\left(\frac{97}{392}\int \frac{1}{(10-3x)\sqrt{12x^2+17x+6}}dx+\frac{3137\sqrt{12x^2+17x+6}}{196(10-3x)}\right)-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}$$

$$\Big\downarrow \text{ 1154}$$

$$\frac{1}{196}\left(\frac{3137\sqrt{12x^2+17x+6}}{196(10-3x)}-\frac{97}{196}\int \frac{1}{7056-\frac{(291x+206)^2}{12x^2+17x+6}}d\left(-\frac{291x+206}{\sqrt{12x^2+17x+6}}\right)\right)-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{196}\left(\frac{97\text{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{16464}+\frac{3137\sqrt{12x^2+17x+6}}{196(10-3x)}\right)-\frac{388x+275}{98(10-3x)\sqrt{12x^2+17x+6}}$$

input
```
Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^2*(30 + 31*x - 12*x^2)^2),x]
```

output
```
-1/98*(275 + 388*x)/((10 - 3*x)*Sqrt[6 + 17*x + 12*x^2]) + ((3137*Sqrt[6 +
 17*x + 12*x^2])/(196*(10 - 3*x)) + (97*ArcTanh[(206 + 291*x)/(84*Sqrt[6 +
 17*x + 12*x^2])])/16464)/196
```

3.136.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1335
```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

3.136.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.68

method	result
risch	$-\frac{37644x^2-98767x-88978}{38416(3x-10)\sqrt{12x^2+17x+6}}+\frac{97\,\operatorname{arctanh}\left(\frac{\frac{206}{3}+97x}{28\sqrt{12\left(x-\frac{10}{3}\right)^2+97x-\frac{382}{3}}}\right)}{3226944}$
trager	$-\frac{\left(37644x^2-98767x-88978\right)\sqrt{12x^2+17x+6}}{38416\left(36x^3-69x^2-152x-60\right)}-\frac{97\ln\left(-\frac{84\sqrt{12x^2+17x+6}-206-291x}{3x-10}\right)}{3226944}$
default	$-\frac{\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{72\left(x+\frac{2}{3}\right)^2}+\frac{\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{288}+\frac{\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{12}}{6912}-\frac{\left(12\left(x-\frac{10}{3}\right)^2+97x-\frac{382}{3}\right)^{\frac{3}{2}}}{67765824\left(x-\frac{10}{3}\right)}$

input
```
int((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x,method=_RETURNVE
RBOSE)
```

output
```
-1/38416*(37644*x^2-98767*x-88978)/(3*x-10)/(12*x^2+17*x+6)^(1/2)+97/32269
44*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^(1/2))
```

3.136.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx$$
$$= \frac{97\,(36\,x^3-69\,x^2-152\,x-60)\log\left(\frac{291\,x+84\,\sqrt{12\,x^2+17\,x+6}+206}{x}\right)-97\,(36\,x^3-69\,x^2-152\,x-60)\log\left(\frac{291\,x\cdots}{}\right)}{6453888\,(36\,x^3-69\,x^2-152\,x-\cdots}$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm
="fricas")
```

output
```
1/6453888*(97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x + 84*sqrt(12*x^2 +
 17*x + 6) + 206)/x) - 97*(36*x^3 - 69*x^2 - 152*x - 60)*log((291*x - 84*s
qrt(12*x^2 + 17*x + 6) + 206)/x) - 168*(37644*x^2 - 98767*x - 88978)*sqrt(
12*x^2 + 17*x + 6))/(36*x^3 - 69*x^2 - 152*x - 60)
```

3.136.6 Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx = \int \frac{\sqrt{(3x+2)\,(4x+3)}}{(3x-10)^2\,(3x+2)^2\,(4x+3)^2}\,dx$$

input
```
integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**2/(-12*x**2+31*x+30)**2,x)
```

output
```
Integral(sqrt((3*x + 2)*(4*x + 3))/((3*x - 10)**2*(3*x + 2)**2*(4*x + 3)**
2), x)
```

3.136.7 Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2(30+31x-12x^2)^2}\,dx = \int \frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)^2(3\,x+2)^2}\,dx$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm
="maxima")
```

output
```
integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^2*(3*x + 2)^2), x)
```

3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.89

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2 (30+31x-12x^2)^2}\,dx$$
$$= \frac{1}{9680832}\sqrt{3}\left(\sqrt{3}\left(175672\sqrt{3}+97\log\left(\frac{7\sqrt{3}-12}{7\sqrt{3}+12}\right)\right)\operatorname{sgn}\left(\frac{1}{3x+2}\right)-\left(97\sqrt{3}\log\left(\frac{\left|-28\sqrt{3}+24\sqrt{\frac{1}{3x+2}+4}\right|}{4\left(7\sqrt{3}+6\sqrt{\frac{1}{3x+2}+4}\right)}\right)\right.\right.$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^2/(-12*x^2+31*x+30)^2,x, algorithm
="giac")
```

output
```
1/9680832*sqrt(3)*(sqrt(3)*(175672*sqrt(3) + 97*log((7*sqrt(3) - 12)/(7*sq
rt(3) + 12)))*sgn(1/(3*x + 2)) - (97*sqrt(3)*log(1/4*abs(-28*sqrt(3) + 24*
sqrt(1/(3*x + 2) + 4))/(7*sqrt(3) + 6*sqrt(1/(3*x + 2) + 4))) + 134456*sqr
t(1/(3*x + 2) + 4) + 28*(221183/(3*x + 2) - 18436)/(12*(1/(3*x + 2) + 4)^(
3/2) - 49*sqrt(1/(3*x + 2) + 4)))*sgn(1/(3*x + 2)))
```

3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^2 (30+31x-12x^2)^2}\,dx = \int \frac{\sqrt{12\,x^2+17\,x+6}}{{\left(3\,x+2\right)}^2\,{\left(-12\,x^2+31\,x+30\right)}^2}\,dx$$

input
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2),x)
```

output
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^2*(31*x - 12*x^2 + 30)^2), x)
```

3.137 $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$

3.137.1 Optimal result

Integrand size = 34, antiderivative size = 139

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx = -\frac{275+388x}{294(10-3x)^2\left(6+17x+12x^2\right)^{3/2}} + \frac{738029+1042556x}{8232(10-3x)^2\sqrt{6+17x+12x^2}} - \frac{50555899\sqrt{6+17x+12x^2}}{19361664(10-3x)^2} - \frac{1634466587\sqrt{6+17x+12x^2}}{7589772288(10-3x)} + \frac{40325\operatorname{arctanh}\left(\frac{206+291x}{84\sqrt{6+17x+12x^2}}\right)}{637540872192}$$

output
```
1/294*(-275-388*x)/(10-3*x)^2/(12*x^2+17*x+6)^(3/2)+40325/637540872192*arc
tanh(1/84*(206+291*x)/(12*x^2+17*x+6)^(1/2))+1/8232*(738029+1042556*x)/(10
-3*x)^2/(12*x^2+17*x+6)^(1/2)-50555899/19361664*(12*x^2+17*x+6)^(1/2)/(10-
3*x)^2-1634466587/7589772288*(12*x^2+17*x+6)^(1/2)/(10-3*x)
```

3.137.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$$
$$= \frac{\sqrt{6+17x+12x^2}(2773753482408+10124325497244x+9848047480070x^2-1096520427663x^3-320682}{7589772288(-10+3x)^2(2+3x)^2(3+4x)^2}$$
$$+\frac{40325\text{arctanh}\left(\frac{6\sqrt{6+17x+12x^2}}{7(2+3x)}\right)}{318770436096}$$

input
```
Integrate[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

output
```
(Sqrt[6 + 17*x + 12*x^2]*(2773753482408 + 10124325497244*x + 9848047480070
*x^2 - 1096520427663*x^3 - 3206824169544*x^4 + 706089565584*x^5))/(7589772
288*(-10 + 3*x)^2*(2 + 3*x)^2*(3 + 4*x)^2) + (40325*ArcTanh[(6*Sqrt[6 + 17
*x + 12*x^2])/(7*(2 + 3*x))])/318770436096
```

3.137.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {1335, 1165, 27, 1235, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{12x^2+17x+6}}{(3x+2)^3\left(-12x^2+31x+30\right)^3}dx$$
$$\Big\downarrow \ 1335$$
$$\int \frac{1}{(10-3x)^3\left(12x^2+17x+6\right)^{5/2}}dx$$
$$\Big\downarrow \ 1165$$
$$-\frac{\int \frac{9(12217-9312x)}{2(10-3x)^3(12x^2+17x+6)^{3/2}}dx}{2646}-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$
$$\Big\downarrow \ 27$$

$$-\frac{1}{588}\int \frac{12217-9312x}{(10-3x)^3\left(12x^2+17x+6\right)^{3/2}}dx-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{1235}$$

$$\frac{1}{588}\left(\frac{1}{882}\int -\frac{63(12510672x+8853659)}{2(10-3x)^3\sqrt{12x^2+17x+6}}dx+\frac{1042556x+738029}{14(10-3x)^2\sqrt{12x^2+17x+6}}\right)-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{588}\left(\frac{1042556x+738029}{14(10-3x)^2\sqrt{12x^2+17x+6}}-\frac{1}{28}\int \frac{12510672x+8853659}{(10-3x)^3\sqrt{12x^2+17x+6}}dx\right)-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{1237}$$

$$\frac{1}{588}\left(\frac{1}{28}\left(\frac{\int -\frac{3(1213341576x+858927841)}{2(10-3x)^2\sqrt{12x^2+17x+6}}dx}{3528}-\frac{50555899\sqrt{12x^2+17x+6}}{1176(10-3x)^2}\right)+\frac{1042556x+738029}{14(10-3x)^2\sqrt{12x^2+17x+6}}\right)-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{588}\left(\frac{1}{28}\left(-\frac{\int \frac{1213341576x+858927841}{(10-3x)^2\sqrt{12x^2+17x+6}}dx}{2352}-\frac{50555899\sqrt{12x^2+17x+6}}{1176(10-3x)^2}\right)+\frac{1042556x+738029}{14(10-3x)^2\sqrt{12x^2+17x+6}}\right)-\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{1228}$$

$$\frac{1}{588}\left(\frac{1}{28}\left(\frac{\frac{40325}{392}\int \frac{1}{(10-3x)\sqrt{12x^2+17x+6}}dx-\frac{1634466587\sqrt{12x^2+17x+6}}{196(10-3x)}}{2352}-\frac{50555899\sqrt{12x^2+17x+6}}{1176(10-3x)^2}\right)+\frac{1042556}{14(10-3x)^2}\right.$$

$$\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$$\Big\downarrow \text{1154}$$

3.137. $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$

$$\frac{1}{588}\left(\frac{1}{28}\left(\frac{-\frac{40325}{196}\int \frac{1}{7056-\frac{(291x+206)^2}{12x^2+17x+6}}d\left(-\frac{291x+206}{\sqrt{12x^2+17x+6}}\right)-\frac{1634466587\sqrt{12x^2+17x+6}}{196(10-3x)}}{2352}-\frac{50555899\sqrt{12x^2+17x+6}}{1176(10-3x)^2}\right)+\right.$$

$$\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{1}{588}\left(\frac{1}{28}\left(\frac{\frac{40325\text{arctanh}\left(\frac{291x+206}{84\sqrt{12x^2+17x+6}}\right)}{16464}-\frac{1634466587\sqrt{12x^2+17x+6}}{196(10-3x)}}{2352}-\frac{50555899\sqrt{12x^2+17x+6}}{1176(10-3x)^2}\right)+\frac{1042556x}{14(10-3x)^2\sqrt{}}\right.$$

$$\frac{388x+275}{294(10-3x)^2\left(12x^2+17x+6\right)^{3/2}}$$

input
```
Int[Sqrt[6 + 17*x + 12*x^2]/((2 + 3*x)^3*(30 + 31*x - 12*x^2)^3),x]
```

output
```
-1/294*(275 + 388*x)/((10 - 3*x)^2*(6 + 17*x + 12*x^2)^(3/2)) + ((738029 +
 1042556*x)/(14*(10 - 3*x)^2*Sqrt[6 + 17*x + 12*x^2]) + ((-50555899*Sqrt[6
 + 17*x + 12*x^2])/(1176*(10 - 3*x)^2) + ((-1634466587*Sqrt[6 + 17*x + 12*
x^2])/(196*(10 - 3*x)) + (40325*ArcTanh[(206 + 291*x)/(84*Sqrt[6 + 17*x +
12*x^2])])/16464)/2352)/28)/588
```

3.137.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

3.137. $\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx$

rule 1335
```
Int[((g_) + (h_.)*(x_))^(m_.)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)*((d_.
) + (e_.)*(x_) + (f_.)*(x_)^2)^(m_.), x_Symbol] :> Int[(d*(g/a) + f*h*(x/c)
)^m*(a + b*x + c*x^2)^(m + p), x] /; FreeQ[{a, b, c, d, e, f, g, h, p}, x]
&& EqQ[c*g^2 - b*g*h + a*h^2, 0] && EqQ[c^2*d*g^2 - a*c*e*g*h + a^2*f*h^2,
0] && IntegerQ[m]
```

3.137.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.52

method	result
risch	$\frac{706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408}{7589772288(12x^2+17x+6)^{\frac{3}{2}}(3x-10)^2}+\frac{40325\operatorname{arctanh}}{}$
trager	$\frac{(706089565584x^5-3206824169544x^4-1096520427663x^3+9848047480070x^2+10124325497244x+2773753482408)\sqrt{12x^2+17x+6}}{7589772288(36x^3-69x^2-152x-60)^2}-$
default	$-\frac{\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{2592\left(x+\frac{2}{3}\right)^3}+\frac{47\left(12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}\right)^{\frac{3}{2}}}{1152\left(x+\frac{2}{3}\right)^2}-\frac{23\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}}{4608}-\frac{23\ln\left(\frac{\left(\frac{17}{2}+12x\right)\sqrt{12}}{12}+\sqrt{12\left(x+\frac{2}{3}\right)^2+x+\frac{2}{3}}\right)\sqrt{12}}{110592}$

input
```
int((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x,method=_RETURNVE
RBOSE)
```

output
```
1/7589772288*(706089565584*x^5-3206824169544*x^4-1096520427663*x^3+9848047
480070*x^2+10124325497244*x+2773753482408)/(12*x^2+17*x+6)^(3/2)/(3*x-10)^
2+40325/637540872192*arctanh(1/28*(206/3+97*x)/(12*(x-10/3)^2+97*x-382/3)^
(1/2))
```

3.137.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.34

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx$$
$$= \frac{40325\,(1296\,x^6-4968\,x^5-6183\,x^4+16656\,x^3+31384\,x^2+18240\,x+3600)\log\left(\frac{291\,x+84\sqrt{12\,x^2+17\,x+6}+20}{x}\right.}{}$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm
="fricas")
```

output
```
1/1275081744384*(40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384
*x^2 + 18240*x + 3600)*log((291*x + 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) -
 40325*(1296*x^6 - 4968*x^5 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x +
 3600)*log((291*x - 84*sqrt(12*x^2 + 17*x + 6) + 206)/x) + 168*(7060895655
84*x^5 - 3206824169544*x^4 - 1096520427663*x^3 + 9848047480070*x^2 + 10124
325497244*x + 2773753482408)*sqrt(12*x^2 + 17*x + 6))/(1296*x^6 - 4968*x^5
 - 6183*x^4 + 16656*x^3 + 31384*x^2 + 18240*x + 3600)
```

3.137.6 Sympy [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx = -\int \frac{\sqrt{12x^2+17x+6}}{46656x^9-268272x^8-76788x^7+1703619x^6+1218456x^5-3669588x^4-6898688x^3-4903920x^2}$$

input
```
integrate((12*x**2+17*x+6)**(1/2)/(2+3*x)**3/(-12*x**2+31*x+30)**3,x)
```

output
```
-Integral(sqrt(12*x**2 + 17*x + 6)/(46656*x**9 - 268272*x**8 - 76788*x**7
+ 1703619*x**6 + 1218456*x**5 - 3669588*x**4 - 6898688*x**3 - 4903920*x**2
 - 1641600*x - 216000), x)
```

3.137.7 Maxima [F]

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3(30+31x-12x^2)^3}\,dx = \int -\frac{\sqrt{12\,x^2+17\,x+6}}{(12\,x^2-31\,x-30)^3(3\,x+2)^3}\,dx$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm
="maxima")
```

output
```
-integrate(sqrt(12*x^2 + 17*x + 6)/((12*x^2 - 31*x - 30)^3*(3*x + 2)^3), x
)
```

3.137.8 Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.67

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx$$
$$= \frac{\sqrt{3}\Big(282273\sqrt{3}\big(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\big)^3-11460924\left(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-37551180\sqrt{3}}{159385218048\left(3\left(2\sqrt{3}x-\sqrt{12\,x^2+17\,x+6}\right)^2-40\sqrt{3}\big(2\sqrt{3}x-\sqrt{12\,x^2+1}\right.}$$
$$+\frac{(8\,(2860316794\,x+6078171227)x+34383350229)x+8090114146}{2213683584\,(12\,x^2+17\,x+6)^{\frac{3}{2}}}$$
$$+\frac{40325}{637540872192}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12\,x^2+17\,x+6}+42\right|\right)$$
$$-\frac{40325}{637540872192}\log\left(\left|-6\sqrt{3}x+20\sqrt{3}+3\sqrt{12\,x^2+17\,x+6}-42\right|\right)$$

input
```
integrate((12*x^2+17*x+6)^(1/2)/(2+3*x)^3/(-12*x^2+31*x+30)^3,x, algorithm
="giac")
```

output
```
1/159385218048*sqrt(3)*(282273*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x +
 6))^3 - 11460924*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6))^2 - 37551180*sqr
t(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x + 6)) - 83365264)/(3*(2*sqrt(3)*x -
 sqrt(12*x^2 + 17*x + 6))^2 - 40*sqrt(3)*(2*sqrt(3)*x - sqrt(12*x^2 + 17*x
 + 6)) - 188)^2 + 1/2213683584*((8*(2860316794*x + 6078171227)*x + 3438335
0229)*x + 8090114146)/(12*x^2 + 17*x + 6)^(3/2) + 40325/637540872192*log(a
bs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) + 42)) - 40325/63
7540872192*log(abs(-6*sqrt(3)*x + 20*sqrt(3) + 3*sqrt(12*x^2 + 17*x + 6) -
 42))
```

3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{6+17x+12x^2}}{(2+3x)^3\left(30+31x-12x^2\right)^3}\,dx=\int \frac{\sqrt{12\,x^2+17\,x+6}}{\left(3\,x+2\right)^3\left(-12\,x^2+31\,x+30\right)^3}\,dx$$

input
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3),x)
```

output
```
int((17*x + 12*x^2 + 6)^(1/2)/((3*x + 2)^3*(31*x - 12*x^2 + 30)^3), x)
```

3.138 $\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx$

3.138.1 Optimal result

Integrand size = 17, antiderivative size = 15

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

output
```
3/5*(x^2-3*x)^(5/3)
```

3.138.2 Mathematica [A] (verified)

Time = 9.28 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}((-3+x)x)^{5/3}$$

input
```
Integrate[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

output
```
(3*((-3 + x)*x)^(5/3))/5
```

3.138.3 Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x-3)\left(x^2-3x\right)^{2/3}dx$$

$$\Big\downarrow \; 1104$$

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

input
```
Int[(-3 + 2*x)*(-3*x + x^2)^(2/3),x]
```

output
```
(3*(-3*x + x^2)^(5/3))/5
```

3.138.3.1 Defintions of rubi rules used

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

3.138.4 Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{3(x^2-3x)^{\frac{5}{3}}}{5}$	12
default	$\frac{3(x^2-3x)^{\frac{5}{3}}}{5}$	12
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$	14
gosper	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$	16
trager	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$	16
risch	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$	18
meijerg	$-\frac{9\,3^{\frac{2}{3}}\,\mathrm{signum}(-3+x)^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{x}{3})}{5(-\mathrm{signum}(-3+x))^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\,\mathrm{signum}(-3+x)^{\frac{2}{3}}x^{\frac{8}{3}}\,{}_2F_1(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3})}{4(-\mathrm{signum}(-3+x))^{\frac{2}{3}}}$	64

input
```
int((2*x-3)*(x^2-3*x)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(x^2-3*x)^(5/3)
```

3.138.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="fricas")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.138.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3x^2(x^2-3x)^{\frac{2}{3}}}{5}-\frac{9x(x^2-3x)^{\frac{2}{3}}}{5}$$

input
```
integrate((-3+2*x)*(x**2-3*x)**(2/3),x)
```

output
```
3*x**2*(x**2 - 3*x)**(2/3)/5 - 9*x*(x**2 - 3*x)**(2/3)/5
```

3.138.7 Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="maxima")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.138.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate((-3+2*x)*(x^2-3*x)^(2/3),x, algorithm="giac")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.138.9 Mupad [B] (verification not implemented)

Time = 12.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (-3+2x)\left(-3x+x^2\right)^{2/3}\,dx=\frac{3\,x\,{\left(x^2-3\,x\right)}^{2/3}\,\left(x-3\right)}{5}$$

input
```
int((2*x - 3)*(x^2 - 3*x)^(2/3),x)
```

output
```
(3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

3.139 $\int ((-3+x)x)^{2/3}(-3+2x)\,dx$

3.139.1 Optimal result

Integrand size = 15, antiderivative size = 16

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx=\frac{3}{5}(-((3-x)x))^{5/3}$$

output
```
3/5*(-(3-x)*x)^(5/3)
```

3.139.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx=\frac{3}{5}((-3+x)x)^{5/3}$$

input
```
Integrate[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

output
```
(3*((-3 + x)*x)^(5/3))/5
```

3.139.3 Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2021}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int ((x-3)x)^{2/3}(2x-3)\,dx$$
$$\Big\downarrow \text{2021}$$
$$\frac{3}{5}(-((3-x)x))^{5/3}$$

input
```
Int[((-3 + x)*x)^(2/3)*(-3 + 2*x),x]
```

output
```
(3*(-((3 - x)*x))^(5/3))/5
```

3.139.3.1 Defintions of rubi rules used

rule 2021
```
Int[(Pp_)*(Qq_)^(m_.), x_Symbol] :> With[{p = Expon[Pp, x], q = Expon[Qq, x
]}, Simp[Coeff[Pp, x, p]*x^(p - q + 1)*(Qq^(m + 1)/((p + m*q + 1)*Coeff[Qq,
 x, q])), x] /; NeQ[p + m*q + 1, 0] && EqQ[(p + m*q + 1)*Coeff[Qq, x, q]*Pp
, Coeff[Pp, x, p]*x^(p - q)*((p - q + 1)*Qq + (m + 1)*x*D[Qq, x])]] /; Free
Q[m, x] && PolyQ[Pp, x] && PolyQ[Qq, x] && NeQ[m, -1]
```

3.139.4 Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{3((-3+x)x)^{\frac{5}{3}}}{5}$	10
default	$\frac{3((-3+x)x)^{\frac{5}{3}}}{5}$	10
gosper	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$	14
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$	14
trager	$\frac{3(-3+x)x(x^{2}-3x)^{\frac{2}{3}}}{5}$	16
risch	$\frac{3(-3+x)^{2}x^{2}}{5((-3+x)x)^{\frac{1}{3}}}$	18
meijerg	$-\frac{9\,3^{\frac{2}{3}}\,\mathrm{signum}(-3+x)^{\frac{2}{3}}x^{\frac{5}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{5}{3};\frac{8}{3};\frac{x}{3}\right)}{5(-\mathrm{signum}(-3+x))^{\frac{2}{3}}}+\frac{3\,3^{\frac{2}{3}}\,\mathrm{signum}(-3+x)^{\frac{2}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(-\frac{2}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{4(-\mathrm{signum}(-3+x))^{\frac{2}{3}}}$	64

input
```
int(((-3+x)*x)^(2/3)*(2*x-3),x,method=_RETURNVERBOSE)
```

output
```
3/5*((-3+x)*x)^(5/3)
```

3.139.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int((-3+x)x)^{2/3}(-3+2x)\,dx=\frac{3}{5}\left(x^{2}-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="fricas")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.139.6 Sympy [A] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.62

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3(x(x-3))^{\frac{5}{3}}}{5}$$

input
```
integrate(((-3+x)*x)**(2/3)*(-3+2*x),x)
```

output
```
3*(x*(x - 3))**(5/3)/5
```

3.139.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}\,((x-3)x)^{\frac{5}{3}}$$

input
```
integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="maxima")
```

output
```
3/5*((x - 3)*x)^(5/3)
```

3.139.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3}{5}\,\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate(((-3+x)*x)^(2/3)*(-3+2*x),x, algorithm="giac")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.139.9 Mupad [B] (verification not implemented)

Time = 12.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int ((-3+x)x)^{2/3}(-3+2x)\,dx = \frac{3\,x\,{\left(x\,\left(x-3\right)\right)}^{2/3}\,\left(x-3\right)}{5}$$

input
```
int((2*x - 3)*(x*(x - 3))^(2/3),x)
```

output
```
(3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

3.140 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx$

3.140.1 Optimal result

Integrand size = 23, antiderivative size = 15

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(-3x+x^2\right)^{5/3}$$

output
```
3/5*(x^2-3*x)^(5/3)
```

3.140.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}((-3+x)x)^{5/3}$$

input
```
Integrate[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

output
```
(3*((-3 + x)*x)^(5/3))/5
```

3.140.3 Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {2162, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^2-9x+9)}{\sqrt[3]{x^2-3x}}\,dx$$
$$\Big\downarrow \text{2162}$$
$$\int -\left((3-2x)\left(x^2-3x\right)^{2/3}\right)dx$$
$$\Big\downarrow \text{25}$$
$$-\int (3-2x)\left(x^2-3x\right)^{2/3}dx$$
$$\Big\downarrow \text{1104}$$
$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

input
```
Int[(x*(9 - 9*x + 2*x^2))/(-3*x + x^2)^(1/3),x]
```

output
```
(3*(-3*x + x^2)^(5/3))/5
```

3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2162
```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Simp[e   Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Poly
nomialRemainder[Pq, b + c*x, x], 0]
```

3.140.4 Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$
trager	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$
risch	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$
gosper	$\frac{3(-3+x)^2x^2}{5(x^2-3x)^{\frac{1}{3}}}$
meijerg	$\frac{2\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{11}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{x}{3}\right)}{11\operatorname{signum}(-3+x)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x))^{\frac{1}{3}}x^{\frac{8}{3}}\,{}_2F_1\left(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3}\right)}{8\operatorname{signum}(-3+x)^{\frac{1}{3}}}+\frac{9\,3^{\frac{2}{3}}(-\operatorname{signum}(-3+x)}{5\operatorname{signum}(}$

input
```
int(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(-3+x)*x*((-3+x)*x)^(2/3)
```

3.140.5 Fricas [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="fricas")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.140.6 Sympy [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx=\int \frac{x(x-3)\,(2x-3)}{\sqrt[3]{x\,(x-3)}}\,dx$$

input
```
integrate(x*(2*x**2-9*x+9)/(x**2-3*x)**(1/3),x)
```

output
```
Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

3.140.7 Maxima [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx=\int \frac{(2\,x^2-9\,x+9)x}{(x^2-3\,x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - 9*x + 9)*x/(x^2 - 3*x)^(1/3), x)
```

3.140.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx=\frac{3}{5}\,(x^2-3\,x)^{\frac{5}{3}}$$

input
```
integrate(x*(2*x^2-9*x+9)/(x^2-3*x)^(1/3),x, algorithm="giac")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.140.9 Mupad [B] (verification not implemented)

Time = 12.44 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{-3x+x^2}}\,dx = \frac{3\,x\,\left(x^2-3\,x\right)^{2/3}\,\left(x-3\right)}{5}$$

input
```
int((x*(2*x^2 - 9*x + 9))/(x^2 - 3*x)^(1/3),x)
```

output
```
(3*x*(x^2 - 3*x)^(2/3)*(x - 3))/5
```

3.141 $\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx$

3.141.1 Optimal result

Integrand size = 21, antiderivative size = 15

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}(-3x+x^2)^{5/3}$$

output
```
3/5*(x^2-3*x)^(5/3)
```

3.141.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}((-3+x)x)^{5/3}$$

input
```
Integrate[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

output
```
(3*((-3 + x)*x)^(5/3))/5
```

3.141.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2048, 2162, 25, 1104}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(2x^2-9x+9)}{\sqrt[3]{(x-3)x}}dx$$

$$\Big\downarrow \text{2048}$$

$$\int \frac{x(2x^2-9x+9)}{\sqrt[3]{x^2-3x}}dx$$

$$\Big\downarrow \text{2162}$$

$$\int -\left((3-2x)\left(x^2-3x\right)^{2/3}\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int (3-2x)\left(x^2-3x\right)^{2/3}dx$$

$$\Big\downarrow \text{1104}$$

$$\frac{3}{5}\left(x^2-3x\right)^{5/3}$$

input
```
Int[(x*(9 - 9*x + 2*x^2))/((-3 + x)*x)^(1/3),x]
```

output
```
(3*(-3*x + x^2)^(5/3))/5
```

3.141.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1104
```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0]
```

rule 2048
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))*((c_) + (d_.)*(x_)^(n_.)))^(p_)
, x_Symbol] :> Int[u*(a*c*e + (b*c + a*d)*e*x^n + b*d*e*x^(2*n))^p, x] /; F
reeQ[{a, b, c, d, e, n, p}, x]
```

rule 2162
```
Int[(Pq_)*((e_.)*(x_))^(m_.)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Simp[e   Int[(e*x)^(m - 1)*PolynomialQuotient[Pq, b + c*x, x]*(b*x + c*x^
2)^(p + 1), x], x] /; FreeQ[{b, c, e, m, p}, x] && PolyQ[Pq, x] && EqQ[Poly
nomialRemainder[Pq, b + c*x, x], 0]
```

3.141.4 Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$\frac{3(-3+x)x((-3+x)x)^{\frac{2}{3}}}{5}$
trager	$\frac{3(-3+x)x(x^2-3x)^{\frac{2}{3}}}{5}$
gosper	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$
risch	$\frac{3(-3+x)^2x^2}{5((-3+x)x)^{\frac{1}{3}}}$
meijerg	$\frac{2\,3^{\frac{2}{3}}(-\mathrm{signum}(-3+x))^{\frac{1}{3}}x^{\frac{11}{3}}\,{}_2F_1(\frac{1}{3},\frac{11}{3};\frac{14}{3};\frac{x}{3})}{11\,\mathrm{signum}(-3+x)^{\frac{1}{3}}}-\frac{9\,3^{\frac{2}{3}}(-\mathrm{signum}(-3+x))^{\frac{1}{3}}x^{\frac{8}{3}}\,{}_2F_1(\frac{1}{3},\frac{8}{3};\frac{11}{3};\frac{x}{3})}{8\,\mathrm{signum}(-3+x)^{\frac{1}{3}}}+\frac{9\,3^{\frac{2}{3}}(-\mathrm{signum}(-3+x}{5\,\mathrm{signum}(}$

input
```
int(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
3/5*(-3+x)*x*((-3+x)*x)^(2/3)
```

3.141.5 Fricas [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx=\frac{3}{5}\left(x^2-3\,x\right)^{\frac{5}{3}}$$

input
```
integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="fricas")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.141.6 Sympy [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \int \frac{x(x-3)\,(2x-3)}{\sqrt[3]{x\,(x-3)}}\,dx$$

input
```
integrate(x*(2*x**2-9*x+9)/((-3+x)*x)**(1/3),x)
```

output
```
Integral(x*(x - 3)*(2*x - 3)/(x*(x - 3))**(1/3), x)
```

3.141.7 Maxima [F]

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \int \frac{(2\,x^2-9\,x+9)x}{((x-3)x)^{\frac{1}{3}}}\,dx$$

input
```
integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((2*x^2 - 9*x + 9)*x/((x - 3)*x)^(1/3), x)
```

3.141.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.73

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3}{5}\,(x^2-3\,x)^{\frac{5}{3}}$$

input
```
integrate(x*(2*x^2-9*x+9)/((-3+x)*x)^(1/3),x, algorithm="giac")
```

output
```
3/5*(x^2 - 3*x)^(5/3)
```

3.141.9 Mupad [B] (verification not implemented)

Time = 12.49 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{x(9-9x+2x^2)}{\sqrt[3]{(-3+x)x}}\,dx = \frac{3\,x\,(x\,(x-3))^{2/3}\,(x-3)}{5}$$

input
```
int((x*(2*x^2 - 9*x + 9))/(x*(x - 3))^(1/3),x)
```

output
```
(3*x*(x*(x - 3))^(2/3)*(x - 3))/5
```

3.142 $\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}(g^2+3h^2x^2)}\,dx$

3.142.1 Optimal result

Integrand size = 40, antiderivative size = 242

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3hx}{g}}}\right)}{2^{2/3}\sqrt{3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} + \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(g^2+3h^2x^2\right)}{6\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}} - \frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3hx}{g}}\right)}{2\ 2^{2/3}h\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}}$$

output
```
1/12*(1-9*h^2*x^2/g^2)^(1/3)*ln(3*h^2*x^2+g^2)*2^(1/3)/h/(-c*g^2/h^2+9*c*x
^2)^(1/3)-1/4*(1-9*h^2*x^2/g^2)^(1/3)*ln((1-3*h*x/g)^(2/3)+2^(1/3)*(1+3*h*
x/g)^(1/3))*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)-1/6*(1-9*h^2*x^2/g^2)^(1/
3)*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*x/g)^(2/3)/(1+3*h*x/g)^(1/3)*3^(
1/2))*2^(1/3)/h/(-c*g^2/h^2+9*c*x^2)^(1/3)*3^(1/2)
```

3.142.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.57

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx$$

$$= \frac{\sqrt[3]{-\frac{2g^2}{h^2}+18x^2}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{g}h^{2/3}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}}{2^{2/3}g-3\ 2^{2/3}hx+\sqrt[3]{g}h^{2/3}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}}\right)-2\log\left(\sqrt[3]{g}\sqrt[3]{-\frac{g^2}{h^2}+9x^2}\right)+\log\left(\right.\right.}{}$$

input
```
Integrate[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x
]
```

output
```
(((-2*g^2)/h^2 + 18*x^2)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*g^(1/3)*h^(2/3)*
(-(g^2/h^2) + 9*x^2)^(1/3))/(2^(2/3)*g - 3*2^(2/3)*h*x + g^(1/3)*h^(2/3)*(
-(g^2/h^2) + 9*x^2)^(1/3))] - 2*Log[g^(1/3)*(-(g^2/h^2) + 9*x^2)^(1/3)] +
Log[g^(2/3)*(-(g^2/h^2) + 9*x^2)^(2/3)] + 2*Log[2^(2/3)*g - 3*2^(2/3)*h*x
- 2*g^(1/3)*h^(2/3)*(-(g^2/h^2) + 9*x^2)^(1/3)] - Log[2^(1/3)*g^2 - 6*2^(1
/3)*g*h*x + 9*2^(1/3)*h^2*x^2 + 2^(2/3)*g^(4/3)*h^(2/3)*(-(g^2/h^2) + 9*x^
2)^(1/3) - 3*2^(2/3)*g^(1/3)*h^(5/3)*x*(-(g^2/h^2) + 9*x^2)^(1/3) + 2*g^(2
/3)*h^(4/3)*(-(g^2/h^2) + 9*x^2)^(2/3)]))/(12*g^(2/3)*h^(1/3)*(c*(-(g^2/h^
2) + 9*x^2))^(1/3))
```

3.142.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {1342, 1341}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{(g^2+3h^2x^2)\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}\,dx$$

$$\Big\downarrow \text{1342}$$

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\int\frac{g+hx}{(g^2+3h^2x^2)\sqrt[3]{1-\frac{9h^2x^2}{g^2}}}dx}{\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

$$\Big\downarrow \text{1341}$$

$$\frac{\sqrt[3]{1-\frac{9h^2x^2}{g^2}}\left(\frac{\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3hx}{g}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3hx}{g}+1}}\right)}{2^{2/3}\sqrt{3}h}+\frac{\log(g^2+3h^2x^2)}{6\ 2^{2/3}h}-\frac{\log\left(\left(1-\frac{3hx}{g}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{\frac{3hx}{g}+1}\right)}{2\ 2^{2/3}h}\right)}{\sqrt[3]{9cx^2-\frac{cg^2}{h^2}}}$$

input
```
Int[(g + h*x)/((-((c*g^2)/h^2) + 9*c*x^2)^(1/3)*(g^2 + 3*h^2*x^2)),x]
```

output
```
((1 - (9*h^2*x^2)/g^2)^(1/3)*(ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*x)/g)^
(2/3))/(Sqrt[3]*(1 + (3*h*x)/g)^(1/3))]/(2^(2/3)*Sqrt[3]*h) + Log[g^2 + 3*
h^2*x^2]/(6*2^(2/3)*h) - Log[(1 - (3*h*x)/g)^(2/3) + 2^(1/3)*(1 + (3*h*x)/
g)^(1/3)]/(2*2^(2/3)*h)))/(-((c*g^2)/h^2) + 9*c*x^2)^(1/3)
```

3.142.3.1 Defintions of rubi rules used

rule 1341
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[Sqrt[3]*h*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - 3*h*(x/g))^
(2/3)/(Sqrt[3]*(1 + 3*h*(x/g))^(1/3)))]/(2^(2/3)*a^(1/3)*f)), x] + (-Simp[3
*h*(Log[(1 - 3*h*(x/g))^(2/3) + 2^(1/3)*(1 + 3*h*(x/g))^(1/3)]/(2^(5/3)*a^(
1/3)*f)), x] + Simp[h*(Log[d + f*x^2]/(2^(5/3)*a^(1/3)*f)), x]) /; FreeQ[{a
, c, d, f, g, h}, x] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] && G
tQ[a, 0]
```

3.142. $\int\frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}(g^2+3h^2x^2)}dx$

rule 1342
```
Int[((g_) + (h_.)*(x_))/(((a_) + (c_.)*(x_)^2)^(1/3)*((d_) + (f_.)*(x_)^2))
, x_Symbol] :> Simp[(1 + c*(x^2/a))^(1/3)/(a + c*x^2)^(1/3)   Int[(g + h*x)
/((1 + c*(x^2/a))^(1/3)*(d + f*x^2)), x], x] /; FreeQ[{a, c, d, f, g, h}, x
] && EqQ[c*d + 3*a*f, 0] && EqQ[c*g^2 + 9*a*h^2, 0] &&  !GtQ[a, 0]
```

3.142.4 Maple [F]

$$\int \frac{hx+g}{\left(-\frac{c\,g^{2}}{h^{2}}+9c\,x^{2}\right)^{\frac{1}{3}}\left(3h^{2}x^{2}+g^{2}\right)}dx$$

input
```
int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

output
```
int((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x)
```

3.142.5 Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \text{Timed out}$$

input
```
integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm=
"fricas")
```

output
```
Timed out
```

3.142.6 Sympy [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \int \frac{g+hx}{\sqrt[3]{c\left(-\frac{g}{h}+3x\right)\left(\frac{g}{h}+3x\right)}\,(g^2+3h^2x^2)}\,dx$$

input
```
integrate((h*x+g)/(-c*g**2/h**2+9*c*x**2)**(1/3)/(3*h**2*x**2+g**2),x)
```

output
```
Integral((g + h*x)/((c*(-g/h + 3*x)*(g/h + 3*x))**(1/3)*(g**2 + 3*h**2*x**
2)), x)
```

3.142.7 Maxima [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm=
"maxima")
```

output
```
integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

3.142.8 Giac [F]

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \int \frac{hx+g}{(3\,h^2x^2+g^2)\left(9\,cx^2-\frac{cg^2}{h^2}\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate((h*x+g)/(-c*g^2/h^2+9*c*x^2)^(1/3)/(3*h^2*x^2+g^2),x, algorithm=
"giac")
```

output
```
integrate((h*x + g)/((3*h^2*x^2 + g^2)*(9*c*x^2 - c*g^2/h^2)^(1/3)), x)
```

3.142.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{-\frac{cg^2}{h^2}+9cx^2}\,(g^2+3h^2x^2)}\,dx = \int \frac{g+h\,x}{(g^2+3\,h^2\,x^2)\,\left(9\,c\,x^2-\frac{c\,g^2}{h^2}\right)^{1/3}}\,dx$$

input
```
int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)),x)
```

output
```
int((g + h*x)/((g^2 + 3*h^2*x^2)*(9*c*x^2 - (c*g^2)/h^2)^(1/3)), x)
```

3.143 $\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\right.}$

3.143.1 Optimal result

Integrand size = 104, antiderivative size = 488

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$
$$= \frac{3\sqrt[6]{3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}}\right)}{f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$+\frac{3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\frac{f(c^2g^2-bcgh+b^2h^2)}{3c^2h^2}+\frac{bfx}{c}+fx^2\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$
$$-\frac{3\,3^{2/3}h\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(cg+bh)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\log\left(\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}+\sqrt[3]{2}\sqrt[3]{1+\frac{3h(b+2cx)}{2cg-bh}}\right)}{2f\sqrt[3]{-\frac{(cg-2bh)(cg+bh)}{ch^2}+9bx+9cx^2}}$$

output
```
-3*3^(1/6)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c
*g)^2)^(1/3)*arctan(-1/3*3^(1/2)+1/3*2^(2/3)*(1-3*h*(2*c*x+b)/(-b*h+2*c*g)
)^(2/3)/(1+3*h*(2*c*x+b)/(-b*h+2*c*g))^(1/3)*3^(1/2))/f/(-(-2*b*h+c*g)*(b*
h+c*g)/c/h^2+9*b*x+9*c*x^2)^(1/3)+1/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+
c*g)/c/h^2-9*b*x-9*c*x^2)/(-b*h+2*c*g)^2)^(1/3)*ln(1/3*f*(b^2*h^2-b*c*g*h+
c^2*g^2)/c^2/h^2+b*f*x/c+f*x^2)/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b*x+9*c
*x^2)^(1/3)-3/2*3^(2/3)*h*(c*h^2*((-2*b*h+c*g)*(b*h+c*g)/c/h^2-9*b*x-9*c*x
^2)/(-b*h+2*c*g)^2)^(1/3)*ln((1-3*h*(2*c*x+b)/(-b*h+2*c*g))^(2/3)+2^(1/3)*
(1+3*h*(2*c*x+b)/(-b*h+2*c*g))^(1/3))/f/(-(-2*b*h+c*g)*(b*h+c*g)/c/h^2+9*b
*x+9*c*x^2)^(1/3)
```

3.143.2 Mathematica [A] (verified)

Time = 2.28 (sec) , antiderivative size = 593, normalized size of antiderivative = 1.22

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

$$= \frac{3^{2/3}\sqrt[3]{c}h^{5/3}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}{-4bh+2c(g-3hx)+\sqrt[3]{c}h^{2/3}\sqrt[3]{2cg-bh}\sqrt[3]{\frac{2b^2}{c}-\frac{cg^2}{h^2}+\frac{bg}{h}+9bx+9cx^2}}\right)-2\log\left(\cdots\right.\right.}{\cdots}$$

input
```
Integrate[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x +
 c*x^2)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2
+ (b*f*x)/c + f*x^2)),x]
```

output
```
(3^(2/3)*c^(1/3)*h^(5/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*c^(1/3)*h^(2/3)*(2*c*g
 - b*h)^(1/3)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))
/(-4*b*h + 2*c*(g - 3*h*x) + c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/
c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))] - 2*Log[Sqrt[h]*((2*b
^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3)] + Log[h*((2*b^2)/c
 - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(2/3)] + 2*Log[Sqrt[c]*(c*g -
2*b*h - 3*c*h*x - c^(1/3)*h^(2/3)*(2*c*g - b*h)^(1/3)*((2*b^2)/c - (c*g^2)
/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3))] - Log[c*(4*b^2*h^2 - 4*b*c*h*(g
- 3*h*x) + c^2*(g - 3*h*x)^2 - 2*b*c^(1/3)*h^(5/3)*(2*c*g - b*h)^(1/3)*((2
*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(1/3) + c^(4/3)*h^(2/3)
*(2*c*g - b*h)^(1/3)*(g - 3*h*x)*((2*b^2)/c - (c*g^2)/h^2 + (b*g)/h + 9*b*
x + 9*c*x^2)^(1/3) + c^(2/3)*h^(4/3)*(2*c*g - b*h)^(2/3)*((2*b^2)/c - (c*g
^2)/h^2 + (b*g)/h + 9*b*x + 9*c*x^2)^(2/3))]))/(2*f*(2*c*g - b*h)^(2/3))
```

3.143.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.029$, Rules used = {1374, 27, 1373}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{g+hx}{\sqrt[3]{\frac{2b^2h^2+bcgh-c^2g^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{2b^2h^2+bcgh-c^2g^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}dx$$

$$\Big\downarrow \text{ 1374}$$

$$\frac{3^{2/3}\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\int \frac{3(g+hx)}{\left(3fx^2+\frac{3bfx}{c}+\frac{f\left(c^2g^2-bchg+b^2h^2\right)}{c^2h^2}\right)\sqrt[3]{-\frac{9c^2x^2h^2}{(2cg-bh)^2}-\frac{9bcxh^2}{(2cg-bh)^2}+\frac{(cg-}{(}}}}{\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{3\ 3^{2/3}\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\int\frac{g+hx}{\left(3fx^2+\frac{3bfx}{c}+\frac{f\left(c^2g^2-bchg+b^2h^2\right)}{c^2h^2}\right)\sqrt[3]{-\frac{9c^2x^2h^2}{(2cg-bh)^2}-\frac{9bcxh^2}{(2cg-bh)^2}+\frac{(cg}{}}}}{\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

$\Big\downarrow$ 1373

$$\frac{3\ 3^{2/3}\sqrt[3]{\frac{ch^2\left(\frac{(cg-2bh)(bh+cg)}{ch^2}-9bx-9cx^2\right)}{(2cg-bh)^2}}\left(\frac{h\arctan\left(\frac{1}{\sqrt{3}}-\frac{2^{2/3}\left(1-\frac{3h(b+2cx)}{2cg-bh}\right)^{2/3}}{\sqrt{3}\sqrt[3]{\frac{3h(b+2cx)}{2cg-bh}+1}}\right)}{\sqrt{3}f}+\frac{h\log\left(\frac{f\left(b^2h^2-bcgh+c^2g^2\right)}{c^2h^2}+\frac{3bfx}{c}+3\right.}{6f}\right.}{\sqrt[3]{-\frac{(cg-2bh)(bh+cg)}{ch^2}+9bx+9cx^2}}$$

input
```
Int[(g + h*x)/(((-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(9*c*h^2) + b*x + c*x^2
)^(1/3)*((f*(b^2 - (-(c^2*g^2) + b*c*g*h + 2*b^2*h^2)/(3*h^2)))/c^2 + (b*f
*x)/c + f*x^2)),x]
```

output
```
(3*3^(2/3)*((c*h^2*(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2) - 9*b*x - 9*c*x^2)
)/(2*c*g - b*h)^2)^(1/3)*((h*ArcTan[1/Sqrt[3] - (2^(2/3)*(1 - (3*h*(b + 2*
c*x))/(2*c*g - b*h))^(2/3))/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))
^(1/3))])/(Sqrt[3]*f) + (h*Log[(f*(c^2*g^2 - b*c*g*h + b^2*h^2))/(c^2*h^2)
 + (3*b*f*x)/c + 3*f*x^2])/(6*f) - (h*Log[(1 - (3*h*(b + 2*c*x))/(2*c*g -
b*h))^(2/3) + 2^(1/3)*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h))^(1/3)])/(2*f))
)/(-(((c*g - 2*b*h)*(c*g + b*h))/(c*h^2)) + 9*b*x + 9*c*x^2)^(1/3)
```

3.143. $\displaystyle\int\frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}dx$

3.143.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1373
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = (-9*c*(h^2/(2*c*g - b
*h)^2))^(1/3)}, Simp[Sqrt[3]*h*q*(ArcTan[1/Sqrt[3] - 2^(2/3)*((1 - (3*h*(b
+ 2*c*x))/(2*c*g - b*h))^(2/3)/(Sqrt[3]*(1 + (3*h*(b + 2*c*x))/(2*c*g - b*h
))^(1/3)))]/f), x] + (-Simp[3*h*q*(Log[(1 - 3*h*((b + 2*c*x)/(2*c*g - b*h))
)^(2/3) + 2^(1/3)*(1 + 3*h*((b + 2*c*x)/(2*c*g - b*h)))^(1/3)]/(2*f)), x] +
 Simp[h*q*(Log[d + e*x + f*x^2]/(2*f)), x])] /; FreeQ[{a, b, c, d, e, f, g,
 h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] && EqQ[c^2*
g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] && GtQ[-9*c*(h^2/(2*c*g - b*h)^2)
, 0]
```

rule 1374
```
Int[((g_.) + (h_.)*(x_))/(((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(1/3)*((d_.)
+ (e_.)*(x_) + (f_.)*(x_)^2)), x_Symbol] :> With[{q = -c/(b^2 - 4*a*c)}, Si
mp[(q*(a + b*x + c*x^2))^(1/3)/(a + b*x + c*x^2)^(1/3)   Int[(g + h*x)/((q*
a + b*q*x + c*q*x^2)^(1/3)*(d + e*x + f*x^2)), x], x]] /; FreeQ[{a, b, c, d
, e, f, g, h}, x] && EqQ[c*e - b*f, 0] && EqQ[c^2*d - f*(b^2 - 3*a*c), 0] &
& EqQ[c^2*g^2 - b*c*g*h - 2*b^2*h^2 + 9*a*c*h^2, 0] &&  !GtQ[4*a - b^2/c, 0
]
```

3.143.4 Maple [F]

$$\int \frac{hx+g}{\left(\frac{2b^2h^2+bcgh-c^2g^2}{9c\,h^2}+bx+c\,x^2\right)^{\frac{1}{3}}\left(\frac{f\left(b^2+\frac{-2b^2h^2-bcgh+c^2g^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+f\,x^2\right)}dx$$

input
```
int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^
2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

output
```
int((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/(f*(b^
2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x)
```

3.143.5 Fricas [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx=\text{Timed out}$$

input
```
integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/
(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algori
thm="fricas")
```

output
```
Timed out
```

3.143.6 Sympy [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$

$$= \frac{3\cdot 3^{\frac{2}{3}}c^2h^2\left(\int \frac{}{b^2h^2\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}-bcgh\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}+9bx-\frac{cg^2}{h^2}+9cx^2}+3bch^2x\sqrt[3]{\frac{2b^2}{c}+\frac{bg}{h}}\right.}{}$$

input
```
integrate((h*x+g)/(1/9*(2*b**2*h**2+b*c*g*h-c**2*g**2)/c/h**2+b*x+c*x**2)*
*(1/3)/(f*(b**2+1/3*(-2*b**2*h**2-b*c*g*h+c**2*g**2)/h**2)/c**2+b*f*x/c+f*
x**2),x)
```

output
```
3*3**(2/3)*c**2*h**2*(Integral(g/(b**2*h**2*(2*b**2/c + b*g/h + 9*b*x - c*
g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b*x - c*g**2/
h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2/h**
2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*b*x - c*g**2
/h**2 + 9*c*x**2)**(1/3)), x) + Integral(h*x/(b**2*h**2*(2*b**2/c + b*g/h
+ 9*b*x - c*g**2/h**2 + 9*c*x**2)**(1/3) - b*c*g*h*(2*b**2/c + b*g/h + 9*b
*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*b*c*h**2*x*(2*b**2/c + b*g/h + 9*b
*x - c*g**2/h**2 + 9*c*x**2)**(1/3) + c**2*g**2*(2*b**2/c + b*g/h + 9*b*x
- c*g**2/h**2 + 9*c*x**2)**(1/3) + 3*c**2*h**2*x**2*(2*b**2/c + b*g/h + 9*
b*x - c*g**2/h**2 + 9*c*x**2)**(1/3)), x))/f
```

3.143.7 Maxima [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \int \frac{3\,(hx+g)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

input
```
integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/
(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algori
thm="maxima")
```

output
```
3*integrate((h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/
(c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*
h^2)/h^2)*f/c^2)), x)
```

3.143.8 Giac [F]

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx = \int \frac{3\,(hx+g)}{\left(cx^2+bx-\frac{c^2g^2-bcgh-2\,b^2h^2}{9\,ch^2}\right)^{\frac{1}{3}}\left(3\,fx^2+\frac{3\,bfx}{c}+\frac{\left(3\,b^2+\frac{c^2g^2-bcgh-2\,b^2h^2}{h^2}\right)f}{c^2}\right)}\,dx$$

input
```
integrate((h*x+g)/(1/9*(2*b^2*h^2+b*c*g*h-c^2*g^2)/c/h^2+b*x+c*x^2)^(1/3)/
(f*(b^2+1/3*(-2*b^2*h^2-b*c*g*h+c^2*g^2)/h^2)/c^2+b*f*x/c+f*x^2),x, algori
thm="giac")
```

output
```
integrate(3*(h*x + g)/((c*x^2 + b*x - 1/9*(c^2*g^2 - b*c*g*h - 2*b^2*h^2)/
(c*h^2))^(1/3)*(3*f*x^2 + 3*b*f*x/c + (3*b^2 + (c^2*g^2 - b*c*g*h - 2*b^2*
h^2)/h^2)*f/c^2)), x)
```

3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{g+hx}{\sqrt[3]{\frac{-c^2g^2+bcgh+2b^2h^2}{9ch^2}+bx+cx^2}\left(\frac{f\left(b^2-\frac{-c^2g^2+bcgh+2b^2h^2}{3h^2}\right)}{c^2}+\frac{bfx}{c}+fx^2\right)}\,dx$$
$$= \int \frac{g+h\,x}{\left(b\,x+c\,x^2+\frac{\frac{2\,b^2\,h^2}{9}+\frac{b\,c\,g\,h}{9}-\frac{c^2\,g^2}{9}}{c\,h^2}\right)^{1/3}\left(f\,x^2-\frac{f\left(\frac{\frac{2\,b^2\,h^2}{3}+\frac{b\,c\,g\,h}{3}-\frac{c^2\,g^2}{3}}{h^2}-b^2\right)}{c^2}+\frac{b\,f\,x}{c}\right)}\,dx$$

input
```
int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/
(c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^
2 - b^2))/c^2 + (b*f*x)/c)),x)
```

output
```
int((g + h*x)/((b*x + c*x^2 + ((2*b^2*h^2)/9 - (c^2*g^2)/9 + (b*c*g*h)/9)/
(c*h^2))^(1/3)*(f*x^2 - (f*(((2*b^2*h^2)/3 - (c^2*g^2)/3 + (b*c*g*h)/3)/h^
2 - b^2))/c^2 + (b*f*x)/c)), x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larg
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
        return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =" "
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


    print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
return grade, grade_annotation
```